[Normal] Standard Arithmetic

Brooks

THE

NORMAL

STANDARD ARITHMETIC,

BY ANALYSIS AND INDUCTION,

DESIGNED FOR

PUBLIC AND PRIVATE SCHOOLS, NORMAL SCHOOLS, ACADEMIES, ETC.

BY

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"Analysis and Induction are the golden keys of science."

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PREFACE.

THE author's first work on arithmetic was published over thirty years ago. The aim of this work was to unfold the science of numbers with logical simplicity, and to base the art of computation upon an intelligent conception of principles. That the method of treating the subject was correct is attested by the widespread popularity of his works and the influence they have had in shaping correct systems of arithmetical instruction.

The fundamental spirit of the method of treatment was that of analysis. This simple principle, running like a golden thread through the entire treatise, simplified arithmetical conceptions and operations, and bound the subject together into a beautiful and symmetrical unity. It gave the pupil a power of thought and comprehension which enabled him to trace his way through the most complex combinations, and to hold the entire subject clearly in his mental grasp. The union with this analysis of the spirit of induction in deriving generalizations from analytical processes placed in the hands of the student the keys of power and mastery in the science. So potent are these two processes that there arises the maxim,—*Analysis and induction are the golden keys of science.*

While the principles of arithmetic remain unchanged, new inventions and new modes of life introduce new objects to which numbers are applied. The standards of the requirements of instruction vary also with the introduction of new branches of study into the elementary schools. Text-books on arithmetic must therefore be revised every few years to keep pace with these changes and requirements. To meet these demands the present book has been written. In its preparation, the aim has been to retain all those methods of teaching the science that gave popularity to the author's former works, and to present such new applications of these methods as may represent the latest forms and requirements of society and business. Especial care has been taken to embody in it the best ideas of modern education, and thus adapt the work to the needs of the modern school-room.

The two leading objects, therefore, in the preparation of the work are—first, that it should be scientific, and second, that it should be practical. Its scientific character consists in its analytical treatment; its simple and concise definitions; its clear, brief, and logical solutions and explanations; and its philosophical arrangements and classifications. It thus stands opposed to the modern tendency to give pupils a disconnected and fragmentary idea of branches of study, which deprives them of the power that comes from a logical and comprehensive grasp of a subject. The motto of the work has been that the more scientific the treatment the greater its simplicity.

The practical character of the work will be seen by an examination of its problems. The applications of the science are not the thought of the author as to what business may be, but they represent the actual business of the day. The solutions in an arithmetic should be, so far as possible, not merely school-room methods, but the actual methods of the counting-room and business-office. To carry out this idea the author has taken special pains to represent the business of real life. Many of the problems and processes are derived from actual business transactions. The Bills and Accounts came out of the stores; the examples in Percentage, Taxes, Banking, Exchange, etc. represent the real business of the day; and the effort has been made to give a spirit of reality to even those parts of the subject which treat mainly of the principles of numbers.

Many of the solutions and explanations are the product of the author's earlier years as a teacher. His purpose was to give to arithmetical reasoning some of that finish and elegance that belonged to geometry. Many of these solutions were tested with his pupils several years before they were published, so that they are largely the outgrowth of the school-room. Based upon a careful study of the human mind, they are believed to represent the natural processes of the mind in clear and logical thinking upon arithmetical subjects. The aim has been that they shall be simple and direct in thought, so that they may be readily understood by the pupil, and yet so concise and logical that they seem to meet the requirements of a finished demonstration.

Taught to reason in this way, pupils become independent of rules or can derive them for themselves. While rules are presented in the work, they are not designed to be committed or blindly followed. They are merely concise expressions of the methods of operation which may aid the pupils in stating their own processes. They are not given to suggest methods of operation, but merely as concise statements of these

operations. In teaching, pupils should be required to see the reason for the different processes, and then to derive their own methods. The rules here given are thus designed as a guide in describing operations, and may sometimes aid the student in remembering them.

The arrangement of the work is believed to be both scientific and practical, being in accordance with the principles of science and adapted to the natural mental growth and development of the pupil. The motto has been from the easy to the difficult, from the simple to the complex, from the known to the unknown. Care has been taken to present the simpler and more practical subjects first, and not to anticipate any principles or processes before the pupil is prepared for them. The proper gradation of the exercises has been kept continually in view. No problem has been inserted, however interesting in its nature, for which it was not thought the pupil had been previously prepared. In the unfolding of subjects each problem was viewed as a stepping-stone to those which followed it, and thus problems which might seem difficult in themselves become easy by virtue of the pupil's previous preparation for them. The proper grading of the path of the science of numbers, with here and there a flower of interest planted along its border, has been a prominent object in the mind of the author.

In the scope and extent of the work judgment has been exercised to present the average requirements of elementary schools. Several subjects heretofore presented in arithmetic have been omitted, while others have been given at the close of the work under the head of a Supplement. The work is regarded as presenting a complete course for elementary schools without the Supplement; but teachers who desire a fuller course will find several topics in the Supplement which are usually found in text-books of this grade. The work is thus adapted to meet the demands of all grades of elementary schools.

With a grateful sense of the appreciation extended to my previous works on arithmetic, I offer the present treatise on the science to teachers and others having charge of the education of youth, with the sincere hope that it may be found a valuable aid in imparting a knowledge of the science of numbers and in developing the thought-powers of the youth of the country.

EDWARD BROOKS.

PHILADELPHIA, *May* 10, 1895.

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NORMAL STANDARD ARITHMETIC.

SECTION I.

ARITHMETICAL LANGUAGE.

1. Arithmetic is the science of numbers and the art of computing with them.

2. A **Unit** is a single thing or *one.* A thing is a *concrete unit; one* is an *abstract unit.*

3. A **Number** is a unit or a collection of units. Numbers are *concrete* and *abstract.*

4. A **Concrete Number** is one in which the kind of unit is named; as, two *yards,* five *books.*

5. An **Abstract Number** is one in which the kind of unit is not named; as, *two, four,* etc.

6. Similar Numbers are those in which the units are alike; as, two *boys* and four *boys.*

7. Dissimilar Numbers are those in which the units are unlike; as, two *boys* and four *books.*

8. Arithmetical Language is the method of expressing numbers.

9. Arithmetical Language is of two kinds, *Oral* and *Written.* The former is called *Numeration* and the latter is called *Notation.*

NUMERATION.

10. Numeration is the method of naming numbers, and of reading them when expressed by characters. It is the *oral expression* of numbers.

11. It would require too many words to give each number a separate name; numbers are therefore named according to the following simple principle:

Principle.— *We name a few of the first numbers, and then form groups or collections, name these groups, and use the names of the first numbers to number these groups.*

12. A single thing is named *one;* one and one more are named *two;* two and one more, *three;* three and one more, *four;* and thus we obtain the simple names,

One, two, three, four, five, six, seven, eight, nine, ten.

13. Now, regarding the collection *ten* as a single thing, we might count *one and ten, two and ten, three and ten,* etc., as far as *ten and ten,* which we would call *two tens.* By this principle were obtained the following numbers:

Eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty.

14. Proceeding in the same way, we would have *two tens and one, two tens and two, two tens and three,* etc. By this principle were obtained the following numbers:

Twenty-one, twenty-two, twenty-three, twenty-four, twenty-five, twenty-six, twenty-seven, twenty-eight, twenty-nine.

15. Continuing in the same manner, we would have *three-tens, four-tens, five-tens,* etc. By this principle were derived the following ordinary names:

Thirty, forty, fifty, sixty, seventy, eighty, ninety.

16. A group of *ten tens* is called a *hundred;* a group of *ten hundreds,* a *thousand.* The next group consists of a *thousand thousands,* and is called a *million;* the next group is a *thousand millions,* called a *billion,* etc.

17. After a *thousand*, the two groups coming between those having a distinct name are numbered by *tens* and *hundreds*, as *ten thousand* and *hundred thousand.*

1. The above shows the *principle* by which the numbers were named. The names we use come from the Saxon or Gothic language.

2. *Eleven* is from the Saxon *endlefen*, or Gothic *ainlif* (*ain*, *one*, and *lif*, *ten*); *twelve* is from the Saxon *twelif*, or Gothic *tvalif* (*tva*, *two*, and *lif*, *ten*).

3. *Twenty* is from the Saxon *twentig* (*twegen*, *two*, and *tig*, a *ten*); *thirty* is from the Saxon *thritig* (*thri*, *three*, and *tig*, a *ten*), etc.

4. *Hundred* is a primitive word; *thousand* is from the Saxon *thusend*, or Gothic *thusundi* (*thus*, ten, and *hund*, hundred); *million*, *billion*, etc. are from the Latin.

NOTATION.

18. **Notation** is the method of writing numbers. Numbers may be written in three ways:

1st. By *words*, or common language.

2d. By *figures*, called the *Arabic Method.*

3d. By *letters*, called the *Roman Method.*

The method by words is that of ordinary written language, and needs no explanation.

ARABIC NOTATION.

19. The **Arabic System** of Notation is the method of expressing numbers by characters called *figures.*

20. In this system numbers are expressed according to the following principle:

We employ different characters to represent the first nine numbers, and then use these characters to number the groups, the group numbered being indicated by the position of the character.

21. **Figures.**—Figures are characters used in expressing numbers. There are ten figures used, as follows:

FIGURES.	1,	2,	3,	4,	5,	6,	7,	8,	9,	0.
NAMES AND VALUES.	one,	two,	three,	four,	five,	six,	seven,	eight,	nine,	naught, cipher, or zero.

22. By the combination of these figures all numbers may be expressed; hence they are appropriately called the *alphabet* of *arithmetic.*

23. Combination.—These figures are combined according to the following principle:

1. A *figure standing alone, or in the first place at the right of other figures, expresses* UNITS *or* ONES.

2. A *figure standing in the second place, counting from the right, expresses* TENS; *in the third place,* HUNDREDS; *in the fourth place,* THOUSANDS, etc.; thus,

10 is 1 ten, or ten.	100 is 1 hundred.
20 " 2 tens, or twenty.	200 " 2 hundred.
30 " 3 tens, or thirty.	300 " 3 hundred.
40 " 4 tens, or forty.	400 " 4 hundred.
50 " 5 tens, or fifty.	1000 " 1 thousand.
60 " 6 tens, or sixty.	2000 " 2 thousand.
90 " 9 tens, or ninety.	4000 " 4 thousand.

24. The name of each of the first twenty-one places is given in the following

NUMERATION TABLE.

	Hundred-quintillions.	Ten-quintillions.	Quintillions.	Hundred-quadrillions.	Ten-quadrillions.	Quadrillions.	Hundred-trillions.	Ten-trillions.	Trillions.	Hundred-billions.	Ten-billions.	Billions.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.
	8	8	8,	8	8	8,	8	8	8,	8	8	8,	8	8	8,	8	8	8,	8	8	8,
PLACES.	21st,	20th,	19th,	18th,	17th,	16th,	15th,	14th,	13th,	12th,	11th,	10th,	9th,	8th,	7th,	6th,	5th,	4th,	3d,	2d,	1st,
PERIODS.		7th.			6th.			5th.			4th.			3d.			2d.			1st.	

25. Periods.—For convenience in writing and reading numbers, the figures are arranged in *periods* of three places each, as shown in the table.

The first three places constitute the *first*, or *units period;* the second three places constitute the *second*, or *thousands period*, etc.

1. Required the names of the following places:

First; third; second; fourth; sixth; eighth; tenth; ninth; twelfth; fifth; seventh; eleventh; thirteenth; seventeenth; fourteenth; sixteenth; eighteenth; fifteenth; nineteenth, etc.

2. Required the places of the following:

Tens; hundreds; thousands; millions; ten-thousands; hundred-thousands; ten-millions; billions; hundred-millions; hundred-billions; units; ten-billions; trillions; quadrillions; hundred-quintillions; ten-trillions; ten-quintillions, etc.

3. Required the names of the following periods:

1. First.	3. Second.	5. Fourth.
2. Third.	4. Fifth.	6. Seventh.

4. Required the period and place of the following:

Thousands; millions; ten-thousands; hundred-millions; billions; hundred-trillions; trillions; ten-trillions; quadrillions; ten-quadrillions; hundred-trillions; quintillions; hundred-quintillions, etc.

26. The combination of figures to express a number forms a *numerical expression.* Thus, 25 is a numerical expression which denotes the same as the common word *twenty-five.*

27. The different figures of a numerical expression are called *terms. Terms* are also used to indicate the *numbers* represented by the figures.

The use of the word *term*, to indicate both the figures and the numbers represented by them, enables us to avoid the error of using the word *figure* for the word *number.*

EXERCISES IN NUMERATION.

28. The pupils are now prepared to learn to *read* numbers when expressed by *figures.* From the preceding explanations we have the following *rule for numeration:*

I. *Begin at the right hand, and separate the numerical expression into periods of three figures each.*

II. *Then begin at the left hand and read each period in succession, giving the name of each period except the last.*

The name of the last period is usually omitted, being understood to be units.

1. What number is expressed by 5463289?

SOLUTION.—Separating the numerical expression into periods of three figures each, beginning at the right hand, we have 5,463,289. The third period is 5 *millions*, the second period is 463 *thousands*, and the first is 289 *units*; hence the number is 5 *million*, 463 *thousand*, 289.

OPERATION.
5463289

Read the following numerical expressions:

2.	2743	**10.**	369421	**18.**	70604081
3.	3246	**11.**	765489	**19.**	54065803
4.	4084	**12.**	904005	**20.**	123456789
5.	5786	**13.**	2345678	**21.**	543219876
6.	11340	**14.**	1020304	**22.**	647892010
7.	60879	**15.**	7708051	**23.**	40367734285
8.	45631	**16.**	65752932	**24.**	12459667384112
9.	806027	**17.**	87098543	**25.**	857364624721396

After pupils are familiar with reading by dividing into periods, the division may be omitted or performed mentally.

EXERCISES IN NOTATION.

29. Having learned to *read* numerical expressions, we are now prepared to *write* them. From the principles which have been explained we derive the following rule:

I. *Begin at the left and write the hundreds, tens, and units of each period in their proper order.*

II. *When there are vacant places, fill them with ciphers.*

1. Express in figures the number *four thousand three hundred four.*

SOLUTION.—We write the 4 thousands in the 4th place, 3 hundreds in the 3d place, a cipher in the 2d place, there being no tens, and 4 units in the 1st place, and we have 4304.

OPERATION.
4304

Express the following numbers in figures:

2. One hundred two.
3. One hundred ten.
4. Two hundred fifty.
5. Two hundred sixty-three.
6. Two hundred seven.
7. Three hundred twelve.
8. Three hundred seven.
9. Five hundred forty-eight.
10. Five hundred eighty-seven.
11. Seven hundred forty-seven.
12. Nine hundred eighty-nine.
13. Eight thousand, five.
14. Five thousand, five hundred thirty-seven.
15. Four thousand, eighty-five.
16. Thirty-three thousand, five hundred forty-six.
17. Two hundred forty-five thousand, five hundred six.
18. Four hundred eight thousand, five hundred nine.
19. Six hundred four thousand, two hundred forty-nine.
20. Three hundred seventy-four thousand, one hundred twenty.
21. Seven hundred seven thousand, seven hundred seven.
22. One million.
23. Three million, ten.
24. Four million, fourteen.
25. Seventy-six million, twenty-four.
26. Nine million, twenty-eight thousand.
27. Seven million, one thousand, sixty-seven.
28. Six million, three hundred fifty-nine thousand.
29. Eighty million, five thousand, forty-five.
30. One million, two hundred fifty thousand, four hundred ninety-nine.
31. Three million, six hundred seven thousand, four hundred ten.
32. Seventeen million, four hundred seven thousand, eighty-four.
33. Twenty-eight million, five hundred ninety-four thousand, sixty-seven.
34. Sixty-five million, thirty-eight thousand, seven hundred eight.
35. Four hundred million, forty-nine hundred twenty-eight.
36. Eleven billion, seven hundred five million, twenty-nine thousand, six.
37. Forty-nine trillion, fifty-eight thousand, seven hundred ninety-eight.

30. Orders.—Since we may have 2 *tens*, 3 *tens*, etc., 2 *hundreds*, 3 *hundreds*, etc., the same as 2 *apples*, 3 *apples*, 2 *books*, 3 *books*, etc., these different groups may be regarded as *units* of different *orders;* thus,

UNITS	are called	Units of the *1st order.*
TENS	" "	Units of the *2d order.*
HUNDREDS	" "	Units of the *3d order.*
THOUSANDS	" "	Units of the *4th order.*
TEN-THOUSANDS	" "	Units of the *5th order.*

31. The table which has been given enables us to read a numerical expression consisting of twenty-one figures; the periods which follow them in order are as follows:

Sextillions, *Septillions*, *Octillions*, *Nonillions*, *Decillions*, *Undecillions*, *Duodecillions*, *Tertio-decillions*. *Quarto-decillions*, *Quinto-decillions*, *Sexto-decillions*, *Septo-decillions*, *Octo-decillions*, *Nono-decillions*, *Vigillions*. With these, and those already given, we can write and read a numerical expression consisting of sixty-six places.

1. The first nine of the Arabic characters are called *digits*, from the Latin word *digitus*, a finger, owing to the fact that the ancients reckoned by counting the fingers. They are also called *significant* figures, because they always indicate a definite number of units. The character 0, called *zero*, *cipher*, or *naught*, always indicates an absence of units.

2. The Arabic Notation is named from the Arabs, who introduced it into Europe by their conquest of Spain during the eleventh century. The Arabs obtained it from the Hindoos, by whom it was probably invented more than 2000 years ago.

3. There are three theories for the origin of the Arabic characters, for which see *Brooks's Philosophy of Arithmetic*.

THE DECIMAL SCALE.

32. It is seen that *ten* units of a lower order make one unit of the next higher order; hence the system of notation is called the *Decimal System*, from the Latin, *decem*, ten.

33. In expressing numbers the value of a figure is *increased tenfold* for every place it is moved from *right to left*, and is *decreased tenfold* for every place it is moved from *left to right.*

34. Since the value of terms decreases from left to right at a tenfold rate, if we fix the place of units by a point (.), we may extend the decimal scale to the right of units.

35. The first place on the right of the point will be one-tenth of units, or *tenths;* the second place, one-tenth of tenths, or *hundredths;* the third place, *thousandths*, etc.

36. Such terms are called *decimals*, and the point is called the *decimal point.* The expression 48.37 is read 48 units, 3 tenths, and 7 hundredths, or 48 and 37 hundredths.

37. The **Money** of the United States is expressed by the decimal system. The *dollar* is the *unit*, and is indicated by the symbol $. The first place at the right of the decimal point is called *dimes;* the second place, *cents*, etc.

38. **Dimes** and **Cents** are usually read as a *number of cents.* Thus, $4.65 is read 4 dollars and 65 cents; and $72.48 is read 72 dollars and 48 cents.

The Decimal system of numeration had its origin in the practice, common to all nations, of counting by groups of tens.

EXAMPLES FOR PRACTICE.

Read the following:

1.	15.7	**4.**	97.50	**7.**	$15.25	**10.**	$642.05
2.	63.25	**5.**	75.267	**8.**	$75.75	**11.**	$763.675
3.	65.75	**6.**	67.375	**9.**	$235.35	**12.**	$896.875

Write the following:

1. Fifteen and five tenths.

2. Seventy-seven and twenty-five hundredths.

3. Three hundred four, and fifty-six hundredths.

4. Four hundred, and three hundred twenty-five thousandths.

5. Forty dollars and nine cents.

6. Ninety dollars and twenty-five cents.

7. Three hundred and twenty-five dollars and fifty cents.

8. Seven hundred eighty-four dollars, thirty cents five mills.

ROMAN NOTATION.

39. The **Roman Method** of Notation employs seven letters of the Roman alphabet. Thus, I represents *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* M, *one thousand.*

40. To express other numbers these characters are combined according to the following principles:

1. *Every time a letter is repeated its value is repeated.*

2. *When a letter is placed before one of greater value, the* DIFFERENCE *of their values is the number represented.*

3. *When a letter is placed after one of greater value, the* SUM *of their values is the number represented.*

4. *A dash placed over an expression increases its value a thousand fold.* Thus, $\overline{\text{VII}}$ denotes seven thousand.

41. These principles are exhibited in the following table:

ROMAN TABLE.

I	One.	L	Fifty.
II	Two.	LX	Sixty.
III	Three.	LXX	Seventy.
IV	Four.	XC	Ninety.
V	Five.	C	One hundred.
VI	Six.	CC	Two hundred.
VII	Seven.	D	Five hundred.
VIII	Eight.	DC	Six hundred.
IX	Nine.	DCCCC	Nine hundred.
X	Ten.	M	One thousand.
XI	Eleven.	MM	Two thousand.
XIV	Fourteen.	$\overline{\text{X}}$	Ten thousand.
XV	Fifteen.	$\overline{\text{C}}$	One hundred thousand.
XIX	Nineteen.	$\overline{\text{M}}$	One million.
XX	Twenty.	MCLX	One thousand one hundred and sixty.
XXX	Thirty.		
XL	Forty.	MDCCCXCV, 1895.	

WRITTEN EXERCISES.

Express the following numbers by the Roman method.

1. Thirty-nine. **2.** Eighty-seven. **3.** Four hundred two. **4.** Seven hundred sixty-six. **5.** One thousand eight hundred seventy-six. **6.** 5749. **7.** 35009. **8.** 406784. **9.** 1245376.

Read the following numbers:

1. LXIX. **2.** LXXXVIII. **3.** MMXL. **4.** MCDXCII. **5.** MDCCLXXVI. **6.** MMCCXXII. **7.** $\overline{\text{IV}}$CDXLIV. **8.** $\overline{\text{MDCL}}$DCLVI. **9.** clxxvii. **10.** xlix. **11.** xcix.

INTRODUCTION TO ADDITION.

ORAL EXERCISES.

1. How many blocks are 3 blocks and 2 blocks?

2. How many apples are 4 apples and 5 apples?

3. Edna spent 8 cents for oranges and 7 cents for peaches; how much did she spend for both?

SOLUTION.—If Edna spent 8 cents for oranges and 7 cents for peaches, for both she spent 8 cents plus 7 cents, which are 15 cents.

4. In a garden there are 9 plum trees and 12 peach trees; how many trees in the garden?

5. A lady gave 18 dollars for a dress and 15 dollars for a fur cape; how much did both cost her?

6. Sarah is 14 years old and her teacher is 17 years older; what is the age of the teacher?

7. If John paid 75 cents for a pair of skates and 25 cents for a ball, what did they both cost him?

8. A lady spent 12 cents for needles, 10 cents for thread, and 18 cents for cambric; what did she pay for her purchases?

9. How many are 7 and 21? 9 and 24? 7 and 31? 10 and 18? 7 and 18? 9 and 27? 11 and 28? 12 and 11? 13 and 11?

10. How many are 6 and 16? 8 and 17? 7 and 18? 9 and 19? 8 and 12? 10 and 22? 13 and 20? 14 and 18? 11 and 31?

11. How many are 5 and 14? 4 and 18? 6 and 15? 9 and 17? 6 and 16? 8 and 18? 9 and 19? 10 and 21? 11 and 20?

12. Count by 2's from 2 to 20; from 20 to 40; from 40 to 60; from 1 to 21; from 21 to 41; from 41 to 61.

13. Count by 3's from 3 to 21; from 21 to 42; from 1 to 22; from 22 to 43; from 2 to 23; from 23 to 44.

14. Count by 4's from 4 to 40; from 40 to 80: from 1 to 25; from 25 to 41; from 2 to 30; from 30 to 50; from 3 to 35; from 35 to 55.

15. Count by 5's from 5 to 60; from 1 to 61; from 2 to 62; from 3 to 63; from 4 to 64. Count also by 6's, 7's, 8's, 9's, 10's, etc.

16. What is the uniting of two or more numbers into one sum called? *Ans. Addition.*

A

SECTION II.

FUNDAMENTAL OPERATIONS.

ADDITION.

42. Addition is the process of finding the *sum* of two or more numbers.

43. The **Sum** of several numbers is a number which contains as many units as the numbers added.

44. The **Sign of Addition** is +, and is read *plus*. It denotes that the numbers between which it is placed are to be added.

45. The **Sign of Equality** is =, and is read *equals*. Thus, $10 = 4 + 6$ is read 10 *equals* 4 *plus* 6.

NOTE.—The symbol + was introduced by *Stifelius*, a German mathematician, in a work published in 1544.

PRINCIPLES.

1. *The numbers added must be similar.*
2. *The sum is a number similar to the numbers added.*

CASE I.

46. To add when the sum of no column exceeds nine units of that column.

1. What is the sum of 21, 43, and 34?

SOLUTION.—We write the numbers so that figures of the same order stand in the same column, draw a line beneath, and begin at the right to add. 4 units and 3 units are 7 units, and 1 unit are 8 units, which we write under the column of units; 3 tens and 4 tens are 7 tens, and 2 tens are 9 tens, which we write under the column of tens. Hence the sum of 21, 43, and 34 is 98.

OPERATION.

$$\begin{array}{r} 21 \\ 43 \\ \underline{34} \\ 98 \end{array}$$

NOTE.—In practice we say 4 and 3 are 7 and 1 are 8; or merely name the results, as 4, 7, 8.

(2)	(3)	(4)	(5)	(6)
437	324	832	473	234
321	653	167	321	365
758				

(7)	(8)	(9)	(10)	(11)
$2.31	$4.25	$21.02	$60.20	$231.25
3.14	3.01	32.25	13.02	542.30
3.20	1.50	40.30	25.46	210.44
$8.65				

What is the sum—

12. Of $12.53, $51.20, and $16.05? *Ans.* $79.78.

13. Of $21.63, $15.02, and $42.32? *Ans.* $78.97.

14. Of 6321312, 1004132, and 2560254? *Ans.* 9885698.

15. William takes 5023 steps in going to school and Samuel takes 1852 steps; how many do both take? *Ans.* 6875.

16. Lizzie's book contains 5136 words and Sallie's book 4721 words; how many words in both books? *Ans.* 9857.

17. A rides 4316 miles in a year and walks 351 miles; how far does he travel in a year? *Ans.* 4667 miles.

18. Henry paid $2.10 for a book, $1.25 for a tennis racket, and 62 cents for a ball; what was the cost of all? *Ans.* $3.97.

19. A farmer's house cost $4610, his barn cost $1270, and his hen-house cost $115; what was the cost of all these buildings? *Ans.* $5995.

20. Mr. Smith gave his son John $24.10, his son James $41.03, and his daughter Emma $34.12; how many dollars did he give all? *Ans.* $99.25.

21. Flossie's pulse beats 183516 times in a day and her mother's pulse beats 114382 times; how many beats do both make in a day? *Ans.* 297898.

22. Thomas bought a geometry for $1.38, a history for $1.40, and an algebra for $1.20; how much did he pay for all? *Ans.* $3.98.

CASE II.

47. To add when the sum of a column exceeds nine units of that column.

1. What is the sum of 685, 483, and 267?

SOLUTION.—We write the numbers so that terms of the same order stand in the same column, and begin at the right to add. 7 units and 3 units are 10 units, and 5 units are 15 units; 15 units equal 1 *ten* and 5 *units;* we write the 5 units under the column of units, and add the 1 ten to the column of tens: 1 ten and 6 tens are 7 tens, and 8 tens are 15 tens, and 8 tens are 23 tens; 23 tens equal 2 *hundreds* and 3 *tens;* we write the 3 tens under the column of tens, and add the 2 hundreds to the column of hundreds; 2 hundreds and 2 hundreds are 4 hundreds, and 4 hundreds are 8 hundreds, and 6 hundreds are 14 hundreds; 14 hundreds equals 1 *thousand* and 4 *hundreds;* we write the 4 hundreds under the column of hundreds, and place the 1 thousand on the left in the place of thousands. Hence, the sum of 685, 483, and 267 is 1435. Hence the following

OPERATION.

$$\begin{array}{r} 685 \\ 483 \\ 267 \\ \hline 1435 \end{array}$$

Rule.—I. *Write the numbers to be added, placing terms of the same order in the same column, and draw a line beneath.*

II. *Begin at the right, add the terms of each column separately, and write the sum underneath if less than ten.*

III. *When the sum of any column exceeds nine, write the units figure only, and add the tens to the next column.*

IV. *Write the entire sum of the last column.*

Proof.—Begin at the top and add the columns downward, and if the work is correct the two sums will be equal.

Or, separate the number into two or more parts, add these parts, and then add the sum of these parts.

1. We write figures of the same order in the same column for *convenience* of adding, since only units of the same order can be directly added.

2. We begin at the *right* to add for *convenience,* so that when the sum of any column exceeds 9, we may add the left-hand term of such sum to the next column.

3. In adding dollars and cents, dollars must be written under dollars and cents under cents, so that the points may be in a vertical line.

WRITTEN EXERCISES.

What is the sum—

2. Of 3462+8247+7645+8357+3112? *Ans.* 30823.

3. Of 2331+5632+7542+7204+8023? *Ans.* 30732.

4. Of 3752+6385+1210+2531+2120? *Ans.* 15998.

5. Of 2136+5103+16287+16302? *Ans.* 39828.

6. Of 3210+3561+3102+520+54231? *Ans.* 64624.

7. Of 1233+403+73250+50673+154632? *Ans.* 280191.

8. What is the sum of 16250 + 4165 + 278756 + 5083 + 614507? *Ans.* 918761.

9. Find the sum of 76085 + 34271 + 25701 + 786435 + 257075 + 843275. *Ans.* 2022842.

10. What is the sum of 76068 + 2751231 + 7587204 + 651302 + 1020304? *Ans.* 12086109.

11. What is the sum of 216407 + 675780 + 5545 + 5367902 + 92712406? *Ans.* 98978040.

12. Find the sum of 326504 + 283127 + 6075 + 7052728 + 549. *Ans.* 7668983.

13. What is the sum of 165372 + 3087584 + 361 + 52167 + 640 + 325? *Ans.* 3306449.

14. Required the sum of 2871785 + 7352810 + 270562 + 34717+321785. *Ans.* 10851659.

15. Find the sum of 758762 + 4803 + 31776 + 708238 + 873213 + 58437985. *Ans.* 60814777.

16. What is the sum of 4283 + 73015 + 273 + 28431 + 39782 + 776889? *Ans.* 922673.

17. Required the sum of 7320 + 48432 + 5865 + 78242 + 78562 + 218783. *Ans.* 437204.

18. Find the sum of 2738 + 785862 + 61243 + 65782 + 71324 + 6078841. *Ans.* 7065790.

19. What is the sum of 34256 + 7891110 + 211341 + 115671 + 8219201? *Ans.* 16471579.

20. Find the sum of 37658 + 78246 + 2813 + 5867 + 7684 + 83215 + 62412. *Ans.* 277895.

21. Find the sum of 74128, 40321, 18000, 916890, 706311, 983634, 752658. *Ans.* 3491942.

22. Find the sum of 15768, 65812, 23418, 562871, 63742, 728105, 2516014. *Ans.* 3975730.

23. Find the sum of 75802, 831746, 765286, 576831, 287250, 783241, 431401. *Ans.* 3751557.

ORAL EXERCISES.

1. Mary gave 15 roses to her brother and 8 roses to her sister; how many did she give to both?

SOLUTION.—If Mary gave 15 roses to her brother and 8 roses to her sister, to both she gave 15 roses plus 8 roses, which are 23 roses.

2. John had 12 cents and earned 10 cents; how many cents did he then have?

3. What is the cost of a suit of clothes if the material cost $24 and the making cost $8?

4. In a certain livery-stable there were 9 black horses, 5 gray horses, and 3 roan horses; how many horses in all?

5. How many are 17 and 7? 18 and 8? 19 and 5? 22 and 8? 26 and 5? 35 and 9? 42 and 8? 49 and 6? 54 and 9?

6. How many are 8 and 23? 9 and 29? 6 and 34? 7 and 37? 5 and 45? 4 and 52? 3 and 66?

7. How many are 109 and 6? 108 and 7? 112 and 8? 122 and 9? 137 and 5? 142 and 7? 154 and 8?

8. How many are 7 and 136? 6 and 155? 8 and 205? 7 and 211? 5 and 227? 4 and 243? 8 and 314? 400 and 700?

9. A boy bought a robin for 10 cents, a gray squirrel for 30 cents, and a white rabbit for 50 cents; what did they all cost?

10. Mary gave 11 cents for tape, 25 cents for sewing-silk, and 10 cents for needles; what was the entire cost?

11. What is the sum of 29 and 45?

SOLUTION.—29 and 40 are 69, and 5 are 74.

12. What is the sum of 25 and 36? 29 and 34? 28 and 43? 57 and 42? 65 and 46? 72 and 49?

13. What is the sum of 27 and 47? 29 and 38? 75 and 45? 79 and 65? 84 and 95? 95 and 67? 125 and 34? 164 and 199?

14. Add by 7's from 7 to 98; from 1 to 99; from 2 to 100; from 3 to 101. Add by 8's from 8 to 120; from 2 to 122; from 3 to 123.

15. Add by 9's from 0 to 108; from 1 to 109; from 2 to 110; from 3 to 111. Add by 11's from 0 to 132; from 1 to 133, etc.

16. Add by 12's from 0 to 144; from 1 to 145; from 2 to 146; from 3 to 147; by 13's from 13 to 130; from 1 to 131; etc.

WRITTEN EXERCISES.

1. A newsboy clears $2.75 one week, $2.87 the next week, and $4.68 the third week; how much does he clear in the three weeks?

	OPERATION.
SOLUTION.—If a newsboy clears $2.75 one week, $2.87 the next week, and $4.68 the third week, in the three weeks he clears the sum of $2.75, $2.87, and $4.68, which we find by adding is $10.30.	$2.75 2.87 4.68 $10.30

2. A conductor of a street car takes in $4.65 on one trip, $3.80 the next trip, and $4.25 on the third trip; how much does he collect on the three trips? *Ans.* $12.70.

3. A farmer's son pays $225 for a horse, $135 for a carriage, and $48.50 for harness and robes; what was the cost of his outfit? *Ans.* $408.50.

4. A young lady paid $5.25 for a pair of shoes, $15.75 for a dress, and $8.50 for a bonnet; what did she expend for these articles? *Ans.* $29.50.

5. In a primary school there are 386 children in first grade, 258 in second grade, 237 in third grade, and 184 in fourth grade; how many pupils are in the four grades? *Ans.* 1065.

6. A lady bought a farm for $8425, a house for $6575, a store for $5750, and had a balance remaining in bank of $7528; what was she worth at first? *Ans.* $28278.

7. A grocer paid \$628 for sweet potatoes, \$235 for white potatoes, and \$437 for turnips; how much did he receive if he sold them all at a gain of \$115? *Ans.* \$1415.

8. A lady bought calico for a dress for \$2.25, a parasol for \$4.75, a necktie for \$0.25, a pair of boots for \$3.50, and a pair of gloves for \$1.75; what was her bill? *Ans.* \$12.50.

9. A speculator paid \$4725 for each of three city lots; what will he receive if he sells them at a gain of \$1367 on the investment? *Ans.* \$15542.

10. If the school session closes on the 29th of June and opens again on the 10th of September, how many days vacation will there be? *Ans.* 72.

11. Mr. Johnson pays his book-keeper \$1250 a year, three salesmen \$800 each, a porter \$520, and a boy \$250; what amount of salaries does he pay? *Ans.* \$4420.

12. How many times does the hammer of a clock strike from 2 o'clock A. M. to 2 o'clock P. M.? From 2 P. M. Monday to 2 P. M. Tuesday? *Ans.* 78; 156.

13. Mr. Wilson paid \$250 for a horse, \$125 for a sleigh, \$75 for harness and bells, and \$27.25 for a fur robe; what was the cost of the outfit? *Ans.* \$477.25.

14. A man gave his son \$5600, his daughter \$575 more than his son, and his wife \$850 more than his daughter; how much did he give to them all? *Ans.* \$18800.

15. A gentleman left by will \$5345 to each of his two sons, \$4575 to each of three daughters, and to his widow as much as the share of a son and a daughter; what was his entire fortune? *Ans.* \$34335.

16. A builder bought a lot for \$650, built upon it a house costing \$5465, and a barn and carriage-house costing \$1074.50; he paid \$215.75 for fencing and \$87.50 for grading; for what must he sell the property to gain \$640? *Ans.* \$8132.75.

INTRODUCTION TO SUBTRACTION.

ORAL EXERCISES.

1. How many blocks are left when 3 blocks are taken from 5 blocks?

SOLUTION.—If 3 blocks are taken from 5 blocks, there will remain 5 blocks minus 3 blocks, or 2 blocks.

2. How many cents remain when 4 cents are taken from a pile of 7 cents?

3. If I have 12 oranges and give my sister 5 of them, how many shall I have remaining?

4. William is 15 years old and John is 6 years younger; how old is John?

5. Mary has 24 apples and Sarah has 13 apples; how many more has Mary than Sarah?

6. In a school numbering 38 pupils 12 are absent; how many pupils are present?

7. A heifer was bought for 15 dollars and sold for 25 dollars; what was the gain?

8. Begin at 2 and count by 2's to 30; begin at 30 and count by 2's backward to 2.

9. Begin at 42 and count by 3's backward to 3; begin at 49 and count by 3's backward to 2.

10. Count by 4's from 48 back to 4; from 55 back to 3; from 54 back to 2; from 53 back to 1.

11. Count by 5's from 60 back to 5; from 64 back to 4; from 63 back to 3; from 62 back to 2; from 61 back to 1.

12. In a similar manner begin at different numbers and count backward by 6's, 7's, 8's, and 9's.

13. Take the number 3, add 5, subtract 6, add 7, subtract 5, add 8, subtract 7, add 9, subtract 4, and name the result.

14. Take the number 11, add 4, subtract 3, add 5, subtract 4, add 6, subtract 5, add 7, subtract 6, add 8, subtract 7, and name the result.

15. What is the process of finding the *difference* between two numbers called? *Ans. Subtraction.*

SUBTRACTION.

48. **Subtraction** is the process of finding the *difference* between two numbers.

49. The **Difference** between two numbers is a number which added to the less equals the greater.

50. The **Minuend** is the number from which we subtract. The **Subtrahend** is the number to be subtracted.

51. The **Sign of Subtraction** is —, and is read *minus*. It denotes that the number immediately following it is to be subtracted from the number preceding it.

The symbol – was introduced by *Stifelius*, a German mathematician, in a work published in 1544.

PRINCIPLES.

1. *The minuend and subtrahend must be similar numbers.*

2. *The difference is a number similar to the minuend and subtrahend.*

CASE I.

52. To subtract when no term of the subtrahend exceeds the corresponding term of the minuend.

1. What is the difference between 568 and 325?

	OPERATION.
	568
	325
	243

SOLUTION.—We write the subtrahend under the minuend, placing terms of the same order in the same column, draw a line beneath, and begin at the right to subtract. 5 units from 8 units leaves 3 units, which we write under the units; 2 tens from 6 tens leave 4 tens, which we write under the tens; 3 hundreds from 5 hundreds leave 2 hundreds, which we write under the hundreds. Tnerefore, the difference between 568 and 325 is 243.

WRITTEN EXERCISES.

	(2)	(3)	(4)	(5)
From	462	695	759	687
Subtract	321	243	443	576

	(6)	(7)	(8)	(9)
From	835	542	948	837
Subtract	210	231	744	421

	(10)	(11)	(12)	(13)
From	964	618	$28.35	$61.84
Subtract	752	315	14.21	50.63

	(14)	(15)	(16)	(17)
From	9881	6977	$38.74	$67.88
Subtract	7450	4865	27.63	56.72

	(18)	(19)	(20)	(21)
From	7642	9871	7478	$99.81
Subtract	6531	7240	4433	45.60

Subtract—

22. 35213 from 65644.

23. 42156 from 75277.

24. 22135 from 96756.

25. 14156 from 75278.

Subtract—

26. 24231 from 35674.

27. 43421 from 48765.

28. $243.12 from $7864.25.

29. $4134.24 from $7654.48.

CASE II.

53. To subtract when terms of the subtrahend exceed the corresponding terms of the minuend.

54. There are two methods of explaining this case, called the *Method of Borrowing* and the *Method of Adding Ten.*

1. From 837 subtract 374.

OPERATION.
837
374
463

SOLUTION BY BORROWING.—We write the subtrahend under the minuend, and begin at the right to subtract. 4 units from 7 units leave 3 units, which we write under the units; we cannot take 7 tens from 3 tens, we will therefore take 1 hundred from the 8 hundreds, and add it to the 3 tens; 1 hundred equals 10 tens, which, added to 3 tens, equals 13 tens; 7 tens from 13 tens leave 6 tens, which we write in tens place; 3 hundreds from 7 hundreds (the number remaining after taking away 1 hundred) leave 4 hundreds, which we write in the hundreds place.

SOLUTION BY ADDING TEN.—4 units from 7 units leave 3 units; we cannot take 7 tens from 3 tens, we will therefore add 10 tens to the 3 tens, making 13 tens; 7 tens from 13 tens leave 6 tens; now, since we have added 10 tens, or 1 hundred, to the minuend, our remainder will be 1 hundred too large; hence to obtain the correct remainder we must add 1 hundred to the subtrahend; 1 hundred and 3 hundreds are 4 hundreds; 4 hundreds from 8 hundreds leave 4 hundreds.

From these solutions we derive the following

Rule.—I. *Write the subtrahend under the minuend, placing terms of the same order in the same column, and draw a line beneath.*

II. *Begin at the right and subtract each term of the subtrahend from the corresponding term of the minuend, writing the remainder beneath.*

III. *If any term of the subtrahend exceeds the corresponding term of the minuend, add* 10 *to the latter, and then subtract.*

IV. *Add* 1 *to the next term of the subtrahend (or subtract* 1 *from the next term of the minuend), and proceed as before.*

Proof.—Add the difference to the subtrahend, and if the work is correct the sum will equal the minuend.

Or, Subtract the difference from the minuend, and if the work is correct the result will equal the subtrahend.

1. Teachers will take either of the above methods of solution which they prefer.
2. The taking 1 from a term of the minuend is called *borrowing*, and the adding 1 to the next term of the subtrahend is called *carrying*.

WRITTEN EXERCISES.

	(2)	(3)	(4)	(5)	(6)	(7)
From	485	362	\$5.32	\$6.17	3231	6537
Take	157	175	1.64	2.21	2465	4489

	(8)	(9)	(10)	(11)	(12)	(13)
From	7815	4712	4057	3987	2014	6441
Take	2066	3805	3268	1998	1909	2528

	(14)	(15)	(16)	(17)
From	$561.32	$604.22	$759.00	$351.00
Take	288.56	519.18	576.75	275.83

Subtract—

18. 7632 from 9724.

19. 5986 from 8133.

20. 8274 from 9981.

21. 6779 from 9792.

22. 9401 from 10320.

23. 8963 from 12542.

Subtract—

24. 57632 from 89724.

25. 35487 from 80522.

26. 21586 from 47305.

27. 33441 from 62784.

28. 20407 from 54727.

29. $231.54 from $305.87.

What is the value—

30. Of $23175 + 4761 - 3745$? *Ans.* 24191.

31. Of $75423 + 2341 - 6789$? *Ans.* 70975.

32. Of $6000664 - 4002543$? *Ans.* 1998121.

33. Of $1000000 + 100000 - 1$? *Ans.* 1099999.

34. Of $8403 - 5264 + 17756$? *Ans.* 20895.

35. Of $3785 - 3467 + 78671$? *Ans.* 78989.

36. Of $\$3.75 - \$1.78 + \$4.64 - \$3.81$? *Ans.* $2.80.

37. From one hundred thousand take nine hundred ninety-nine. *Ans.* 99001.

38. From two million and two take four thousand and four. *Ans.* 1995998.

39. From six hundred thousand take six hundred sixty-six. *Ans.* 599334.

40. From seventy-seven thousand seven take six thousand sixty-six. *Ans.* 70941.

41. From eighty-eight thousand eighty take eighty thousand eighty-eight. *Ans.* 7992.

42. From sixty-six thousand sixty take forty-four thousand forty-four. *Ans.* 22016.

43. From forty-four thousand take four thousand and forty-four. *Ans.* 39956.

44. From one hundred million and one take three million and six. *Ans.* 96999995.

45. From 7 billion, 7 million, 7 thousand, and 7 take 90 million, 90 thousand, and 9. *Ans.* 6916916998.

ORAL EXERCISES.

1. Anna bought 20 pears and gave Kate 8 of them; how many did Anna then have?

SOLUTION.—If Anna bought 20 pears and gave Kate 8 of them, Anna then had the difference between 20 pears and 8 pears, which is 12 pears.

2. In a school numbering 30 pupils, 9 are absent; how many pupils are present?

3. James found 27 eggs in the barn and broke 6 of them; how many remained unbroken?

4. How many are 19 minus 8? 17 minus 9? 23 minus 7? 25 minus 6? 27 minus 9? 33 minus 4?

5. How many are 34 minus 5? 41 minus 4? 50 minus 5? 48 minus 7? 75 minus 9? 100 minus 8? 123 minus 6?

6. Required the value of 115 minus 7; 120 minus 9; 125 minus 6; 140 minus 4; 132 minus 6; 150 minus 8; 160 minus 10.

7. Eliza, having 15 cents, spent 10 cents, after which her mother gave her 20 cents; how much had she then?

8. George had $12 in the savings bank; he earned $15, of which he spent $7, putting the remainder in bank; what amount had he deposited?

9. A merchant paid $75 for goods and $7 for transportation; for what must he sell them to gain $15?

10. Subtract 25 from 54.

SOLUTION.—20 from 54 leaves 34, and 5 from 34 leaves 29.

11. Subtract 25 from 41; 36 from 54; 39 from 61; 34 from 65; 45 from 72; 49 from 75; 55 from 82; 57 from 85.

12. Subtract 121 from 144; 145 from 257; 155 from 289; 207 from 367; 309 from 463; 337 from 475.

13. A lady bought some elastic for 8 cents, some edging for 15 cents, and some pins for 12 cents; what change will she receive out of half a dollar?

14. A man bought a horse for $125, paid $25 for his keeping, and sold him for $187; what was the gain?

15. A man went into a clothing-store and bought a vest for $6, a coat for $20, and a pair of trowsers for $11; he handed the clerk 4 ten-dollar bills; what change did he receive?

16. Take the number 12, add 3, subtract 2, add 4, subtract 3, add 5, subtract 4, add 6, subtract 5, add 7, subtract 6, and name the result.

17. How many are 15 plus 8, minus 7, plus 9, minus 4, plus 12, minus 10, plus 16, minus 19, plus 20, minus 18?

18. Henry solved 127 problems in a month, and William solved 119 problems; how many more did Henry solve than William?

WRITTEN EXERCISES.

1. A man having 625 acres of land sold 362 acres; how many acres had he remaining?

OPERATION.

$$\begin{array}{r} 625 \\ \underline{362} \\ 263 \end{array}$$

SOLUTION.—If a man had 625 acres of land and sold 362 acres, he had remaining the difference between 625 acres and 362 acres, which is 263 acres.

2. Mr. Mason borrowed $4054, and has repaid $2565; how much does he still owe? *Ans.* $1489.

3. A farmer has 1543 sheep and 945 lambs; how many more sheep has he than lambs? *Ans.* 598.

4. B and C have each 742 acres of land; if B sells C 364 acres, how much will each then have? *Ans.* 378 and 1106.

5. A man sold a city lot for $3785, which was $275 more than he paid for it; what did he pay for it? *Ans.* $3510.

6. My brother and I own 2534 head of cattle; if I own 1425 head, how many does my brother own? *Ans.* 1109.

7. During the summer vacation James rode 135 miles on his bicycle and John rode 162; how many miles did John ride more than James? *Ans.* 27.

8. A and B together have $7795; how much money has A if B has $3475? *Ans.* $4320.

9. George Washington was born in 1732 and died in 1799; what was his age when he died? *Ans.* 67 years.

10. Andrew Jackson was born in 1767 and died in 1845; how old was he when he died? *Ans.* 78 years.

11. How many years from the birth of Milton in 1608 to the birth of Walter Scott in 1771? *Ans.* 163 years.

12. How many years from the battle of Lexington in 1775 to the firing on Fort Sumter in 1861? *Ans.* 86 years.

13. Socrates was born 468 B. C., and died when he was 70 years old; in what year did he die? *Ans.* 398 B. C.

14. How many years from the birth of Washington in 1732 to the birth of Lincoln in 1809? *Ans.* 77 years.

15. How many years from the birth of Wm. Penn in 1644 to the birth of Benjamin Franklin in 1706? *Ans.* 62 years.

16. How many years from the discovery of America in 1492 to the founding of Jamestown in 1607? *Ans.* 115 years.

17. Watches were invented at Nuremberg in 1477; how many years from that time to 1893? *Ans.* 416 years.

18. Printing from movable type is supposed to have been invented in 1447; how long has the invention been in use?

19. Lanterns were invented by King Alfred in 890; how many years has the invention been in use?

20. The circulation of the blood was discovered by Harvey in 1619; how many years since that time?

21. Spectacles were invented in 1299; how long was it before the invention of the telescope in 1610? *Ans.* 311 years.

22. The first steamship crossed the ocean in 1819; how many years is it since that time?

23. The first line of telegraph was established in the United States in 1844; how long has the invention been in use?

24. A produce-dealer had in bank $5021, and checked out $1250 on one day and $1297 on the next day; how much remained then in bank? *Ans.* $2474.

25. Mr. Johnson bought a house for $7515, and paid for it in yearly installments of $1252.50; how many years did it require to pay for the house? *Ans.* 6 years.

WRITTEN EXERCISES

IN ADDITION AND SUBTRACTION.

1. A merchant gave his note for $5000. He paid at one time $2475, and at another $1225; how much remained to be paid? *Ans.* $1300.

2. A man having $7500 in bank gave two checks, one for $2427 and one for $2487; how much had he remaining in the bank when these checks were cashed? *Ans.* $2586.

3. An estate worth $45000 has two mortgages upon it, one for $17000, and the other for $9000; what is the estate worth above the incumbrances? *Ans.* $19000.

4. A farmer raised 725 bushels of potatoes, of which he sold 215 bushels to B, 310 bushels to C, and 117 bushels to D; how many bushels did he retain? *Ans.* 83.

5. Mr. Watson's gross profits last year were $8000; he paid for rent $1050, insurance $175, salaries $2500, and incidental expenses $375; what was his net profit? *Ans.* $3900.

6. A Western railroad owning a tract of land of 5760 acres sold at one time 1250 acres; at another time, 2365 acres; at another time, 720 acres; how much of the tract then remained? *Ans.* 1425 acres.

7. A clerk receives $1000 a year; he pays $175 a year for house-rent, his butcher's bill is $187.50 and grocer's bill $167.25, and he spends for clothes, etc. $325; what does he save in the year? *Ans.* $145.25.

8. I received for goods a check on the First National Bank for $12,097.25; on presenting it at the bank I made a deposit

of $10350; what balance did the bank pay me on the check? *Ans.* $1747.25.

9. An estate of $20000 was divided as follows: the cost of settling the estate was $1076; the widow received $9462; the younger son $2419; the daughter $3631; the elder son the remainder; what was the share of the latter? *Ans.* $3412.

10. In five granaries there are 8796 bushels of corn; in the first there are 1865 bushels; in the second, 1279 bushels; in the third, 1644 bushels; in the fourth, 1949 bushels; how many bushels are contained in the fifth? *Ans.* 2059.

11. Mr. Harris bought two houses, paying for one $5775 and for the other $7050. After making alterations costing $1175, and paying taxes amounting to $237.26, he sold them both for $15000. How much did he gain? *Ans.* $762.74.

12. I bought in a fancy store the day before Christmas a writing-desk for $4.25, a glove-box for $1.25, a handkerchief-box for $1.50, a case of scissors for $2.75, and a gentleman's dressing-case for $6.75; what change should I receive out of a twenty-dollar bill? *Ans.* $3.50.

13. A farmer took to the store 3 dozen eggs, $0.90; 15 pounds of butter, $6; 3 barrels of apples, $7; he bought 25 pounds crushed sugar, $1.75; 4 gallons molasses, $1.20; 10 yards gray flannel, $3.75; and 1 piece Utica sheeting, $7.50; how much cash must he pay? *Ans.* $0.30.

14. I had in bank on Monday morning, April 4, $9225; I checked out Monday afternoon $625; Tuesday I deposited $1200; Wednesday I deposited $450; Thursday I checked out $957.25; Friday I again checked out $475.25; Saturday I deposited $640; what is the amount in bank Monday morning, April 11? *Ans.* $9457.50.

INTRODUCTION TO MULTIPLICATION.

ORAL EXERCISES.

1. How many blocks are there in 2 piles, each pile containing 3 blocks?

SOLUTION.—If in 1 pile there are 3 blocks, in 2 piles there are 2 times 3 blocks, or 6 blocks.

2. How many apples are there in 3 groups, each containing 4 apples? How many are three 4's? three 2's? two 4's? etc.

3. Repeat the table of 2 times; of 3 times; of 4 times; of 5 times; of 6 times; of 7 times; of 8 times, etc.

4. If a melon costs 12 cents, what will 6 melons cost at the same rate?

SOLUTION.—If 1 melon costs 12 cents, 6 melons will cost 6 times 12 cents, or 72 cents.

5. What will 8 chairs cost at the rate of 6 dollars each?

6. What will 7 spelling-books cost at 20 cents apiece?

7. My orchard has 10 rows of trees and 12 trees in a row; how many trees in the orchard?

8. A jeweller sold 15 gold rings at 4 dollars apiece; how much did he receive for them?

9. Joseph has 25 dollars, and James has 6 times as much; how many dollars has James?

10. Mary bought 12 yards of sateen at 15 cents a yard; what was the cost of the sateen?

11. Ernest earned 15 dollars a week, and paid 8 dollars a week for board; how much could he save in 9 weeks?

12. If 6 men mow a field of grass in 18 days, how long will it take one man to mow it?

13. Henry sold 19 quarts of chestnuts to a merchant at 8 cents a quart; what did he receive for his chestnuts?

14. How many are 4 times 5, plus 3? 5 times 7, plus 6? 6 times 8, plus 7? 7 times 9, plus 8? 8 times 7, plus 10? 10 times 11, plus 12?

15. What is the process of taking one number as many times as there are units in another called? *Ans. Multiplication.*

16. What is the result in multiplying called? *Ans.* The *Product.*

MULTIPLICATION.

55. Multiplication is the process of finding the *product* of two numbers.

56. The **Product** of two numbers is the result of taking one number as many times as there are units in the other.

57. The **Multiplicand** is the number to be multiplied.

58. The **Multiplier** is the number by which we multiply.

59. The **Sign of Multiplication** is $\times$, and is read *multiplied by*, or *times*. When placed between two numbers it denotes that one is to be multiplied by the other.

The symbol $\times$ was introduced by *Wm. Oughtred*, an English mathematician born in 1754.

PRINCIPLES.

1. *The multiplier is always an abstract number.*
2. *The multiplicand may be abstract or concrete.*
3. *The product is always similar to the multiplicand.*

CASE I.

60. When the multiplier is not greater than twelve.

1. Multiply 467 by 7.

OPERATION.	
Multiplicand,	467
Multiplier,	7
Product,	3269

SOLUTION.—We write the multiplier under the multiplicand, draw a line beneath, and begin at the right to multiply. 7 times 7 units are 49 units, or 9 units and 4 tens; we write the 9 units in units place in the product, and reserve the 4 tens to add to the next product; 7 times 6 tens are 42 tens, plus the 4 tens, equal 46 tens, or 6 tens and 4 hundreds; we write the 6 tens in tens place, and reserve the 4 hundreds to add to the next product; 7 times 4 hundreds are 28 hundreds, plus the 4 hundreds, equal 32 hundreds, or 2 hundreds and 3 thousands, which we write in thousands and hundreds places in the product. Hence the product is 3269. Hence the following

Rule.—I. *Write the multiplier under the multiplicand, and draw a line beneath.*

II. *Begin at the right, and multiply each term of the multiplicand by the multiplier, carrying as in addition.*

WRITTEN EXERCISES.

(2)	(3)	(4)	(5)	(6)
476	785	867	438	509
3	2	4	7	6
1428	1570	3468	3066	3054

(7)	(8)	(9)	(10)	(11)
\$4.08	\$6.84	\$13.75	\$45.06	\$32.50
6	8	5	9	12
\$24.48	\$54.72	\$68.75	\$405.54	\$390.00

Multiply

12. 7581 by 7. *Ans.* 53067.
13. 4217 by 6. *Ans.* 25302.
14. 9208 by 5. *Ans.* 46040.
15. 6178 by 8. *Ans.* 49424.
16. 16234 by 4. *Ans.* 64936.
17. 12785 by 7. *Ans.* 89495.
18. 42198 by 3. *Ans.* 126594.
19. 98789 by 5. *Ans.* 493945.
20. 57276 by 6. *Ans.* 343656.
21. 69477 by 4. *Ans.* 277908.
22. 90872 by 9. *Ans.* 817848.
23. 90047 by 8. *Ans.* 720376.

24. If a steamer sails 345 miles in one day, how far will it sail in 8 days?

OPERATION.

345
8
2760 *Ans.*

SOLUTION.—If a steamer sails 345 miles in 1 day, in 8 days it will sail 8 times 345 miles, which we find by multiplying is 2760 miles. Therefore, etc.

25. If sound moves 1092 feet in a second, how far will it move in 5 seconds? *Ans.* 5460 feet.

26. If an acre of land cost \$225, what will 8 acres cost at the same rate? *Ans.* \$1800.

27. If there are 231 cubic inches in one gallon, how many cubic inches in 7 gallons? *Ans.* 1617.

28. Light moves about 186,000 miles in a second; how far will it move in 5 seconds? *Ans.* 930,000 miles.

29. The distance of the moon from the earth is nearly 240,000 miles; what is six times the distance? *Ans.* 1,440,000 miles.

30. If the distance of the earth from the sun is about 91,430,000 miles, what is seven times the distance from the earth to the sun? *Ans.* 640,010,000 miles.

CASE II.

61. When the multiplier is greater than twelve.

1. Multiply 676 by 27.

SOLUTION.—We write the multiplier under the multiplicand, units under units, tens under tens, etc. Since 27 equals 7 units and 2 tens, it is evident that 27 times a number equals 7 times the number plus 2 tens times the number. Seven times 676 equals 4732; 2 times 676 equals 1352, hence 2 ten times 676 equals 1352 tens. Taking the sum of the partial products, we have 18252. Hence, etc.

OPERATION.

$$\begin{array}{r} 676 \\ \underline{27} \\ 4732 \\ \underline{1352} \\ 18252 \end{array}$$

Rule.—I. *Write the multiplier under the multiplicand, placing terms of the same order in the same column, and draw a line beneath.*

II. *Begin at the right, and multiply each term of the multiplicand by each term of the multiplier, writing the first term of each product under the term of the multiplier used to obtain it.*

III. *Add the partial products, and their sum will be the entire product.*

Proof.—Multiply the multiplier by the multiplicand; if the work is correct, this product will equal the first product.

1. When there are ciphers between the significant terms of the multiplier, pass over them and multiply by the significant terms alone.

2. We begin at the right to multiply, so that when any product exceeds nine, we may add the left-hand term to the next product.

2. Multiply 356 by 45; also by 405.

OPERATION.

$$\begin{array}{r} 356 \\ \underline{45} \\ 1780 \\ \underline{1424} \\ 16020 \end{array}$$

OPERATION.

$$\begin{array}{r} 356 \\ \underline{405} \\ 1780 \\ \underline{1424} \\ 144180 \end{array}$$

WRITTEN EXERCISES.

(3)	(4)	(5)	(6)
437	247	365	465
14	15	16	18
6118	3705	5840	8370

(7)	(8)	(9)	(10)
$5.18	$6.14	$3.54	$1.77
21	22	24	25
$108.78	$135.08	$84.96	$44.25

Multiply

11. 237 by 26. *Ans.* 6162.
12. 345 by 28. *Ans.* 9660.
13. 842 by 27. *Ans.* 22734.
14. 764 by 29. *Ans.* 22156.
15. 368 by 33. *Ans.* 12144.
16. 418 by 35. *Ans.* 14630.
17. 794 by 36. *Ans.* 28584.
18. 587 by 38. *Ans.* 22306.
19. 912 by 39. *Ans.* 35568.
20. 645 by 42. *Ans.* 27090.
21. 1529 by 44. *Ans.* 67276.
22. 1892 by 45. *Ans.* 85140.
23. 1234 by 46. *Ans.* 56764.
24. 1781 by 48. *Ans.* 85488.
25. 8182 by 52. *Ans.* 425464.
26. 7864 by 57. *Ans.* 448248.
27. 1821 by 58. *Ans.* 105618.
28. 3644 by 59. *Ans.* 214996.
29. 4071 by 64. *Ans.* 260544.
30. 5105 by 87. *Ans.* 444135.
31. 6671 by 75. *Ans.* 500325.
32. 4749 by 94. *Ans.* 446406.

Find the value of

33. 9621×121.
34. 4353×234.
35. 5167×358.
36. 7862×632.
37. 1781×1048.
38. 3549×2044.
39. 1895×4005.
40. 7334×1336.
41. 4781×1803.
42. 2545×1564.
43. 4532×1765.
44. 8372×3033.
45. 2775×4061.
46. 2196×6012.
47. 81864×4015.
48. 43152×5254.

Find the value of

49. 61876 × 5025.

50. 43641 × 4792.

51. 29841 × 2032.

52. 28531 × 3024.

53. 10747 × 2008.

54. 23561 × 1842.

55. 25242 × 2663.

56. 87574 × 3814.

57. 63314 × 2177.

58. 45831 × 4728.

59. 97564 × 81273.

60. 170082 × 34567.

61. 971563 × 30807.

62. 628506 × 75019.

63. 270044 × 270044.

64. 536009 × 748020.

ORAL EXERCISES.

1. What will 5 yards of ribbon cost at 12 cents a yard?

SOLUTION.—If 1 yard of ribbon costs 12 cents, 5 yards of ribbon will cost 5 times 12 cents, or 60 cents.

2. What will 7 pounds of beef cost at 15 cents a pound?

3. What will 10 yards of linen cost at 75 cents a yard?

4. If a railroad train runs 32 miles an hour, how far will it run at that rate in 5 hours?

5. If a newsboy clears 45 cents a day for 7 days, how much will he clear in a week?

6. If there are 16 spokes in a wheel, how many spokes are there in 8 wheels?

7. A saleswoman earned $18 a week and paid $6 for her board; how much could she save in 7 weeks?

8. A huckster had 25 watermelons, sold 15 of them, and then bought 5 times as many as he sold; how many had he then?

9. Multiply 68 by 6.

SOLUTION.—6 times 8 units are 48 units, which equals 4 *tens* and 8 *units;* 6 times 6 *tens* are 36 *tens*, which increased by the 4 tens equals 40 *tens;* hence the product is 408.

10. Multiply 27 by 3; 26 by 4; 34 by 5; 42 by 6; 55 by 7; 72 by 8; 55 by 12; 77 by 11; 112 by 10.

11. Multiply 123 by 4; 345 by 3; 634 by 5; 277 by 6; 604 by 7; 634 by 8; 4003 by 6; 1603 by 5; 604 by 11; 506 by 12.

12. A jeweller bought 4 watches at 25 dollars apiece, and sold them at a gain of 10 dollars each; what did he receive for them?

13. How much are 5 loads of flour worth, each load containing 12 barrels, at $6 a barrel?

14. A man bought 5 pairs of rubbers at 50 cents a pair, and 2 pairs of shoes at $1.25 a pair; what change will he receive out of a $5 bill?

15. A farmer's wife took to a store 3 pounds of butter at 35 cents a pound, and bought 12 yards of calico at 8 cents a yard; what balance is due her?

16. A lady went "a-shopping," and bought 12 yards of silk at $2 a yard, 3 yards of satin at $3 a yard, and 3 pairs of gloves at $1.50 a pair; she handed the clerk four $10 bills; what change should she receive?

WRITTEN EXERCISES.

1. How many boxes of oranges can be raised on 45 trees if each tree yields 184 boxes?

OPERATION.

$$\begin{array}{r} 184 \\ 45 \\ \hline 920 \\ 736 \\ \hline 8280 \end{array}$$

SOLUTION.—If 1 tree yields 184 boxes, 45 trees will yield 45 times 184 boxes, which, by multiplying, we find to be 8280 boxes.

2. How many scholars in a graded school of 26 classes if there are 48 scholars in each class? *Ans.* 1248 scholars.

3. In a row of 20 houses there are 24 windows in each house; how many windows in the row? *Ans.* 480 windows.

4. If a clerk deposits $625 annually in a savings bank, how much will he deposit in 25 years? *Ans.* $15625.

5. Mr. Jones bought a farm of 98 acres at $125 an acre; what was the cost of the farm? *Ans.* $12250.

6. What will a hogshead of wine containing 63 gallons cost at the rate of $5.75 a gallon? *Ans.* $362.25.

7. If a trolley-car requires 35 seconds to go 1 square, how long will it take to go 25 squares? *Ans.* 875 seconds.

8. How much will it cost to heat a school-building a year if it takes 58 tons of coal at $5.25 a ton? *Ans.* $304.50.

9. If there are 16 windows in a street-car, how many will there be in 56 cars? *Ans.* 896 windows.

NOTE.—In this and similar problems we multiply by the smaller number, using both numbers abstractly, though in the explanation we should use the proper number as the multiplier.

10. If it takes 8 nails to fasten a horseshoe, how many nails will be required to shoe 4 two-horse teams? *Ans.* 256 nails.

11. A young clerk earned $24 a week, and paid $6 for his board; what would he save in a year? *Ans.* $936.

12. If a type-writer spends 15 cents a day for car fare, how much will she spend in the week-days of March, April, and May, deducting 13 Sundays? *Ans.* $11.85.

13. A lady has spent the months of July and August at the seashore for the last 16 years; how many days has she spent at the seashore? *Ans.* 992 days.

14. A windmill pumps 24 gallons of water an hour; after pumping 12 hours, how much does a tank holding 450 gallons lack of being full? *Ans.* 162 gal.

15. John takes 1434 steps in going to school; if he goes and returns twice a day, how many steps will he take in 24 days? *Ans.* 137664 steps.

16. If Henry takes 1539 steps going to school in one day, how many steps will he take going to school in a school-year of 40 weeks, 5 school-days in a week? *Ans.* 307800.

17. If the driving-wheels of a locomotive revolve 378 times in going 1 mile, how many times will they revolve in 347 miles? *Ans.* 131166 times.

18. A ferry-boat goes 28 miles a day, and another ferry-boat goes 36 miles a day; how far will both boats go in 185 days? *Ans.* 11840 miles.

19. The President's cabinet consists of 8 members, each of whom receives a salary of $8000; what is the amount of their salaries? *Ans.* $64000.

20. If a military division consists of 6 regiments, each regiment of 10 companies, and each company of 95 men, how many men in the division? *Ans.* 5700 men.

21. If George sells 56 papers a day and Edmund 64 papers, how many more papers does Edmund sell than George in 87 days? *Ans.* 696 papers.

22. In a row of houses there are 54 rooms, in each room 3 windows, and in each window 4 panes of glass; how many panes in all the houses? *Ans.* 648 panes.

23. A railroad conductor makes a trip of 175 miles every day except Sundays; how many miles will he travel in a year, 52 Sundays in the year? *Ans.* 54775 miles.

24. Two ships start from the same place and sail in opposite directions, one going 52 miles a day, the other 75 miles a day; how far apart will they be in 54 days? *Ans.* 6858 miles.

25. Mr. Thomson's annual income is $5475, and his average daily expenditure is $8.25; what can he save in a year of 365 days? *Ans.* $2463.75.

26. A shoe-dealer bought 42 cases of ladies' French kid boots, each case containing 12 pairs, at $4.25 a pair; what did the bill amount to? *Ans.* $2142.

27. In a block of houses are 54 buildings, each building containing 28 windows, and each window 12 panes of glass; how many panes of glass in the block? *Ans.* 18144.

28. A college professor receives $3500 salary, and spends $480 for board, $275 for clothing, $265 for books, and $347.25 for travelling and other expenses annually; how much will he save in 10 years? *Ans.* $21327.50.

29. Two travellers are 5327 miles apart; if they travel toward each other, one at the rate of 49 miles a day, and the other at the rate of 37 miles, how far apart will they be at the end of 54 days? *Ans.* 683 miles.

CONTRACTIONS IN MULTIPLICATION.

62. Contractions in Multiplication are abbreviated methods of multiplying.

63. A Composite Number is the product of two or more numbers, each greater than a unit, called *factors*.

Thus, 24 is a composite number, whose factors are 4 and 6, or 3 and 8, or 2, 3, and 4.

CASE I.

64. When the multiplier is a composite number.

1. Multiply 47 by 36.

SOLUTION.—36 equals 6 times 6, hence 36 times 47 equals 6 times 6 times 47. 6 times 47 equals 282, and 6 times 6 times 47 equals 6 times 282, which equals 1692; therefore 47 multiplied by 36 equals 1692. Hence the following rule:

OPERATION.
$$\begin{array}{r} 47 \\ \underline{6} \\ 282 \\ \underline{6} \\ 1692 \end{array}$$

Multiply the multiplicand by one factor, this product by another factor, and thus continue until all the factors have been used; the last product will be the result required.

Multiply

2. 78 by 25.	*Ans.* 1950.	**8.** 463 by 45.	*Ans.* 20835.
3. 97 by 36.	*Ans.* 3492.	**9.** 794 by 49.	*Ans.* 38906.
4. 75 by 72.	*Ans.* 5400.	**10.** 3692 by 35.	*Ans.* 129220.
5. 98 by 48.	*Ans.* 4704.	**11.** 4189 by 27.	*Ans.* 113103.
6. 113 by 63.	*Ans.* 7119.	**12.** 9917 by 42.	*Ans.* 416514.
7. 448 by 54.	*Ans.* 24192.	**13.** 40709 by 81.	*Ans.* 3297429.

14. What cost 45 books at 35 cents apiece?

SOLUTION.—45 equals 5 times 9. If 1 book costs 35 cents, 9 books will cost 9 times 35 cents, which are 315 cents; and 45 books, which are 5 times 9 books, will cost 5 times 315 cents, which are 1575 cents. Therefore, etc.

OPERATION.
$$\begin{array}{r} 35 \\ \underline{9} \\ 315 \\ \underline{5} \\ 1575 \end{array}$$

15. What cost 27 sheep at $11 each? *Ans.* $297.

16. What cost 25 horses at $95 each? *Ans.* $2375.

17. What cost 28 pianos at $175 each? *Ans.* $4900.

18. What will 36 watches cost at $45 apiece? *Ans.* $1620.

19. What cost 72 books at $2.25 each? *Ans.* $162.

20. If 1 bushel of oats is worth $0.72, how much are 56 bushels worth? *Ans.* $40.32.

21. If a yoke of oxen cost $145, what will 42 yoke cost at the same rate? *Ans.* $6090.

22. What cost 64 acres of land at $245 an acre; and what cost 81 acres at the same rate? *Ans.* $15680; $19845.

CASE II.

65. When there are ciphers at the right of one or both factors.

Principle.—*Annexing one cipher to a number multiplies it by* 10; *annexing two ciphers multiplies it by* 100; *annexing three ciphers, multiplies it by* 1000, *etc.*

For, adding one cipher removes each term one place to the left, and thus makes it denote ten times as many units as before, hence the entire number is ten times as great as before.

1. Multiply 24 by 160, also 2400 by 160.

OPERATION.	OPERATION.
24	2400
160	160
144	144
24	24
3840	384000

SOLUTION 1ST.—24 multiplied by 16 equals 384; hence 24 multiplied by 160, which is 10 times 16, equals 10 times 384, which, by annexing one cipher, equals 3840.

SOLUTION 2D.—16 times 24 equals 384, hence 16 times 24 *hundred* equals 100 times as much, which by annexing two ciphers, is 38400; and 160 times 2400 equals 10 times 38400, which, by annexing one cipher, is 384000. Hence the following rule:

Take the product of the numbers denoted by the significant figures, and annex as many ciphers to the result as are found at the right of both factors.

WRITTEN EXERCISES.

Multiply

2. 527 by 70.
3. 947 by 60.
4. 2085 by 200.
5. 2107 by 400.
6. 6070 by 500.
7. 8345 by 7000.
8. 3170 by 8000.
9. 7524 by 1500.
10. 8641 by 6300.
11. 4500 by 3500.
12. 8700 by 6500.
13. 56300 by 25100.
14. 72800 by 3200.
15. 70900 by 40300.
16. 687000 by 36500.
17. 907000 by 4900.
18. 807000 by 24300.
19. 378000 by 20500.

WRITTEN EXERCISES

IN ADDITION, SUBTRACTION, AND MULTIPLICATION.

1. Susan and Jane tried which could count the greater number in 15 minutes; Susan counted 65 and Jane 74 a minute; how many did Jane count more than Susan? *Ans.* 135.

2. A weaver, desiring employment in a silk-mill at Paterson, started from a point 312 miles distant, and walked 15 miles a day for 12 days; how far was he then from Paterson? *Ans.* 132 miles.

3. Mrs. Brown bought 12 yards of oil-cloth at 65 cents a yard, 15 yards of drugget at 45 cents a yard, and 32 yards of ingrain carpet at 75 cents a yard; what did she pay for her purchases? *Ans.* $38.55.

4. A cashier in a store has $1200 a year, out of which he pays $6 a week for board, $0.75 a week for washing, $2.50 a month for car-fare, and about $10 a week for his other expenses; what can he save in a year? *Ans.* $299.

5. A builder bought 7 thousand shingles at $8.25 per thousand, 250 pounds white paint at $0.15 a pound, 150 gallons linseed oil at $0.45 a gallon, and 20 gallons white varnish at $1.37 a gallon; what was his entire bill? *Ans.* $190.15.

6. A lady bought two cottage sets of chamber furniture at $35 each, 2 wire mattresses at $4.50, 2 hair mattresses at $22, a chiffonier for $15, and an easy-chair for $8.75; what was her bill? *Ans.* $146.75.

7. The light from a certain star is 1 year in reaching the earth; if light moves at the rate of 186,000 miles in a second, and there are 86,400 seconds in one day and 365 days in a year, what is its distance? *Ans.* 5,865,696 million miles.

8. I bought for my sitting-room 8 rolls of wall-paper at $0.45 a roll, 7 rolls at $0.25, and 30 yards of bordering at 6 cents a yard; the paper-hanger charged $3.50 for putting it on; what was the cost of papering the room? *Ans.* $10.65.

9. For the above room I bought 52 yards of body Brussels carpet at $1.25 a yard, 3 shades at $0.75 each, 3 pairs of lace curtains at $4.25 a pair, and 3 curtain-poles at $0.75 each; what was my bill? *Ans.* $82.25.

10. Mrs. Jackson bought 1 dozen pillow-cases at 10 cents each, half a dozen bolster-cases at 20 cents each, 1 dozen sheets at 45 cents each, 3 pair of blankets at $5.25 a pair, and a dozen huckaback towels at 25 cents each; what was the whole bill? *Ans.* $26.55.

11. An importer sold three lots of silks, the first containing 15 pieces of 48 yards each, at $1.75 a yard; the second 17 pieces of 45 yards each, at $2 a yard; the third 18 pieces of 51 yards each, at $2.25 a yard; what did he receive for the whole? *Ans.* $4855.50.

12. Samuel Goodwin & Co. sold to Wood and Wilson, Oct. 9, 1894, the following: 450 bushels wheat, at 54 cents a bushel; 250 bushels oats, at 31 cents; 750 bushels corn, at 55 cents. Nov. 12, S. G. & Co. bought of Wood and Wilson 375 bushels rye, at 45 cents, and 160 bushels barley, at 64 cents; required the balance of the account.

Ans. $461.85, in favor of S. Goodwin & Co.

INTRODUCTION TO DIVISION.

ORAL EXERCISES.

1. How many pounds of sugar, at 4 cents a pound, can be bought for 36 cents?

SOLUTION.—If 1 pound of sugar cost 4 cents, for 36 cents we can buy as many pounds as 4 is contained times in 36, which are 9.

2. How many marbles, at 9 cents a dozen, can be bought for 81 cents?

3. How many quarts of cherries, at 8 cents a quart, can be bought for 72 cents?

4. There are 7 days in a week; how many weeks in 98 days?

5. If I deposit $12 in a savings bank every month, in how many months shall I deposit $132?

6. If a man earns $3 a day, how many days will it take him to earn $75?

7. How many are 20 plus 5, divided by 5? 21 plus 7, divided by 7? 45 plus 9, divided by 9? 66 plus 11, divided by 11?

8. How many are 4 times 9, divided by 3? 5 times 8, divided by 4? 7 times 10, divided by 5? 4 times 12, divided by 8?

9. How many are 3 times 44, divided by 11? 5 times 35, divided by 7? 4 times 25, divided by 5? 6 times 45, divided by 9?

10. How many cords of wood, at $4 a cord, can be bought for 7 barrels of flour, at $8 a barrel?

11. James bought chestnuts at 7 cents a quart; how many quarts would he receive for 5 pecks of apples at 35 cents a peck?

12. A man built 15 rods of fence at $8 per rod; how many tons of coal, at $6 a ton, will settle his account?

13. How many kegs of nails, at $4 a keg, can be bought for $10 in money and 10 barrels of apples at $3 a barrel?

14. What is the process of finding how often one number is contained in another called? *Ans. Division.*

15. What is the result called? *Ans.* The *Quotient.*

16. What is the number which sometimes remains called? *Ans.* The *Remainder.*

DIVISION.

66. **Division** is the process of finding the *quotient* of two numbers.

67. The **Quotient** of two numbers is a number which expresses how often one number is contained in another.

68. The **Dividend** is the number to be divided.

69. The **Divisor** is the number by which we divide.

70. The **Remainder** is the number which is sometimes left after dividing.

71. The **Sign of Division** is ÷, and is read *divided by*. It denotes that the number preceding it is to be divided by the number following it.

1. The symbol ÷ was introduced by Dr. John Pell, an English mathematician born in 1610.

2. Division is also indicated by writing the divisor beneath the dividend, with a straight line between them; or by writing the divisor at the left of the dividend, with a curved line between them; thus $\frac{27}{9}$, and 9)27, mean 27 divided by 9.

PRINCIPLES

1. *When the divisor and dividend are similar numbers, the quotient is an abstract number.*

2. *In dividing a number into equal parts, the dividend and quotient are similar, and the divisor is abstract.*

72. There are **Two Methods** of performing division, called *Short Division* and *Long Division*.

SHORT DIVISION.

73. **Short Division** is that method in which only the dividend, divisor, and quotient are written.

Short division is generally employed when the divisor does not exceed *twelve*, the largest multiplier in the multiplication table.

1. How many times is 3 contained in 375?

OPERATION.

$$3\overline{)375}$$
$$\overline{125}$$

SOLUTION.—We write the divisor at the left of the dividend, with a curved line between them, draw a line beneath the dividend, and begin at the left to divide. 3 is contained in 3 hundreds 1 hundred times; 3 is contained in 7 tens 2 tens times, with a remainder of 1 ten; 1 ten equals 10 units, which with 5 units equals 15 units; 3 is contained in 15 units 3 times; hence we have for the quotient 115. Hence we have the following

Rule.—I. *Write the divisor at the left of the dividend, with a curved line between them and a line beneath the dividend.*

II. *Begin at the left, divide each term of the dividend by the divisor, and write the quotient beneath.*

III. *If there is a remainder after any division, regard it as prefixed to the next term of the dividend, and divide as before.*

IV. *If any partial dividend is less than the divisor, write a cipher in the quotient and prefix the dividend to the next term.*

V. *When there is a final remainder, annex it, with the divisor written beneath, to the integral part of the quotient.*

Proof.—Multiply the quotient by the divisor, and add the remainder, if any, to the product; if the work is correct, the result will equal the dividend.

In practice we need not name the denomination of the different partial dividends. Thus, in the above solution we say 3 is contained in 3 once; 3 is contained in 7 twice and 1 remaining.

WRITTEN EXERCISES.

(2)	(3)	(4)	(5)	(6)
2)474	2)486	3)609	3)843	3)681
237	243	203	281	227

(7)	(8)	(9)	(10)	(11)
4)868	4)476	4)884	5)650	5)760
217	119	221	130	152

Divide	Answers.	Divide	Answers.
12. 1564 by 4.	391.	**25.** 2422 by 7.	346.
13. 4225 by 5.	845.	**26.** 3080 by 7.	440.
14. 7896 by 4.	1974.	**27.** 5348 by 7.	764.
15. 9188 by 4.	2297.	**28.** 6248 by 8.	781.
16. 9765 by 5.	1953.	**29.** 72344 by 8.	9043.
17. 8345 by 5.	1669.	**30.** 85784 by 8.	10723.
18. 5675 by 5.	1135.	**31.** 24579 by 9.	2731.
19. 8556 by 6.	1426.	**32.** 81729 by 9.	9081.
20. 7218 by 6.	1203.	**33.** 3479562 by 6.	579927.
21. 8166 by 6.	1361.	**34.** 5467894 by 7.	$781127\frac{5}{7}$.
22. 9672 by 6.	1612.	**35.** 5187004 by 8.	$648375\frac{4}{8}$.
23. 8448 by 6.	1408.	**36.** 6979891 by 9.	$775543\frac{4}{9}$.
24. 8561 by 7.	1223.	**37.** 9326701 by 9.	$1036300\frac{1}{9}$.

38. If a pound of sugar cost 4 cents, how many pounds can you buy for 356 cents?

SOLUTION.—If 1 pound of sugar cost 4 cents, for 356 cents we can buy as many pounds as 4 cents are contained times in 356 cents, which are 89. Therefore, etc.

OPERATION.

$$4\overline{)356}$$
$$89$$

39. If a man walks 3 miles an hour, how long will it take him to walk 426 miles? *Ans.* 142.

40. I divided \$576 equally among some men, giving to each man \$8; how many men were there? *Ans.* 72 men.

41. If the circumference of a wheel is 12 inches, how many times will it revolve in moving 1728 inches? *Ans.* 144 times.

42. I bought some muslin at 7 cents a yard, and paid \$3.15 for it; how many yards did I buy? *Ans.* 45 yards.

43. If a freight-train averages 12 miles an hour, how long will it take to go 348 miles? *Ans.* 29 hours.

44. If it require 1 sheet of paper to print 8 pages of a book, how many sheets will be required for a book of 376 pages? *Ans.* 47 sheets.

LONG DIVISION.

74. Long Division is the method of dividing when the work is written out in full. It is generally used when the divisor exceeds 12.

1. Divide 6754 by 23.

Solution.—23 is not contained in 6 thousands any thousands times, hence there are no thousands in the quotient. 6 thousands and 7 hundreds are 67 hundreds; 23 is contained in 67 hundreds 2 hundreds times: 2 hundreds times 23 are 46 hundreds, which subtracted from 67 hundreds leave 21 hundreds: 21 hundreds and 5 tens are 215 tens; 23 is contained in 215 tens 9 tens times: 9 tens times 23 are 207 tens, which subtracted from 215 tens leave 8 tens: 8 tens and 4 units are 84 units; 23 is contained in 84 units 3 times; 3 times 23 equals 69: subtracting, there is a remainder of 15, which will not contain 23; hence the quotient is 293, with a remainder of 15.

OPERATION.

```
23)6754(293
   46
   ---
   215
   207
   ---
     84
     69
     --
     15
```

Rule.—I. *Draw curved lines at both sides of the dividend, and place the divisor at the left.*

II. *Divide the number expressed by the fewest terms at the left of the dividend that will contain the divisor, and place the quotient at the right of the dividend.*

III. *Multiply the divisor by this quotient, write the product under the partial dividend, subtract, and to the remainder annex the next term of the dividend.*

IV. *Divide as before, and thus continue until all the terms of the dividend have been used.*

V. *If any partial dividend will not contain the divisor, place a cipher in the quotient, annex the next term of the dividend, and proceed as before.*

VI. *When there is a final remainder, write it after the quotient, or, with the divisor under it, as part of the quotient.*

Proof.—Multiply the integral part of the quotient by the divisor, and add the remainder, if any, to the product; if the work is correct, the result will be equal to the dividend.

I. The pupils will notice that there are *five* operations: 1st. *Write the number;* 2d. *Divide;* 3d. *Multiply;* 4th. *Subtract;* 5th. *Bring down.*

II. Pupils often have difficulty in finding the correct quotient figure; this difficulty can be greatly diminished by attention to the following suggestions:

1st. Notice how often the left-hand term of the divisor is contained in the term or terms of the partial dividend, as far from the right-hand term as the left-hand term in the divisor is from the right-hand term.

2d. If, when we multiply, the product is greater than the partial dividend, the quotient term must be diminished.

3d. If, when we subtract, the remainder is greater than the divisor, the quotient term must be increased.

III. We commence at the left to divide, so that the remainder can be united to the number of units of the next lower order, giving a new partial dividend. The sign + is used to denote a remainder.

WRITTEN EXERCISES.

Divide

1. 559 by 13. *Ans.* 43.
2. 882 by 21. *Ans.* 42.
3. 864 by 24. *Ans.* 36.
4. 4844 by 14. *Ans.* 346.
5. 3465 by 15. *Ans.* 231.
6. 4032 by 16. *Ans.* 252.
7. 5760 by 20. *Ans.* 288.
8. 5670 by 18. *Ans.* 315.
9. 8652 by 21. *Ans.* 412.
10. 9637 by 23. *Ans.* 419.
11. 4248 by 24. *Ans.* 177.
12. 5675 by 25. *Ans.* 227.
13. 9568 by 26. *Ans.* 368.
14. 8033 by 29. *Ans.* 277.
15. 9960 by 30. *Ans.* 332.
16. 11151 by 27. *Ans.* 413.
17. 17584 by 28. *Ans.* 628.
18. 23200 by 32. *Ans.* 725.
19. 13940 by 34. *Ans.* 410.
20. 24696 by 36. *Ans.* 686.
21. 25574 by 38. *Ans.* 673.
22. 31746 by 39. *Ans.* 814.
23. 29807 by 41. *Ans.* 727.
24. 73125 by 45. *Ans.* 1625.
25. 81282 by 46. *Ans.* 1767.
26. 60564 by 49. *Ans.* 1236.
27. 251650 by 50. *Ans.* 5033.
28. 321412 by 52. *Ans.* 6181.
29. 333828 by 54. *Ans.* 6182.
30. 105168 by 56. *Ans.* 1878.
31. 126962 by 58. *Ans.* 2189.
32. 174581 by 59. *Ans.* 2959.
33. 179172 by 61. *Rem.* 15.
34. 187488 by 63. *Ans.* 2976.

Divide

35. 1569100 by 65.
36. 1381080 by 68.
37. 692346 by 69.
38. 2356692 by 78.
39. 2046035 by 85.
40. 5040411 by 97.
41. 6414982 by 98.
42. 635292 by 126.
43. 239458 by 134.
44. 460484 by 146.
45. 815894 by 421.
46. 1206528 by 192.
47. 2674965 by 453.
48. 1901808 by 562.
49. 2884716 by 681.
50. 4011642 by 774.
51. 2043693 by 873.
52. 1114155 by 945.
53. 8255456 by 1034.
54. 9325814 by 2042.
55. 7379936 by 4372.
56. 14464920 by 2235.
57. 27227704 by 6472.
58. 44848924 by 4981.
59. 97547337 by 3891.
60. 43035915 by 34567.
61. 78543671 by 23478.
62. 899400844 by 567891.

PROBLEMS IN DIVISION.

75. In **Division** there are two classes of practical problems:

1st. To find the number of equal parts of a number.

2d. To divide a number into equal parts.

76. To find the number of equal parts of a number.

ORAL EXERCISES.

1. If the fare on a street-car is 5 cents, how many trips can I take for 60 cents?

Solution.—If one trip costs 5 cents, for 60 cents I can take as many trips as 5 is contained in 60, or 12.

2. If a girl's weekly expenses at boarding-school are $7, how many weeks can she go for $98?

3. How many 5-cent pieces in a dollar? How many 10-cent pieces in 2 dollars? How many "quarters" in 5 dollars?

4. If Mary spells 12 words each day, in how many days will she spell 144 words?

5. If a boy solves 5 problems a day, in how many school-days will he solve 60 problems?

6. If 1 ton of coal cost $6, how many tons can be bought for 12 cords of wood at $7 a cord?

7. How many bananas, at 10 cents a dozen, can be bought for 6 dozen oranges, at 25 cents a dozen?

8. How many yards of lace, at 8 cents a yard, may be bought for 6 yards of muslin, at 12 cents a yard?

9. If sugar is worth $10 a barrel, how many barrels could be exchanged for $11 in cash and 7 barrels of flour at $7 a barrel?

10. Divide 76 by 4.

SOLUTION.—4 is contained in 7 tens 1 ten times and 3 tens remaining; 3 tens and 6 units equal 36 units; 4 is contained in 36 units 9 units times; hence the quotient is 1 ten and 9 units, or 19.

Divide

11. 66 by 2.	**15.** 92 by 4.	**19.** 640 by 8.	**23.** 1208 by 8.
12. 72 by 3.	**16.** 112 by 7.	**20.** 450 by 9.	**24.** 7209 by 9.
13. 64 by 4.	**17.** 111 by 3.	**21.** 168 by 12.	**25.** 1308 by 12.
14. 80 by 5.	**18.** 119 by 7.	**22.** 176 by 11.	**26.** 1728 by 12.

27. A builds 3 rods of fence in 1 day, and B builds 4 rods; how many days will it take them to build 42 rods?

WRITTEN EXERCISES.

1. If the circumference of a bicycle-wheel is 8 feet, how many times will it turn in going a mile, or 5280 feet?

SOLUTION.—If the circumference of the wheel is 8 feet, in turning once it passes over 8 feet, and in 5280 feet it will turn as many times as 8 feet are contained in 5280 feet, which are 660.

OPERATION.

$$\begin{array}{r} 8)\underline{5280} \\ 660 \end{array}$$

2. If a conductor of a street-car collects $15 a day, how many days will it take for him to collect $1275? *Ans.* 85 days.

3. The driving wheels of a locomotive are 15 feet in circumference; how many revolutions will they make in running a mile, 5280 feet? *Ans.* 352.

4. A farmer pays a farm-hand $25 a month; in how many months will he receive $450? *Ans.* 18 months.

5. If a farmer can lay up $425 a year, how many years will it take him to lay up $6800? *Ans.* 16 years.

6. If an ocean-steamer averages 250 miles a day, how long will it take to cross the Atlantic, 3000 miles? *Ans.* 12 days.

7. A man buys a piano for $750, paying $525 cash; how long will it take to pay the balance at $25 a month? *Ans.* 9 mos.

8. A drover spent $5664 in the purchase of cattle at an average cost of $24 a head; how many head of cattle did he buy? *Ans.* 236.

9. If the post-office sends 13,125 pounds of mail-matter in bags, each holding 75 pounds, how many bags will it require? *Ans.* 175 bags.

10. If the schools of Philadelphia average 45 pupils to a teacher, how many teachers are required if there are 126,000 children in the public schools of the city? *Ans.* 2800.

11. A seamstress bought a sewing-machine for $52.50; how long will it take to pay for it in weekly installments of $1.75? *Ans.* 30 weeks.

12. How many years will it take a man to save $5475, if his savings average a dollar a day, reckoning 365 days to the year? *Ans.* 15 years.

13. A farmer raised 9810 bushels of wheat; if the average yield per acre was 45 bushels, how many acres did he have in wheat? *Ans.* 218 acres.

14. A train running 55 miles an hour goes a certain distance in 18 hours; how long would it take to run the distance at the rate of 45 miles an hour? *Ans.* 22 hours.

15. A man and his son receive $216 for 24 days' work; if the son earns $1.50 a day, what does the father earn a day? *Ans.* $7.50.

16. If a farm-hand works 84 days at $1.50 a day, how much corn, at 56 cents a bushel, will it take to pay for his labor? *Ans.* 225 bushels.

17. Pennsylvania contains 45,215 square miles and Delaware contains 2050 square miles; how many states the size of Delaware could be made from Pennsylvania? *Ans.* 22+.

18. The circumference of the earth is 25,000 miles; how long would it take a vessel to sail around it, at the rate of 125 miles a day? *Ans.* 200 days.

19. Mr. Miller, having $10,000, invests $3750 in a house, and the remainder in land at $125 an acre; how much land does he buy? *Ans.* 50 acres.

20. If a man saves $25 a month, how many years will it take to save enough to buy a lot for $500, and build a house upon it costing $2500? *Ans.* 10 years.

21. If the distance from the earth to the sun is 91,430,000 miles, how long will it take light to reach us from the sun, moving 186,000 miles a second? *Ans.* 491+ seconds.

22. The moon is about 240,000 miles from the earth; how long would it require a cannon-ball to reach it if it could move at the rate of 48 miles a minute? *Ans.* 5000 minutes.

77. To divide a number into equal parts.

ORAL EXERCISES.

1. What is one of the 3 equal parts of 18?

SOLUTION.—One of the 3 equal parts of 18 is 6, since 6 taken 3 times is 18.

2. Find one of the 4 equal parts of 16; of 20; of 24; of 32.

3. Find one of the 5 equal parts of 20; of 25; of 35; of 45; of 60; of 65.

4. Find one of the 6 equal parts of 24; of 36; of 48; of 54; of 72; of 90.

5. Find one of the 7 equal parts of 21; of 35; of 49; of 84; of 70; of 98.

6. Find one of the 8 equal parts of 32; of 48; of 56; of 88; of 96; of 120.

7. What is one-third of anything? *Ans.* One-third of anything is one of the 3 equal parts into which it can be divided.

8. What is one-half of anything? one-fourth? one-fifth? one-sixth? one-seventh? one-eighth? one-ninth? one-tenth?

9. What is one-fourth of 24 dollars?

SOLUTION.—One-fourth of \$24 is \$6, since \$6 taken 4 times equals \$24.

10. What is one-fourth of 20? of 36? of 48? of 56? of 72?

11. What is one-third of 21? of 27? of 30? of 36? of 48?

12. What is one-fifth of 15? of 30? of 40? of 45? of 60?

13. What is one-sixth of 48? of 72? of 84? of 90? of 96?

14. One-half is expressed thus, $\frac{1}{2}$; one-third, $\frac{1}{3}$; one-fourth, $\frac{1}{4}$.

15. A railroad-train went 75 miles in 3 hours; how far did it go in 1 hour?

SOLUTION.—If a train went 75 miles in 3 hours, in 1 hour it went $\frac{1}{3}$ of 75 miles, or 25 miles.

16. A teacher bought 8 reading-books for \$3.20; how much is that apiece?

17. Mary's mark in arithmetic is 85, in grammar 75, and in composition 65; what is her average in these studies?

18. A farmer paid \$30 for each of two cows, and \$45 for a third; what was the average price of the three cows?

19. A dealer bought 12 pairs of shoes at \$4 a pair, and sold them for \$51; how much does he gain on a pair?

20. A boy buys 4 base-balls for \$3, paying \$1.25 for one and 50 cents each for two; what did he pay for the fourth?

21. A steam yacht ran up a river for 6 hours at 12 miles an hour, and returned in 4 hours; what was the rate of coming down the river?

22. Two yachts set out from the same place, and sailed in the same direction, one at the rate of 8 miles an hour, and the other at the rate of 6 miles an hour; how soon will the faster yacht be 20 miles ahead of the slower yacht?

WRITTEN EXERCISES.

1. Divide 372 into 4 equal parts.

SOLUTION.—If we divide 372 into 4 equal parts, each part is $\frac{1}{4}$ of 372; $\frac{1}{4}$ of 37 tens is 9 tens and 1 ten remaining; 1 ten and 2 units equal 12 units; $\frac{1}{4}$ of 12 units is 3 units; hence $\frac{1}{4}$ of 372 is 93, or 93 is one of the 4 equal parts of 372.

OPERATION.
4)372
93

2. Divide 345 into 5 equal parts. *Ans.* 69.

3. Divide 234 into 6 equal parts. *Ans.* 39.

4. Divide 343 into 7 equal parts. *Ans.* 49.

5. Divide 736 into 8 equal parts. *Ans.* 92.

6. Find one-17th of 391. *Ans.* 23.

7. Find one-15th of 675. *Ans.* 45.

8. Find one-9th of 2106. *Ans.* 234.

9. Find one-12th of 1728. *Ans.* 144.

10. Find one-11th of 2816. *Ans.* 256.

11. Find one-25th of 3925. *Ans.* 157.

12. Find one-38th of 15580. *Ans.* 410.

13. Find one-49th of 15582. *Ans.* 318.

14. If the fare on a railroad is 54 cents for 18 miles, what is the rate of fare per mile?

SOLUTION.—If the fare for 18 miles is 54 cents, for 1 mile it is $\frac{1}{18}$ of 54 cents, or 3 cents.

OPERATION.
18)54(3
54

15. If 12 coal-miners earn \$672 in one month, how much does each earn in a month? *Ans.* \$56.

16. If 9 public libraries contain 43,677 volumes, what is the average number of volumes in each library? *Ans.* 4853.

17. A lady bought 25 shares of bank-stock for \$3125; what was the cost per share? *Ans.* \$125.

18. A steamer makes a trip of 3975 miles in 15 days; what is her average rate per day? *Ans.* 265 miles.

19. A house is rented at $720 a year; how much rent does the owner receive in 8 months? *Ans.* $480.

20. The expense of building a railroad 53 miles long was $383,985; what was the average cost per mile? *Ans.* $7245.

21. A nursery-man paid $143.75 for 125 fruit-trees; what did the trees cost apiece? *Ans.* $1.15.

22. The salary of the President of the United States is $50,000 a year; how much is that a day? *Ans.* $137, nearly.

23. A man left an estate of $87,465 to be divided equally among his widow and 6 children; how many dollars did each receive? *Ans.* $12,495.

24. A has 75 Holstein cows, valued at $11,250; B has 80 horses, valued at $10,800; which is more valuable, one of A's cows or one of B's horses? *Ans.* A's cow, $15.

25. The circulation of a public library is 76,014 volumes in a year; what is the average circulation per day, if the library is open every day except 52 Sundays and 4 legal holidays? *Ans.* 246.

CONTRACTIONS IN DIVISION.

78. Contractions in Division are abbreviated forms of dividing.

79. When the divisor is a composite number.

1. Divide 3294 by 18, using the factors 3 and 6.

OPERATION.

$$\begin{array}{r} 3)\underline{3294} \\ 6)\underline{1098} \\ 183 \end{array}$$

SOLUTION 1ST.—To *multiply* by 18 we may multiply by 6, and then multiply that product by 3; hence, to *divide* by 18 we may divide by 3, and then divide that quotient by 6. Dividing by 3 we have 1098, and dividing 1098 by 6 we have 183; hence, etc.

SOLUTION 2D.—Since 18 times a number equals 6 times 3 times the number, $\frac{1}{18}$ of the number equals $\frac{1}{6}$ of $\frac{1}{3}$ of the number; $\frac{1}{3}$ of 3294 is 1098, and $\frac{1}{6}$ of 1098 is 183; hence, etc.

Rule.—*Divide the dividend by one factor of the divisor, the quotient by another factor, and thus continue for all the factors used; the last quotient will be the quotient required.*

WRITTEN EXERCISES.

Divide the following, using the factors:

2. 645 by 15. *Ans.* 43.
3. 396 by 12. *Ans.* 33.
4. 972 by 18. *Ans.* 54.
5. 3260 by 20 *Ans.* 163.
6. 3598 by 14. *Ans.* 257.
7. 12360 by 30. *Ans.* 412.
8. 4096 by 32. *Ans.* 128.
9. 22590 by 45. *Ans.* 502.
10. 13104 by 56. *Ans.* 234.
11. 41760 by 96. *Ans.* 435.
12. 70840 by 110. *Ans.* 644.
13. 67584 by 132. *Ans.* 512.

TO FIND THE TRUE REMAINDER.

80. The **True Remainder** in successive division is not the last remainder, nor the sum of all the remainders; it is necessary, therefore, to explain the method of finding the true remainder.

1. Divide 791 by 24, using the factors 2, 3, and 4.

Solution.—Dividing by 2, we find that 791 equals 395 *twos* and 1 remaining; dividing 395 *twos* by 3, we find 395 *twos* equal 131 *sixes* and 2 *twos*, or 4, remaining; dividing by 4, we find that 131 *sixes* consist of 32 *twenty-fours* and 3 *sixes*, or 18, remaining. Hence the true remainder is 18+4+1, which is 23. Hence, to find the correct remainder we have the following

OPERATION.

2)791
3)395 1
4)131, 2 *twos* = 4
32, 3 *sixes* = 18
True remainder, 23.

Rule.—*Multiply each remainder by all the divisors preceding the one which obtained it, and take the sum of the products and the remainder arising from the first division.*

WRITTEN EXERCISES.

Divide the following and find the true remainder:

2. 642 by 14. *Rem.* 12.
3. 783 by 18. *Rem.* 9.
4. 2344 by 15. *Rem.* 4.
5. 5955 by 20. *Rem.* 15.
6. 7846 by 30. *Rem.* 16.
7. 18750 by 90. *Rem.* 30.
8. 43257 by 150. *Rem.* 57.
9. 64742 by 288. *Rem.* 230.
10. 88964 by 310. *Rem.* 304.
11. 98781 by 560. *Rem.* 221.

81. When there are ciphers at the right of the divisor.

1. Divide 7466 by 600.

SOLUTION.—6 *hundreds* are contained in 74 *hundreds* 12 times, and 200 remaining; 600 is not contained in 66, hence the entire remainder is 200+66, or 266. From this solution we may derive the following

OPERATION.

$$6|00)74|66$$
$$\overline{12 - 266}$$

Rule.—I. *Cut off the ciphers at the right of the divisor and as many terms at the right of the dividend.*

II. *Divide the remaining part of the dividend by the remaining part of the divisor.*

III. *Prefix the remainder to the part of the dividend cut off, and the result will be the true remainder.*

1. When the divisor is a unit of any order with ciphers, the remainder will be the figures cut off at the right, and the quotient the figures at the left.

2. When the part of the divisor at the left of the naughts is greater than 12, divide by Long Division.

Divide

2. 789 by 60.
3. 859 by 400.
4. 1853 by 500.
5. 4774 by 700.
6. 8847 by 800.
7. 10972 by 900.
8. 24789 by 1800.
9. 270640 by 2200.
10. 120765 by 13600.
11. 347974 by 249000.
12. 6799432 by 643000.
13. 8744561 by 825000.
14. 1045764 by 102000.
15. 9784563 by 978000.
16. 1179654 by 325000.
17. 7894356 by 220000.

EXERCISE UPON THE PARENTHESIS.

82. The **Parenthesis** ( ) denotes that the quantities included are to be subjected to the same operation.

Thus, $(8+6-4)\times 3$ denotes that the value of $8+6-4$, which is 10, is to be multiplied by 3.

83. The *vinculum*, thus $\overline{8+6-4}\times 3$, is often used in place of the parenthesis.

1. What is the value of $(11+8-6)\times 4$?

SOLUTION.—$11+8$ equals 19; 19 minus 6 equals 13; and 13 multiplied by 4 equals 52. Therefore, etc.

Required the value of

2. $(27+22-19)\times 6$.	**10.** $(45+16\times 5)\div 25$.
3. $(52+87-78)\times 8$.	**11.** $(96-84\div 7)\times 10$.
4. $(87-65+88)\times 15$.	**12.** $(175+75\div 5)\div 19$.
5. $(188+99-84)\div 7$.	**13.** $(224\div 14+32)\times 15$.
6. $(152+119-118)\div 9$.	**14.** $(615-75)\div(324-270)$.
7. $(369-115+327)\times 24$.	**15.** $(320+64)\div(496-480)$.
8. $(780-360+210)\div 45$.	**16.** $(760-640)\times(560\div 140)$.
9. $(240-76)\times(840-225)$.	**17.** $(361-201)\times(320-312)$.

In a series of numbers connected with symbols the sign $\times$ denotes the closest connection, the sign $\div$ next, thus, $12+8\div 2-5\times 2=6$; also, $16\div 4\times 2=2$, rather than 8.

ARITHMETICAL ANALYSIS.

84. **Arithmetical Analysis** is a process of solving problems by comparing corresponding elements of the problem.

85. In analyzing we usually reason *to the unit* and *from the unit*, the unit being the basis of the comparison.

ORAL EXERCISES.

1. If 3 oranges cost 12 cents, what will 5 oranges cost?

SOLUTION.—If 3 oranges cost 12 cents, 1 orange costs one-third of 12 cents, or 4 cents; and 5 oranges will cost 5 times 4 cents, or 20 cents.

2. If 5 yards of velvet cost \$20, how much will 8 yards of velvet cost at the same rate?

3. If the fare in the cars is 18 cents for 6 miles, how much carfare must I pay for riding 8 miles?

4. If a school-boy solves 25 problems in 5 school-days, how many problems at this rate will he solve in 10 school-days?

5. If a postman in delivering mail walks 48 miles in 4 days, how far will he walk in 10 days?

6. If a newsboy clears $2.50 in 5 days, how much would he clear at this rate in 8 days?

7. If a servant-girl can save $3.50 in 7 weeks, how much at this rate can she save in 12 weeks?

8. If 6 school-dictionaries are bought for $18, how much will 10 school-dictionaries cost at the same rate?

9. If a trolley-car runs 24 "squares" in a city in 8 minutes, how many squares will it run at this rate in 20 minutes?

10. If it takes 5 men 12 days to do a certain piece of work, how many men will it require to do it in 6 days?

11. In how many days can 4 men do as much work as 6 men can do in 8 days?

12. Florence bought 6 car-tickets for 25 cents; how many tickets at this rate could she buy for 75 cents?

13. A farmer's wife took to market 8 turkeys, for which she received $3.60; how much would she have received at this rate for a dozen turkeys?

14. A farmer's son paid $2.40 for the shoeing of a span of horses; how much would he pay at this rate for the shoeing of four span of horses and one riding-horse?

WRITTEN EXERCISES.

1. If 25 cows are worth $1325, how many dollars are 36 cows worth?

SOLUTION.—If 25 cows are worth $1325,
1 cow is worth $\frac{1}{25}$ of $1325 = $53,
And 36 cows are worth $53 × 36 = $1908.

2. If a railway train runs 245 miles in 5 hours, how far at this rate would it run in 8 hours? *Ans.* 392 miles.

3. A drover sold 126 cows at the rate of $162 for 6 cows; how much did he receive for them? *Ans.* $3402.

4. If a factory uses 256 tons of coal in a year, what is the cost at the rate of 15 tons for $73.50? *Ans.* $1254.40.

5. The erection of 4 houses cost $5280; required the cost of erecting a row of 16 similar houses. *Ans.* $21,120.

6. A man sold 13 horses for $2405; for how much would he sell 37 horses at the same rate? *Ans.* $6845.

7. Harry is in school 5 hours a day for 5 days in a week; how many hours will he be in school during a session of 24 weeks? *Ans.* 600 hours.

8. If a coasting-steamer consumes 725 tons of coal in a 5-days' voyage, how many tons will it consume in 3 voyages of 6 days each? *Ans.* 2610 tons.

9. A carriage-maker sold 15 carriages for $1875; how much would he receive for 25 carriages, selling them at the same rate? *Ans.* $3125.

10. If the salary of a book-keeper is $725 for 5 months, how much will he earn in 3 years at the same rate per month? *Ans.* $5220.

11. If a laborer's expenses are $16.25 a month, and he earns $342 in 9 months, how much could he save at this rate in 4 years? *Ans.* $1044.

12. There is in a factory a Worthington pump which pumps 2548 gallons in 28 hours; how many gallons will it pump at this rate in 5 days? *Ans.* 10920 gallons.

13. The sales of tickets at a railroad-station amounted to $36,580 in the month of March; how much at the same rate per day will be received during the months of April, May, and June? *Ans.* $107380.

14. If the receipts of a trolley-road during the month of January, 1895, are $6324, how much will the receipts be at same rate per day for the whole year? *Ans.* $74460.

15. The quotient of one number divided by another is 367, the divisor is 445, and the remainder 189; what is the dividend? *Ans.* 163504.

16. There are two numbers, the greater of which is 25 times 670, and their difference 55 times 81; what is the lesser number? *Ans.* 12295.

WRITTEN EXERCISES

ON THE FOUR FUNDAMENTAL RULES.

1. The minuend is 2167 and the subtrahend is 1526; what is the remainder? *Ans.* 641.

2. The minuend is 8273 and the remainder is 4412; what is the subtrahend? *Ans.* 3861.

3. The subtrahend is 1984 and the remainder 5276; what is the minuend? *Ans.* 7260.

4. The multiplicand is 856 and the multiplier 345; what is the product? *Ans.* 295320.

5. The multiplicand is 2372 and the product 1259532; what is the multiplier? *Ans.* 531.

6. The product is 790209 and the multiplier 999; what is the multiplicand? *Ans.* 791.

7. The dividend is 621712 and the divisor 637; what is the quotient? *Ans.* 976.

8. The dividend is 875776 and the quotient 3421; what is the divisor? *Ans.* 256.

9. The divisor is 469 and the quotient 7531; what is the dividend? *Ans.* 3532039.

10. The dividend is 61344 and divisor 63; required the quotient and remainder. *Ans.* 973; 45.

11. The dividend is 8749, the quotient is 364, and the remainder 13; what is the divisor? *Ans.* 24.

12. The divisor is 32, the quotient 231, and the remainder 21; what is the dividend? *Ans.* 7413.

13. Twice the difference between two numbers is 1120, and the smaller number is 1775; what is the larger of the two numbers? *Ans.* 2335.

14. One-third of the larger of two numbers is 5940, and one-half of their difference is 943; what is the smaller number? *Ans.* 15934.

15. Twice the sum of two numbers is 36768, and one-half of one number is 8756; what is the other? *Ans.* 872.

16. The sum of two numbers is 458, and the greater is 346; what is the product of the two numbers? *Ans.* 38752.

17. What number multiplied by one-half of 908 will produce three times 147550? *Ans.* 975.

18. One-fourth of the product of two numbers is 25068, and three times one of the numbers is 6267; what is the other number? *Ans.* 48.

19. What number is that which being multiplied by 6, the product divided by 8, and 240 added to the quotient, equals 360? *Ans.* 160.

20. What number is that which, divided by 12, the quotient multiplied by 8, and 580 added to the product, equals 740? *Ans.* 240.

21. If we multiply a number by 12, divide the product by 4, add 240 to this quotient, and then subtract 580, the result will be 395; what is the number? *Ans.* 245.

22. Find a number which being divided by 7, the quotient multiplied by 12, the product divided by 5, and the quotient increased by 30, the result will be 150. *Ans.* 350.

PRACTICAL PROBLEMS

IN THE FUNDAMENTAL RULES.

1. If the total weight of 125 bags of mail matter is 9375 pounds, what is the average weight of a bag? *Ans.* 75 pounds.

2. Which are worth more, 56 cows at $45 apiece or 42 horses at $65 apiece? *Ans.* Horses, $210.

3. An engine ran 750 miles one week, and three times as far the next; how far did it run both weeks? *Ans.* 3000 miles.

4. In one year the War Department expended $1765.25 for mucilage at $5.75 a dozen quarts; how many quarts were purchased? *Ans.* 3684.

5. If a boarding-school uses 4 quarts of milk a day for 9 pupils, how many quarts will be needed in a week for 63 pupils? *Ans.* 196.

6. A pupil being told to multiply a certain number by 23, mistook the 3 for a 5, and his answer was 150; what was the correct answer? *Ans.* 138.

7. Mrs. Clark bought half a dozen chairs, an extension-table for $11.25, and a lounge for $7.75; her bill was $26.50; what did the chairs cost apiece? *Ans.* $1.25.

8. A storekeeper bought from one farmer 24 dozen of eggs, 6 dozen from another, 18 dozen from a third, 7 dozen from a fourth, and 12 dozen from another; what did he pay for them at 15 cents a dozen? *Ans.* $10.05.

9. Harry bought a bicycle for $75; he rented it to a friend for 6 months at $2 a month, and then sold it for $70; did he gain or lose, and how much? *Ans.* Gain, $7.

10. A newsboy sold a lot of Sunday papers for $1.75; if he had sold 5 more he would have received $2; how many did he sell? *Ans.* 35.

11. The fare from New York to Philadelphia is $2.50; how much does the railroad company receive from a train of 8 cars with 35 through passengers in each car? *Ans.* $700.

12. A farm-hand agreed to work for $250 per year and a horse worth $50; how much is due him if he leaves at the end of 8 months, if he has already received $50 and the horse? *Ans.* $100.

13. A village postmaster made requisition for the following stamps: 24 sheets of 1-cent, 89 of 2-cent, 20 of 5-cent, and 17 of 10-cent stamps; what was the value of the stamps, there being 100 in each sheet? *Ans.* $472.

14. A lady paid a store-bill of $784, giving 30 twenty-dollar bills, 10 ten-dollar bills, 4 one-dollar bills, and the

remainder in five-dollar bills; how many five-dollar bills did she use? *Ans.* 16.

15. A grain-dealer paid $402.80 for 760 bushels of corn; at what rate must he sell in order to gain 14 cents a bushel? *Ans.* 67 cents a bushel.

16. Two trains leave New York for Chicago, 900 miles, at the same hour, one averaging 30 miles an hour, the other 45 miles an hour; how long will the second train reach Chicago before the first? *Ans.* 10 hours.

17. If a newsboy buys papers at 18 cents a dozen, and sells them at 2 cents apiece, how much does he clear a year, 52 weeks, if he averages a sale of 72 papers a day, 6 days a week? *Ans.* $112.32.

18. John Smith went to Kentucky with $1850 to buy horses; he bought 25 horses, his expenses were $75, and he had remaining $150; what was the average cost of each horse? *Ans.* $65.

19. A dealer in live-stock bought 150 sheep for $1275, and sold them at $7.25 each; how many pigs must he sell, at a gain of $1.25 apiece, to make up his loss? *Ans.* 150.

20. If a newsboy buys papers at 8 cents a dozen, and sells them at a cent apiece, how much can he clear in March if he averages 120 papers a day, including Sundays? *Ans.* $12.40.

21. A boy's father gave him $600 for his first year at college (40 weeks); he spends $65 for books and stationery, $30 for travelling expenses, $150 for clothing, $25 for incidentals, and had $30 left at the end of the year; what did he pay for board and washing a week? *Ans.* $7.50.

22. Mr. Walton left by will $7500 to each of 3 sons, $6250 to each of two daughters, and the balance of his estate, $25,000, to a hospital; the will was set aside, however, and the property equally divided among his children; what did each receive? *Ans.* $12,000.

GENERAL PRINCIPLES
OF THE FUNDAMENTAL RULES.

PRINCIPLES OF ADDITION.

1. The sum of all the parts equals the whole.

2. The whole diminished by one or more parts equals the sum of the other parts.

PRINCIPLES OF SUBTRACTION.

1. The Remainder equals the Minuend minus the Subtrahend.

2. The Minuend equals the Subtrahend plus the Remainder.

3. The Subtrahend equals the Minuend minus the Remainder.

PRINCIPLES OF MULTIPLICATION.

1. The Product equals the Multiplicand into the Multiplier.

2. The Multiplicand equals the Product divided by the Multiplier.

3. The Multiplier equals the Product divided by the Multiplicand.

PRINCIPLES OF DIVISION.

1. The Quotient equals the Dividend divided by the Divisor.

2. The Dividend equals the Divisor multiplied by the Quotient.

3. The Divisor equals the Dividend divided by the Quotient.

4. The Dividend equals the Divisor multiplied by the Quotient plus the Remainder.

5. The Divisor equals the Dividend minus the Remainder divided by the Quotient.

OTHER PRINCIPLES OF DIVISION.

1. Multiplying the Dividend or dividing the Divisor by any number multiplies the Quotient by that number.

2. Dividing the Dividend or multiplying the Divisor by any number divides the Quotient by that number.

3. Multiplying or dividing both Dividend and Divisor by the same number does not change the Quotient.

4. *A General Law.*—A change in the Dividend by multiplication or division produces a *similar* change in the Quotient; but such a change in the Divisor produces an *opposite* change in the Quotient.

NOTE TO TEACHER.—Let the pupils be required to show the reason for the above principles, and give illustrations of them.

INTRODUCTION TO SECONDARY OPERATIONS.

ORAL EXERCISES.

1. What numbers multiplied together will produce 6? 8? 10? 12? 14? 16? 18? 20? 24? 26? 28? 30?

2. What numbers can be produced out of the numbers 2 and 3? 3 and 5? 2, 3, and 5? 3, 4, and 5? 2, 3, 4, and 5?

3. Will the product of any two numbers, each greater than a unit, produce 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and 37?

4. What may we call a number which is *composed* by multiplying several numbers together? *Ans.* A *Composite Number.*

5. What shall we call numbers that cannot be produced by multiplying several numbers together? *Ans. Prime Numbers.*

6. Which are prime and which composite numbers in the following list: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15?

7. What may we call the numbers whose product makes a composite number? *Ans. Makers* of the numbers.

8. If the word *Factor* means the same as *maker*, what may we call the *makers* of a composite number? *Ans.* The *Factors.*

9. Form composite numbers out of the factors 3 and 4; 3, 4, and 5; 4, 5, and 6.

10. What are the factors of 12? 15? 18? 20? 21? 24? 25? 27? 30? 32? 33? 35?

11. Form a composite number by using 2 twice as a factor; 3 twice as a factor; 2 three times as a factor; 3 four times as a factor.

12. Required *one* of the *two equal* factors of 9; of 16; of 25; of 36; *one* of the *three equal* factors of 8; of 27; of 64; of 125.

13. What would it seem natural to call the process of making composite numbers? *Ans. Composition.*

14. What would it seem natural to call the process of finding the factors of a number? *Ans. Factoring.*

15. What are the first four operations of arithmetic called?
Ans. The *Fundamental* or *Primary Operations* of arithmetic.

16. What would it be natural to call those operations which are derived from the fundamental operations? *Ans.* The *Derivative* or *Secondary Operations.*

SECTION III.

SECONDARY OPERATIONS.

86. The **Primary Operations** of arithmetic are those of synthesis and analysis, including the four fundamental rules.

87. The **Secondary, or Derivative Operations,** are those which arise from or grow out of the primary operations of synthesis and analysis.

88. The **Secondary Operations** are *Composition, Factoring, Greatest Common Divisor, Least Common Multiple, Involution,* and *Evolution.*

COMPOSITION.

89. **Composition** is the process of forming composite numbers when their factors are given.

90. A **Composite Number** is a number which can be produced by multiplying together two or more numbers, each greater than a unit; as 8, 12, 15, etc.

91. The **Factors** of a composite number are the numbers which, multiplied together, will produce it; thus, 4 and 2 are the factors of 8.

92. A **Prime Number** is one that cannot be produced by multiplying together two or more numbers, each greater than a unit; as, 2, 5, 7, 11, etc.

93. An **Even Number** is one that is exactly divisible by 2; as, 2, 4, 6, etc. An **Odd Number** is one that is not exactly divisible by 2; as, 1, 3, 5, etc.

94. An **Exact Divisor** of a number is a number that will divide it without a remainder.

ORAL AND WRITTEN EXERCISES.

1. Tell which of the following numbers are prime or composite: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16.

2. Name the composite numbers from 1 to 50.

3. Name the prime numbers from 1 to 50.

4. Write a list of the prime numbers from 47 to 103.

5. Write the numbers from 1 to 100, and cut out all the composite numbers, leaving the primes.

PRINCIPLES.

1. *Every composite number is equal to the product of its factors.*

2. *A factor of a number is a factor of any number of times that number.*

95. To form composite numbers out of their factors.

1. Form a composite number out of 3, 5, and 7.

SOLUTION.—A composite number formed out of the factors 3, 5, and 7 is equal to their product, which is $3 \times 5 \times 7 = 105$.

WRITTEN EXERCISES.

Form composite numbers out

2. Of 3, 4, 5, and 7. *Ans.* 420.

3. Of two 2's, 5, and 7. *Ans.* 140.

4. Of four 2's, three 3's, and two 5's. *Ans.* 10800.

5. Find a number whose factors are five 5's. *Ans.* 3125.

6. Form a composite number out of the first four prime numbers after unity. *Ans.* 210.

7. Form a composite number out of all the prime numbers between 6 and 20. *Ans.* 323323.

8. Form all the composite numbers you can out of 3, 5, and 7. *Ans.* 15; 21; 35; 105.

9. Form all the composite numbers you can out of 3, 5, 7, 11, and 13. *Ans.* 15; 21; 33; 39; 35; 55, etc.

10. Find a composite number consisting of three factors, the first being 3, the second being twice as great, and the third three times as great. *Ans.* 162.

DIVISIBILITY OF COMPOSITE NUMBERS.

96. Composite Numbers can be divided by the factors which produce them.

97. The Factors of many composite numbers may be seen by inspection from the following principles:

1. TWO *is an exact divisor of any even number.*

2. THREE *is an exact divisor of a number when the sum of the digits is divisible by* 3.

This may be shown by trying several numbers, and, seeing that it is true with these, we infer that it is true with all. A rigid demonstration is too difficult for this place.

3. FOUR *is an exact divisor of a number when the two right-hand terms are ciphers, or when the number they express is divisible by* 4.

Thus, $924 = 900 + 24$, or $9 \times 100 + 24$. Now, since 100 is divisible by 4, 9 times 100, or 900, is divisible by 4; and since 900 and 24 are both divisible by 4, their sum, or 924, is divisible.

4. FIVE *is an exact divisor of a number when its right-hand term is* 0 *or* 5.

When the unit figure is 0, the last partial dividend must be 0, 10, 20, 30, or 40, each of which is divisible by 5. When the unit figure is 5, the last partial dividend must be 5, 15, 25, 35, or 45, each of which is divisible by 5.

5. SIX *is an exact divisor of a number when it is even and the sum of the digits is divisible by* 3.

Since the number is even, it is divisible by 2, and since the sum of the digits is divisible by 3, the number is divisible by 3, and since it contains both 2 and 3, it will contain their product, 3×2, or 6.

6. EIGHT *is an exact divisor of a number when the three right-hand terms are ciphers, or when the number expressed by them is divisible by* 8.

Thus, $9432 = 9000 + 432 = 9 \times 1000 + 432$. Now, since 1000 is divisible by 8, 9 times 1000, or 9000, is divisible by 8; and since 9000 and 432 are both divisible by 8, their sum, or 9432, is divisible.

7. NINE *is an exact divisor of a number when the sum of the digits is divisible by* 9.

This may be shown by trying several numbers, and, seeing that it is true with these, we can infer that it is true with all. It may also be rigidly demonstrated.

1. *A number is divisible by* 7 *when the sum of the odd numerical periods, minus the sum of the even numerical periods, is divisible by* 7.
2. *A number is divisible by* 11 *when the difference between the sums of the digits in the odd places and in the even places is* 0, *or is divisible by* 11.
3. These two principles are rather curious than useful. For their demonstration see *Higher Arithmetic.*

EXAMPLES FOR PRACTICE.

1. Illustrate Prin. 2 with 321, 408, 612, 2040, 3456.

2. Illustrate Prin. 3 with 524, 632, 740, 2948, 5764.

3. Illustrate Prin. 4 with 270, 485, 870, 2675, 4360.

4. Illustrate Prin. 5 with 336, 444, 786, 3672, 4458.

5. Illustrate Prin. 6 with 3024, 7000, 6256, 7512, 7840.

6. Illustrate Prin. 7 with 414, 369, 729, 8190, 5679.

7. Show that 369, 408, and 708 are divisible by 3.

8. Show that 624, 632, and 840 are divisible by 4.

9. Show that 1632, 2128, and 2048 are divisible by 8.

10. Show that 657, 702, and 2583 are divisible by 9.

INTRODUCTION TO FACTORING.

ORAL EXERCISES.

1. Name some of the factors or makers of 10, 12, 15, 28, 36, 50, 72.

2. Name the prime numbers which are factors of 12, of 14, of 20, of 28, of 42, of 54, and of 66.

3. What shall we call the factors of numbers when these factors are prime numbers? *Ans.* The *Prime Factors.*

4. Name the prime factors of 10, of 12, of 14, of 21, of 24, of 30, of 32, of 36, of 40, of 45, of 50, of 60, and of 80.

5. Illustrate the principle that the factors of a number are *divisors* of the number.

6. How, then, can we find the factors of a number?

Ans. By finding the divisors of a number.

7. How can we find the divisors of a number?

Ans. By trial, aided by the principles of Art. 97.

8. What may we call that subject of arithmetic which treats of finding the factors of numbers? *Ans. Factoring.*

FACTORING.

98. Factoring is the process of finding the factors of composite numbers. Unity and the number itself are not regarded as factors.

99. The **Factors** of a composite number are the numbers which multiplied together will produce it. Thus, 3 and 4 are factors of 12.

100. The **Prime Factors** of a composite number are the prime numbers which multiplied together will produce it. Thus, 2, 2, and 3 are the prime factors of 12.

PRINCIPLES.

1. *A divisor of a number, excepting unity and itself, is a factor of that number.*

2. *A divisor of a factor of a number, excepting unity, is a factor of the number.*

3. *A number is divisible by its prime factors or by any product of them.*

4. *A number is divisible only by its prime factors or by some product of them, or by unity.*

CASE I.

101. To resolve a number into its prime factors.

1. Find the prime factors of 165.

Solution.—Dividing by 3, we find that 3 is a factor of 165 (Prin. 1). Dividing the quotient by 5, we find that 5 and 11 are factors of 55 (Prin. 2), and since these numbers, 3, 5, and 11, are prime numbers, they are the prime factors of 165.

OPERATION.

$$\begin{array}{r} 3\underline{)165} \\ 5\underline{)55} \\ 11 \end{array}$$

Rule.—I. *Divide the given number by any prime number greater than 1 that will exactly divide it.*

II. *Divide the quotient, if composite, in the same manner, and thus continue until the quotient is prime.*

III. *The divisors and last quotient will be the prime factors required.*

What are the prime factors

2. Of 36?	*Ans.* 2, 2, 3, 3.	**9.** Of 429?	*Ans.* 3, 11, 13.
3. Of 60?	*Ans.* 2, 2, 3, 5.	**10.** Of 660?	*Ans.* 3, 5, 11, etc.
4. Of 105?	*Ans.* 3, 5, 7.	**11.** Of 925?	*Ans.* 5, 5, 37.
5. Of 210?	*Ans.* 2, 3, 5, 7.	**12.** Of 1470?	*Ans.* 3, 5, 7, etc.
6. Of 252?	*Ans.* 2, 3, 7, etc.	**13.** Of 1452?	*Ans.* 2, 3, 11, etc.
7. Of 385?	*Ans.* 5, 7, 11.	**14.** Of 1584?	*Ans.* 2, 3, 11, etc.
8. Of 420?	*Ans.* 2, 3, 5, etc.	**15.** Of 2310?	*Ans.* 3, 7, 11, etc.

CASE II.

102. To resolve a number into equal factors.

1. Find the two equal factors of 484.

SOLUTION.—We first resolve the number into its prime factors. Now, since there are *two* 2's, we take *one* 2 for each factor; and since there are *two* 11's, we take *one* 11 for each factor; hence each of the two equal factors is 2 × 11, or 22. Therefore 22 is one of the two equal factors of 484.

OPERATION

$$
\begin{array}{r|l}
2 & 484 \\
2 & 242 \\
11 & 121 \\
 & 11
\end{array}
$$

$$2 \times 11 = 22$$

Rule.—I. *Resolve the number into its prime factors.*

II. *Take the continued product of one of each of the two equal factors when we wish the two equal factors, one of each of the three for the three equal factors, etc.*

2. Find one of the two equal factors of 64, 81, 144, 225, 576, 1296, 2704. *Ans.* 8; 9; 12; 15; 24; 36; 52.

3. Find one of the three equal factors of 27, 64, 125, 216, 512, 1000, 1728. *Ans.* 3; 4; 5; 6; 8; 10; 12.

4. Find one of the four equal factors of 16, 81, 256, 625, 1296, 4096, 20736. *Ans.* 2; 3; 4; 5; 6; 8; 12.

INTRODUCTION TO COMMON DIVISOR.

ORAL EXERCISES.

1. Name an exact divisor of 8; of 9; of 10; of 16; of 18; of 21; of 27; of 32.

2. What exact divisors are common to 6 and 9? to 10 and 15? to 15 and 18? to 24 and 30? to 32 and 40? to 48 and 72?

3. What may a divisor *common* to two or more numbers be called?
Ans. Their *Common Divisor.*

4. What is a common divisor of 18 and 24? of 16 and 20? of 24 and 80? of 48 and 56?

5. What is the greatest divisor common to 28 and 36? to 42 and 56? to 56 and 72? to 80 and 96?

6. What may the greatest divisor common to two or more numbers be called? *Ans.* Their *Greatest Common Divisor.*

7. What is the greatest common divisor of 25 and 30? of 45 and 50? of 64 and 80? of 72 and 96?

8. What prime factors are common to 16 and 24? 20 and 30? 28 and 35? 32 and 40?

9. The product of what two prime factors of 12 and 18 will divide both? of 20 and 30?

GREATEST COMMON DIVISOR.

103. A **Divisor** of a number is a number that exactly divides it.

Thus, 4 is a divisor of 20.

104. A **Common Divisor** of two or more numbers is a number that exactly divides each of them.

Thus, 4 is a common divisor of 16 and 20.

105. The **Greatest Common Divisor** of two or more numbers is the greatest number that exactly divides each of them.

Thus, 8 is the greatest common divisor of 16 and 24.

The greatest common divisor may be represented by the initials G. C. D.

PRINCIPLES.

1. *A common factor of two or more numbers is a factor of their greatest common divisor.*

Thus, 3, which is a factor of 12 and 18, is a factor of their G. C. D.

2. *The product of all the common prime factors of two or more numbers is their greatest common divisor.*

Thus, the common factors of 12 and 18 are 2 and 3; and 2×3, or 6, is their G. C. D.

3. *A common divisor of two numbers is a divisor of their sum and also of their difference.*

Thus, take any two numbers, as 12 and 20, of which 4 is a common divisor. Now, 12 equals *three* times 4, and 20 equals *five* times 4, and their sum equals *three* times 4 plus *five* times 4, or *eight* times 4, which contains 4, or is divisible by 4. Their difference is *five* times 4 minus *three* times 4, or *two* times 4, which is also divisible by 4.

CASE I.

106. When the numbers are small and can be readily factored.

1. Find the greatest common divisor of 42, 84, and 126.

SOLUTION.—The factors of 42 are 2, 3, and 7; the factors of 84 are 2, 2, 3, and 7; the factors of 126 are 2, 3, 3, and 7. We see that 2, 3, and 7 are all the prime factors common to the three numbers; hence their product, which is 42, is the greatest common divisor of the numbers (Prin. 2). Hence the following

OPERATION.

$$42 = 2 \times 3 \times 7$$
$$84 = 2 \times 2 \times 3 \times 7$$
$$126 = 2 \times 3 \times 3 \times 7$$
$$2 \times 3 \times 7 = 42$$

Rule.—*Resolve the numbers into their prime factors, and take the product of all the common factors.*

NOTE.—For another method, see Supplement.

WRITTEN EXERCISES.

What is the greatest common divisor of

2. 21, 35, 56?	*Ans.* 7.	5. 42, 56, 168?	*Ans.* 14.
3. 36, 48, 84?	*Ans.* 12.	6. 63, 105, 315?	*Ans.* 21.
4. 32, 48, 128?	*Ans.* 16.	7. 90, 120, 210?	*Ans.* 30.

8. 182, 196, 294? *Ans.* 14.	**11.** 216, 360, 1296? *Ans.* 72.
9. 225, 375, 500? *Ans.* 25.	**12.** 330, 495, 660? *Ans.* 165.
10. 210, 315, 420? *Ans.* 105.	**13.** 1344, 1536, 1728? *Ans.* 192.

CASE II.

107. When the numbers are large and cannot be readily factored.

1. Find the greatest common divisor of 32 and 88.

Solution.—We divide 88 by 32, the divisor 32 by 24, and the divisor 24 by the remainder 8, and have no remainder; then will 8 be the greatest common divisor of 32 and 88.

For, since 32 and 88 are each a number of times the G. C. D., their difference, 24, is a number of times the G. C. D., by Prin. 3; and since 24 and 32 are each a number of times the G. C. D., their difference, 8, is also a *number of times* the G. C. D.

Since 8 divides 24, it will divide 24+8, or 32 (Prin. 3); and since it divides 32 and 24, it will divide $2\times32+24$, or 88.

Now, since 8 divides 32 and 88, and is a *number of times the G. C. D.*, it must be *once* the G. C. D.

OPERATION.

```
32)88(2
   64
   24)32(1
      24
       8)24(3
         24
```

SHORTER FORM.

32	88	2
24	64	1
8	24	3
	24	

Rule.—*Divide the greater number by the less, the divisor by the remainder, and thus continue to divide the last divisor by the last remainder until there is no remainder. The last divisor will be the greatest common divisor.*

When there are more than two numbers, find the greatest common divisor of two of them, then of that divisor and one of the other numbers, etc.

WRITTEN EXERCISES.

Find the greatest common divisor of

2. 91 and 143. *Ans.* 13.	**6.** 333 and 592. *Ans.* 37.
3. 115 and 161. *Ans.* 23.	**7.** 423 and 752. *Ans.* 47.
4. 119 and 187. *Ans.* 17.	**8.** 697 and 820. *Ans.* 41.
5. 126 and 231. *Ans.* 21.	**9.** 901 and 1060. *Ans.* 53.

PRACTICAL EXAMPLES.

1. What is the length of the longest pole with which you can measure the three lengths, 132, 156, and 168 feet? *Ans.* 12 feet.

2. A merchant has three pieces of cloth measuring respectively 48, 56, and 80 yards, and wishes to cut them into pieces of equal lengths as long as possible; how many yards will each contain? *Ans.* 8 yards.

3. On a newly laid out street A owns 96 yards front, B 120 yards, and C 168 yards; they agree to divide their land into lots of equal sizes; what will be the width of the largest lots into which their land can be divided? *Ans.* 24 yards.

4. Robert Thomas has 453 bushels of wheat, 315 bushels of oats, and 270 bushels of rye which he wishes to send to market in bags of equal size; what is the capacity of the largest bag he can use, and how many bags will he require? *Ans.* 3 bu.; 346 bags.

5. A benevolent society distributed $840, $968, and $1120 in equal sums to the poor of three wards of a city, the equal sums being as large as possible; required the amount of the equal sums, and the number receiving relief in each ward. *Ans.* $8; 105; 121; 140.

INTRODUCTION TO COMMON MULTIPLE.

ORAL EXERCISES.

1. What number is three times 4? four times 5? five times 6? six times 7?

2. A number which is one or more times another number is called a *multiple* of that number.

3. What is a multiple of 4? of 5? of 6? of 7? of 8? of 9? of 10? of 11? of 12?

4. What multiple is common to 2 and 3? to 3 and 4? to 4 and 6? to 6 and 8? to 6 and 9?

5. What may we call a multiple common to two or more numbers? *Ans.* A *Common Multiple.*

6. What is a common multiple of 3 and 4? of 4 and 8? of 6 and 9; of 6 and 8? of 8 and 10?

7. What is the least multiple common to 2 and 4? to 4 and 6? to 4 and 8? to 6 and 9? to 8 and 12?

8. What may we call the least multiple common to two or more numbers? *Ans.* Their *Least Common Multiple.*

9. What is the least common multiple of 4 and 6? 8 and 12? 12 and 16? 20 and 30? 25 and 30?

LEAST COMMON MULTIPLE.

108. A **Multiple** of a number is one or more times the number. Thus, 4 times 5, or 20, is a multiple of 5.

109. A **Common Multiple** of two or more numbers is a number which is a multiple of each of them. Thus, 24 is a common multiple of 2, 3, and 4.

110. The **Least Common Multiple** of two or more numbers is the least number which is a multiple of each of them. Thus, 12 is the least common multiple of 2, 3, and 4.

The least common multiple may be represented by the initials L. C. M.

PRINCIPLES.

1. *A multiple of a number is exactly divisible by that number.*

Thus, 24, which is 4 times 6, is exactly divisible by 6.

2. *A multiple of a number must contain all the prime factors of that number.*

Thus, 24, which contains 6, must contain 2 and 3, the two factors of 6.

3. *A common multiple of two or more numbers must contain all the prime factors of each of those numbers.*

Thus, 24, which is a common multiple of 4 and 6, must contain the factors 2 and 2 of 4, and the factors 2 and 3 of 6.

4. *The least common multiple of two or more numbers must contain all the prime factors of each number, and no other factors.*

Thus, 30, the least common multiple of 6 and 15, contains 2 and 3, the factors of 6, and 3 and 5, the factors of 15, and no other factors.

CASE I.

111. When the numbers are small and easily factored.

1. Find the least common multiple of 12, 30, and 70.

SOLUTION.—We first resolve the numbers into their prime factors. A multiple of 12 must contain 2, 2, 3, the factors of 12; a multiple of 30 must contain 2, 3, 5, the factors of 30; a multiple of 70 must contain 2, 5, 7, the factors of 70; hence the common multiple of 12, 30, and 70	OPERATION. $12 = 2 \times 2 \times 3$ $30 = 2 \times 3 \times 5$ $70 = 2 \times 5 \times 7$ $2 \times 2 \times 3 \times 5 \times 7 = 420$

must contain all these different factors and no others; therefore $2 \times 2 \times 3 \times 5 \times 7$, or 420, is the L. C. M. of 12, 30, and 70 (Prin. 4). Hence the following

Rule.—I. *Resolve the numbers into their prime factors.*

II. *Take the product of all the different factors, using each factor the greatest number of times it occurs in either number.*

1. Any numbers which are divisors of the others may be omitted, since the multiple of the other numbers will be a multiple of these.
2. For another method, see Supplement.

WRITTEN EXERCISES.

Find the least common multiple of

2. 16, 24, 36.	*Ans.* 144.	**7.** 30, 32, 36.	*Ans.* 1440.
3. 15, 45, 60.	*Ans.* 180.	**8.** 56, 72, 96.	*Ans.* 2016.
4. 45, 30, 72.	*Ans.* 360.	**9.** 26, 39, 52, 65.	*Ans.* 780.
5. 20, 24, 27.	*Ans.* 1080.	**10.** 18, 26, 117, 312.	*Ans.* 936.
6. 35, 40, 42.	*Ans.* 840.	**11.** 32, 36, 49, 56, 128.	*Ans.* 56448.

CASE II.

112. When the numbers are large and cannot be readily factored.

1. Find the least common multiple of 112 and 133.

SOLUTION.—The greatest common divisor of these numbers is 7; 112 equals 16 times 7, and 133 equals 19 times 7; hence the L. C. M., as found in the first	OPERATION. $112 = 16 \times 7$; $133 = 19 \times 7$ L. C. M. $= 16 \times 7 \times 19$ $= 112 \times \frac{133}{7}$

method, is $16 \times 7 \times 19$, which equals 112 multiplied by 133 divided by 7, or *the first number multiplied by the second divided by their greatest common divisor.*

Rule.—*Find the greatest common divisor of two numbers; divide one number by it, and multiply the other number by the quotient.*

NOTE.—When there are more than two numbers, find the least common multiple of two of them, then of this multiple and the third number, etc.

WRITTEN EXERCISES.

Find the least common multiple of

2. 91 and 117. *Ans.* 819.
3. 135 and 144. *Ans.* 2160.
4. 169 and 221. *Ans.* 2873.
5. 357 and 612. *Ans.* 4284.

PRACTICAL EXAMPLES.

1. What is the smallest vessel that can be filled by using either a 3-quart measure, a 4-quart measure, a 5-quart measure, or a 6-quart measure? *Ans.* 60 quarts.

2. A silk manufacturer wished to make his goods so that they might cut without waste into dress-patterns of either 12, 15, 20, or 30 yards; what is the shortest length he can make a piece of silk? *Ans.* 60 yards.

3. What is the smallest number of bushels of potatoes that will exactly fill either barrels containing 3 bushels each, sacks containing 4 bushels each, boxes containing 15 bushels each, or bins containing 56 bushels each? *Ans.* 840 bushels.

CANCELLATION.

113. **Cancellation** is a process of abbreviating arithmetical operations by rejecting equal factors in both dividend and divisor.

ORAL AND WRITTEN EXERCISES.

1. If we omit the factor 2 from 12 and 6, what factors will remain?

2. Divide 24 by 6. Divide 24 by ½ of 6. Divide ½ of 24 by 6.

3. Divide ½ of 24 by ½ of 6. Divide 36 by 18, first taking out the common factor 6.

4. Is there any difference in the quotient of 48 divided by 12, and ¼ of 48 divided by ¼ of 12?

5. Divide 72 by 48, first omitting common factors. Divide 90 by 60, first omitting common factors.

6. Divide $2\times2\times2$ by 2×2; $3\times3\times4$ by 3×3; $3\times4\times5$ by 3×5.

7. Divide $2\times3\times7$ by 2×7; $2\times3\times4$ by 2×3; $3\times5\times8$ by 3×8; $6\times5\times3$ by 3×6; $2\times7\times9\times10$ by 9×2.

PRINCIPLES.

1. *The cancelling of a factor from any number divides the number by that factor.*

Thus, if we take the factor 3 out of 24, we shall divide 24 by 3.

2. *The cancelling of a factor in both dividend and divisor will not change the quotient.*

Cancelling a factor in both dividend and divisor is the same as dividing them both by the same number, which, by the principles of division, does not change the quotient.

WRITTEN EXERCISES.

1. Divide 66×63 by 27×44.

SOLUTION.—We cancel the common factor 22 from 66 and 44, writing 3, the other factor of 66, above 66, and 2, the other factor of 44, below 44; we then cancel 3 and the 3 in 27, writing 9, the other factor of 27, below 27; we then cancel 9 and the 9 in 63, writing 7 above 63; the product of the remaining factors of the dividend is 7, the product of the remaining factors of the divisor is 2; hence the quotient is 7 divided by 2, or $3\frac{1}{2}$.

OPERATION.

$$\frac{\overset{\cancel{3}}{\cancel{66}}\times\overset{7}{\cancel{63}}}{\underset{\cancel{9}}{\cancel{27}}\times\underset{2}{\cancel{44}}}=\frac{7}{2}=3\tfrac{1}{2}$$

Rule.—I. *Cancel the common factors from the dividend and the divisor.*

II. *Then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

1. The unit 1 takes the place of a cancelled factor, but need not be written, except in the dividend of the quotient when there are no other factors of the dividend remaining.

2. A factor in one term will cancel two or more factors in the other term when their product is equal to the former.

2. Divide $14\times16\times27$ by $7\times8\times9$. *Ans.* 12.

3. Divide $18\times24\times35$ by $6\times8\times7$. *Ans.* 45.

4. Divide $20 \times 35 \times 225$ by $14 \times 15 \times 50$. *Ans.* 15.

5. Divide $48 \times 40 \times 80$ by $12 \times 16 \times 32$. *Ans.* 25.

6. Divide $52 \times 75 \times 196$ by $12 \times 28 \times 25$. *Ans.* 91.

7. Divide $36 \times 75 \times 80$ by $30 \times 40 \times 18$. *Ans.* 10.

8. Divide $49 \times 81 \times 91$ by $35 \times 63 \times 39$. *Ans.* $4\frac{1}{5}$.

9. Divide $56 \times 77 \times 160$ by $49 \times 25 \times 33$. *Ans.* $17\frac{1}{15}$.

10. Divide $150 \times 84 \times 99$ by $75 \times 91 \times 121$. *Ans.* $1\frac{73}{143}$.

11. Divide $70 \times 98 \times 231$ by $77 \times 105 \times 196$. *Ans.* 1.

PRACTICAL EXAMPLES.

1. How many books, at 15 cents apiece, may be exchanged for 12 reams of paper, at 55 cents a ream?

SOLUTION.—If 1 ream of paper is worth 55 cents, 12 reams are worth 12×55 cents; for 12×55 cents, at 15 cents apiece, we can get as many books as 15 is contained times in 12×55, which we find, by cancellation, to be 44.

OPERATION.

$$\frac{\overset{4}{\cancel{12}} \times \overset{11}{\cancel{55}}}{\underset{\cancel{5}}{\cancel{15}}} = 44$$

2. A lady exchanged cloaks, at \$15 each, for 18 jackets, at \$5 each; how many jackets did she get? *Ans.* 6.

3. How many pairs of shoes, at \$2.50 a pair, can be bought for 30 pounds of leather, at 75 cents a pound? *Ans.* 9.

4. How many barrels of sugar of 240 pounds each, at 5 cents a pound, can be exchanged for 8 pieces of sheeting of 45 yards each, at 10 cents a yard? *Ans.* 3.

5. A farmer exchanged 12 firkins of butter, each containing 56 pounds, at 25 cents a pound, for 28 boxes of soap, each box containing 50 pounds; what was the price of the soap per pound? *Ans.* 12 cents.

6. If 25 Jersey cows each give 8 quarts of milk a day, at 5 cents a quart, how many pieces of matting of 40 yards each, at 50 cents a yard, will pay for the milk of 12 days? *Ans.* 6.

INTRODUCTION TO FRACTIONS.

ORAL EXERCISES.

1. If we divide an apple into two equal parts, what is one of these parts called? What are two of these parts called?

2. If we divide an apple into 3 equal parts, what is one of these parts called? If into 4 equal parts? 5 equal parts? etc.

3. How many fourths make a whole? How many fifths make a whole? Sixths? Sevenths? Eighths? Ninths? Tenths?

4. How many thirds in *one* apple? In *two* apples? How many 4ths in 1 apple? In 2 apples?

5. To get 4 thirds, how many apples must we cut into thirds? To get 6 fourths, how many apples must we cut into fourths?

6. To fracture is to break or divide into equal parts. These equal parts are called *Fractions*. What, then, is a fraction?

7. The number of equal parts into which a unit may be divided is represented by a figure below the line; thus, $\frac{\ }{2}$ represents *halves*; $\frac{\ }{3}$, *thirds*; $\frac{\ }{4}$, *fourths*, etc.

8. The number of fractional parts considered is represented by a figure above the line; thus, $\frac{2}{3}$ represents 2 *thirds*; $\frac{3}{4}$, 3 *fourths*.

9. The number written below the line is called the *denominator* of the *fraction*, because it gives the *name* to the parts.

10. The number written above the line is called the *numerator*, because it *numbers* the parts.

11. Name the numerator and denominator in the following fractions: $\frac{3}{4}$, $\frac{4}{5}$, $\frac{2}{6}$, $\frac{7}{8}$, $\frac{5}{7}$, $\frac{8}{9}$, $\frac{11}{12}$, $\frac{12}{13}$.

12. Fractions that are greater than a unit are called *improper fractions*, because they were not thought to be properly fractions.

13. Fractions less than a unit are called *proper fractions*, because they are properly a part of a unit.

14. Tell which are proper and which are improper fractions in the following: $\frac{3}{4}$, $\frac{5}{4}$, $\frac{4}{5}$, $\frac{7}{5}$, $\frac{8}{6}$, $\frac{5}{7}$, $\frac{4}{8}$, $\frac{11}{7}$, $\frac{7}{11}$.

15. If we unite, or "mix together," an integer and a fraction, as 4 and $\frac{2}{3}$—thus, $4\frac{2}{3}$—we have what is called a *mixed number*.

16. If we combine one fraction with another, forming "a compound" of two or more fractions—as, $\frac{1}{2}$ of $\frac{2}{3}$—we have what is called a *compound fraction*.

SECTION IV.

COMMON FRACTIONS.

114. A **Fraction** is a number of the equal parts of a unit.

115. A **Common Fraction** is one in which the unit is divided into *any number* of equal parts.

116. A **Common Fraction** is expressed by two figures, one written above the other, with a short line between them. Thus, $\frac{3}{4}$ expresses 3 *fourths*.

117. The **Denominator** of a fraction denotes the number of equal parts into which the unit is divided.

118. The **Numerator** of a fraction denotes the number of equal parts which are taken.

119. The *Numerator* is written above the line, and the *Denominator* below it. The numerator and denominator are called the *Terms* of the fraction.

Thus, in the fraction $\frac{7}{8}$, the 8 is the denominator; it denotes that the unit is divided into 8 equal parts. The 7 is the numerator; it denotes that 7 of the 8 equal parts are taken.

120. A **Simple Fraction** is a fraction having a single integral numerator and denominator; as, $\frac{2}{3}$, $\frac{4}{5}$, etc.

121. A **Proper Fraction** is a simple fraction whose value is less than a unit; as, $\frac{2}{3}$, $\frac{3}{4}$.

122. An **Improper Fraction** is a simple fraction whose value is equal to or greater than a unit; as, $\frac{5}{5}$, $\frac{7}{6}$, $\frac{12}{8}$, etc.

123. A **Compound Fraction** is a fraction of a fraction; as, $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{7}{8}$, etc.

124. An **Integer** is a number of entire units. The word *Number*, as generally used, means an integer.

125. A **Mixed Number** consists of an integer and a fraction; as, $4\frac{2}{3}$, $5\frac{3}{4}$, etc.

126. The **Reciprocal** of a number is a unit divided by that number; thus, the reciprocal of 3 is $\frac{1}{3}$.

127. The **Number of Cases** of common fractions here treated is six. They are as follows:

1. Reduction.	**4.** Multiplication.
2. Addition.	**5.** Division.
3. Subtraction.	**6.** Relation of Fractions.

Note.—Each fractional part is used as a single thing, and is therefore a unit; hence, we have *Units* and *Fractional units*.

NUMERATION AND NOTATION.

128. **Numeration of Fractions** is the art of reading a fraction when expressed by figures.

Rule.—*Read the number of fractional units expressed by the numerator, and give them the name indicated by the denominator.*

Name the kind and read the following:

1. $\frac{3}{4}$.	**4.** $\frac{8}{9}$.	**7.** $2\frac{1}{7}$.	**10.** $\frac{2}{3}$ of $\frac{3}{4}$.	**13.** $\frac{18}{3}$.
2. $\frac{5}{6}$.	**5.** $\frac{7}{2}$.	**8.** $5\frac{2}{3}$.	**11.** $\frac{5}{6}$ of $\frac{9}{10}$.	**14.** $\frac{5}{12}$ of $\frac{7}{8}$ of $\frac{3}{4}$.
3. $\frac{7}{4}$.	**6.** $\frac{15}{16}$.	**9.** $\frac{18}{18}$.	**12.** $\frac{2}{7}$ of $3\frac{1}{2}$.	**15.** $\frac{2}{3}$ of $\frac{6}{7}$ of $4\frac{1}{2}$.

129. **Notation of Fractions** is the art of expressing fractions by means of figures.

Rule.—*Write the number of fractional units, draw a line beneath, under which write the number which indicates the kind of fractional units.*

Write the following fractions:

1. Three-fourths.	**5.** Eight-twelfths.
2. Five-sevenths.	**6.** Nine-thirteenths.
3. Seven-eighths.	**7.** Eight-fifteenths.
4. Ten-elevenths.	**8.** Eleven-twentieths.

130. To **Analyze** a fraction is to explain what is expressed by the fractional notation.

1. Explain the fractional expression $\frac{3}{4}$.

Solution.—In the fraction $\frac{3}{4}$ the denominator 4 indicates that the unit is divided into 4 equal parts, and the numerator 3 denotes that 3 of these parts are taken.

Explain the following fractions:

2. $\frac{4}{5}$.	4. $\frac{8}{9}$.	6. $\frac{14}{15}$.	8. $\frac{17}{12}$.
3. $\frac{6}{7}$.	5. $\frac{9}{11}$.	7. $\frac{7}{10}$.	9. $\frac{19}{20}$.

131. Fractions may be treated in two ways: 1st, by analyzing problems and inferring principles or rules; 2d, by establishing some general principles and deriving the rules from these principles.

In the Oral Exercises the first method is used; in the Written Exercises the second method is often used, based on the principles derived by the Oral Exercises.

REDUCTION OF FRACTIONS.

132. The **Reduction of Fractions** is the process of changing their form without altering their value.

133. There are **Six Cases** of reduction:

1st. Numbers to fractions.	4th. To lower terms.
2d. Fractions to numbers.	5th. Compound to simple.
3d. To higher terms.	6th. Complex to simple.

Note.—Reducing to a *Common Denominator* is included in these six cases.

CASE I.

134. To reduce whole or mixed numbers to improper fractions.

ORAL EXERCISES.

1. How many thirds are there in $5\frac{2}{3}$?

Solution.—In *one* there are 3 *thirds*, and in 5 there are 5 times 3 thirds, or 15 thirds, which, added to 2 thirds, equal 17 thirds; therefore $5\frac{2}{3} = \frac{17}{3}$.

2. How many fourths in $5\frac{1}{4}$? in $7\frac{3}{4}$? in $8\frac{1}{4}$? in $11\frac{3}{4}$?

3. How many fifths in $4\frac{3}{5}$? in $8\frac{1}{5}$? in $9\frac{2}{5}$? in $12\frac{4}{5}$?

4. How many sixths in $3\frac{5}{6}$? in $7\frac{2}{6}$? in $9\frac{1}{6}$? in $11\frac{5}{6}$?

5. How many sevenths in $3\frac{4}{7}$? in $8\frac{1}{7}$? in $6\frac{5}{7}$? in $10\frac{3}{7}$?

6. Describe the operation we perform in reducing a mixed number to a fraction.

WRITTEN EXERCISES.

1. Reduce $18\frac{7}{8}$ to eighths.

OPERATION.

$$\begin{array}{l} 18\frac{7}{8} \\ \underline{8\phantom{\frac{7}{8}}} \\ \frac{144}{8}+\frac{7}{8}=\frac{151}{8} \end{array}$$

SOLUTION.—In *one* there are 8 eighths, and in 18 there are 18 times 8 *eighths*, or $\frac{144}{8}$; $\frac{144}{8}$ plus $\frac{7}{8}$ equals $\frac{151}{8}$. Therefore, etc. Hence the following

Rule.—*Multiply the integer by the denominator, add the numerator to the product, and write the denominator under the sum.*

Reduce to improper fractions:

2. $16\frac{3}{4}$. *Ans.* $\frac{67}{4}$.	**7.** $18\frac{5}{9}$. *Ans.* $\frac{167}{9}$.	**12.** $62\frac{13}{18}$. *Ans.* $\frac{1129}{18}$.
3. $18\frac{4}{5}$. *Ans.* $\frac{94}{5}$.	**8.** $41\frac{7}{8}$. *Ans.* $\frac{335}{8}$.	**13.** $76\frac{19}{20}$. *Ans.* $\frac{1539}{20}$.
4. $20\frac{5}{6}$. *Ans.* $\frac{125}{6}$.	**9.** $35\frac{11}{12}$. *Ans.* $\frac{431}{12}$.	**14.** $85\frac{39}{40}$. *Ans.* $\frac{3439}{40}$.
5. $24\frac{4}{7}$. *Ans.* $\frac{172}{7}$.	**10.** $45\frac{12}{13}$. *Ans.* $\frac{597}{13}$.	**15.** $49\frac{49}{50}$. *Ans.* $\frac{2499}{50}$.
6. $35\frac{6}{11}$. *Ans.* $\frac{391}{11}$.	**11.** $51\frac{7}{15}$. *Ans.* $\frac{772}{15}$.	**16.** $235\frac{42}{65}$ *Ans.* $\frac{15317}{65}$.

CASE II.

135. To reduce improper fractions to whole or mixed numbers.

ORAL EXERCISES.

1. How many units in $\frac{25}{4}$?

SOLUTION.—In *one* there are $\frac{4}{4}$, hence in 25 fourths there are as many ones as 4 is contained times in 25, which is $6\frac{1}{4}$. Therefore $\frac{25}{4}=6\frac{1}{4}$.

How many units

2. In $\frac{14}{3}$?	**5.** In $\frac{16}{3}$?	**8.** In $\frac{40}{9}$?	**11.** In $\frac{27}{15}$?
3. In $\frac{17}{4}$?	**6.** In $\frac{23}{4}$?	**9.** In $\frac{42}{12}$?	**12.** In $\frac{32}{16}$?
4. In $\frac{27}{5}$?	**7.** In $\frac{27}{6}$?	**10.** In $\frac{44}{11}$?	**13.** In $\frac{54}{25}$?

14. Describe the operation in reducing an improper fraction to a mixed number.

WRITTEN EXERCISES.

1. How many units in $\frac{89}{6}$?

SOLUTION.—In a unit there are $\frac{6}{6}$, hence in $\frac{89}{6}$ there are as many units as $\frac{6}{6}$ are contained times in $\frac{89}{6}$, or as 6 is contained times in 89, which are $14\frac{5}{6}$ times. Hence the following

OPERATION.

$$\frac{89}{6} = 14\frac{5}{6}$$

Rule.—*Divide the numerator by the denominator, and the quotient will be the whole or mixed number.*

Reduce to whole or mixed numbers

2. $\frac{75}{3}$. *Ans.* 25.	7. $\frac{285}{15}$. *Ans.* 19.	12. $\frac{825}{35}$. *Ans.* $23\frac{20}{35}$.
3. $\frac{94}{5}$. *Ans.* $18\frac{4}{5}$.	8. $\frac{384}{17}$. *Ans.* $22\frac{10}{17}$.	13. $\frac{477}{30}$. *Ans.* $15\frac{27}{30}$.
4. $\frac{125}{6}$. *Ans.* $20\frac{5}{6}$.	9. $\frac{976}{19}$. *Ans.* $51\frac{7}{19}$.	14. $\frac{736}{42}$. *Ans.* $17\frac{22}{42}$.
5. $\frac{172}{7}$. *Ans.* $24\frac{4}{7}$.	10. $\frac{1309}{16}$. *Ans.* $81\frac{13}{16}$.	15. $\frac{997}{48}$. *Ans.* $20\frac{37}{48}$.
6. $\frac{391}{11}$. *Ans.* $35\frac{6}{11}$.	11. $\frac{1500}{75}$. *Ans.* 20.	16. $\frac{6789}{246}$. *Ans.* $27\frac{147}{246}$.

CASE III.

136. **To reduce fractions to higher terms.**

137. **Reducing a fraction to higher terms** is the process of changing it to an equal fraction having a larger numerator and denominator.

ORAL EXERCISES.

1. How many 12ths are there in $\frac{3}{4}$?

SOLUTION.—In *one* there are $\frac{12}{12}$, and in $\frac{1}{4}$ there are $\frac{1}{4}$ of $\frac{12}{12}$, which are $\frac{3}{12}$; and in $\frac{3}{4}$ there are 3 times $\frac{3}{12}$, which are $\frac{9}{12}$. Therefore $\frac{3}{4} = \frac{9}{12}$.

Reduce

2. $\frac{4}{5}$ to 10ths.	6. $\frac{4}{5}$ to 20ths.	10. $\frac{5}{6}$ to 48ths.
3. $\frac{5}{6}$ to 12ths.	7. $\frac{5}{6}$ to 30ths.	11. $\frac{6}{7}$ to 70ths.
4. $\frac{6}{7}$ to 14ths.	8. $\frac{6}{7}$ to 28ths.	12. $\frac{9}{10}$ to 60ths.
5. $\frac{7}{8}$ to 16ths.	9. $\frac{7}{8}$ to 24ths.	13. $\frac{11}{12}$ to 48ths.

14. Describe the operation of reducing a fraction to higher terms.

From the above analysis we derive the following principle:

Principle 1.—*Multiplying both terms of a fraction by the same number does not change the value of the fraction.*

WRITTEN EXERCISES.

1. How many fifteenths in $\frac{4}{5}$?

SOLUTION.—Since multiplying both terms of a fraction by the same number does not change its value (Prin. 1), we multiply both terms of $\frac{4}{5}$ by the number which will give the required denominator, which we see, by dividing 15 by 5, is 3; hence, $\frac{4}{5} = \frac{12}{15}$. Hence the following

OPERATION.

$15 \div 5 = 3$

$\frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$

Rule.—*Multiply both numerator and denominator by the number which will give the required denominator.*

Reduce

2. $\frac{7}{8}$ to 56ths.	**7.** $\frac{13}{14}$ to 126ths.	**12.** $\frac{17}{18}$ to 270ths.
3. $\frac{7}{9}$ to 99ths.	**8.** $\frac{15}{16}$ to 240ths.	**13.** $\frac{21}{22}$ to 308ths.
4. $\frac{11}{12}$ to 108ths.	**9.** $\frac{14}{17}$ to 255ths.	**14.** $\frac{23}{24}$ to 408ths.
5. $\frac{12}{13}$ to 117ths.	**10.** $\frac{13}{18}$ to 216ths.	**15.** $\frac{25}{26}$ to 520ths.
6. $\frac{12}{13}$ to 156ths.	**11.** $\frac{16}{19}$ to 247ths.	**16.** $\frac{34}{41}$ to 820ths.

CASE IV.

138. To reduce fractions to lower terms.

139. Reducing a fraction to lower terms is the process of changing it to an equal fraction having a smaller numerator and denominator.

A fraction is in its lowest terms when the numerator and denominator are prime to each other.

ORAL EXERCISES.

1. How many thirds are there in $\frac{8}{12}$?

SOLUTION.—*One* equals $\frac{12}{12}$, and $\frac{1}{3}$ equals $\frac{1}{3}$ of $\frac{12}{12}$, or $\frac{4}{12}$; since $\frac{4}{12}$ equals $\frac{1}{3}$, $\frac{8}{12}$ equals as many thirds as 4 is contained times in 8, which is 2; hence, $\frac{8}{12}$ equals $\frac{2}{3}$.

Reduce

2. $\frac{9}{12}$ to 4ths.	**5.** $\frac{18}{24}$ to 8ths.	**8.** $\frac{16}{56}$ to 7ths.
3. $\frac{12}{20}$ to 5ths.	**6.** $\frac{24}{30}$ to 10ths.	**9.** $\frac{48}{64}$ to 8ths.
4. $\frac{25}{30}$ to 6ths.	**7.** $\frac{16}{36}$ to 9ths.	**10.** $\frac{84}{108}$ to 9ths.

11. Describe the operation we perform in reducing a fraction to lower terms. From the above analysis we derive the following principle:

Principle 2.—*Dividing both terms of a fraction by the same number does not change the value of the fraction.*

WRITTEN EXERCISES.

1. Reduce $\frac{18}{30}$ to fifths.

SOLUTION.—Since dividing both terms of a fraction by the same number does not change its value (Prin. 2), we may reduce $\frac{18}{30}$ to lower terms by dividing both numerator and denominator by 6; dividing, we have $\frac{18}{30}$ equal to $\frac{3}{5}$; and since the terms 3 and 5 are prime to each other, the fraction is in its lowest terms. Hence the following

OPERATION.

$$\frac{18}{30} = \frac{18 \div 6}{30 \div 6} = \frac{3}{5}$$

Rule.—*Divide both terms successively by their common factors.*

Or, divide both terms by their greatest common divisor.

NOTE.—When the factors are not readily seen, use the second rule.

Reduce the following fractions to lowest terms:

2. $\frac{12}{18}$, $\frac{15}{21}$.	*Ans.* $\frac{2}{3}$, $\frac{5}{7}$.	**10.** $\frac{192}{216}$, $\frac{792}{864}$.	*Ans.* $\frac{8}{9}$, $\frac{11}{12}$.
3. $\frac{18}{24}$, $\frac{16}{20}$.	*Ans.* $\frac{3}{4}$, $\frac{4}{5}$.	**11.** $\frac{650}{760}$, $\frac{1760}{1920}$.	*Ans.* $\frac{5}{6}$, $\frac{11}{12}$.
4. $\frac{24}{36}$, $\frac{36}{48}$.	*Ans.* $\frac{2}{3}$, $\frac{3}{4}$.	**12.** $\frac{288}{864}$, $\frac{648}{720}$.	*Ans.* $\frac{1}{3}$, $\frac{9}{10}$.
5. $\frac{42}{63}$, $\frac{36}{45}$.	*Ans.* $\frac{2}{3}$, $\frac{4}{5}$.	**13.** $\frac{288}{1024}$, $\frac{864}{1296}$.	*Ans.* $\frac{9}{32}$, $\frac{2}{3}$.
6. $\frac{84}{105}$, $\frac{150}{180}$.	*Ans.* $\frac{4}{5}$, $\frac{5}{6}$.	**14.** $\frac{726}{792}$, $\frac{1680}{4312}$.	*Ans.* $\frac{11}{12}$, $\frac{30}{77}$.
7. $\frac{210}{252}$, $\frac{462}{504}$.	*Ans.* $\frac{5}{6}$, $\frac{11}{12}$.	**15.** $\frac{1694}{1848}$, $\frac{9660}{11040}$.	*Ans.* $\frac{11}{12}$, $\frac{7}{8}$.
8. $\frac{240}{288}$, $\frac{324}{356}$.	*Ans.* $\frac{5}{6}$, $\frac{81}{89}$.	**16.** $\frac{3003}{3696}$, $\frac{10605}{11445}$.	*Ans.* $\frac{13}{16}$, $\frac{101}{109}$.
9. $\frac{336}{378}$, $\frac{525}{735}$.	*Ans.* $\frac{8}{9}$, $\frac{5}{7}$.	**17.** $\frac{4770}{6678}$, $\frac{6161}{7171}$.	*Ans.* $\frac{5}{7}$, $\frac{61}{71}$.

CASE V.

140. **To reduce compound fractions to simple ones.**

ORAL EXERCISES.

1. What is $\frac{1}{3}$ of $\frac{1}{4}$?

SOLUTION.—$\frac{1}{3}$ of $\frac{1}{4}$ is *one* of the *three* equal parts into which $\frac{1}{4}$ may be divided; if each fourth is divided into 3 equal parts, 4 fourths, or the unit, will be divided into 4 times 3, or 12 equal parts; hence each part is $\frac{1}{12}$ of a unit. Therefore $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{12}$.

What is

2. $\frac{1}{2}$ of $\frac{1}{3}$?	6. $\frac{1}{7}$ of $\frac{1}{8}$?	10. $\frac{2}{3}$ of $\frac{5}{6}$?
3. $\frac{1}{3}$ of $\frac{1}{5}$?	7. $\frac{2}{3}$ of $\frac{1}{4}$?	11. $\frac{5}{6}$ of $\frac{7}{8}$?
4. $\frac{1}{5}$ of $\frac{1}{6}$?	8. $\frac{4}{5}$ of $\frac{2}{3}$?	12. $\frac{5}{6}$ of $\frac{12}{13}$?
5. $\frac{1}{6}$ of $\frac{1}{7}$?	9. $\frac{1}{11}$ of $\frac{1}{12}$?	13. $\frac{4}{5}$ of $\frac{15}{16}$?

14. In reducing a compound fraction to a simple one, what do we do with the numerators, and what with the denominators?

15. Since, as the analysis shows, $\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{4 \times 3}$, we infer the following principle:

Principle 3.—*Multiplying the denominator of a fraction by an integer divides the fraction by the integer.*

WRITTEN EXERCISES.

1. What is the value of $\frac{4}{5}$ of $\frac{7}{8}$?

OPERATION.

$\frac{4}{5}$ of $\frac{7}{8} = \frac{4 \times 7}{5 \times 8} = \frac{7}{10}$. *Ans.*

SOLUTION.—$\frac{1}{5}$ of $\frac{7}{8}$ equals $\frac{7}{40}$ (Prin. 3), and since $\frac{1}{5}$ of $\frac{7}{8}$ equals $\frac{7}{40}$, $\frac{4}{5}$ of $\frac{7}{8}$ equals 4 times $\frac{7}{40}$, which equals $\frac{28}{40}$, or $\frac{7}{10}$. Hence the following

Rule.—*Multiply the numerators together and the denominators together, cancelling factors common to both terms.*

1. Reduce whole or mixed numbers to fractions before commencing the reduction to a simple fraction.
2. Reduction of compound fractions prepares for multiplication of fractions, to which it is closely related in practice, though logically different.

What is

2. $\frac{3}{4}$ of $\frac{5}{6}$? *Ans.* $\frac{5}{8}$.	10. $\frac{3}{4}$ of $5\frac{1}{3}$? *Ans.* 4.
3. $\frac{4}{5}$ of $\frac{11}{12}$? *Ans.* $\frac{11}{15}$.	11. $\frac{7}{8}$ of $7\frac{1}{5}$? *Ans.* $6\frac{3}{10}$.
4. $\frac{5}{9}$ of $\frac{14}{20}$? *Ans.* $\frac{7}{18}$.	12. $\frac{6}{7}$ of $\frac{8}{15}$ of $\frac{21}{28}$? *Ans.* $\frac{12}{35}$.
5. $\frac{7}{8}$ of $\frac{24}{28}$? *Ans.* $\frac{3}{4}$.	13. $\frac{4}{9}$ of $\frac{12}{15}$ of $\frac{25}{64}$? *Ans.* $\frac{5}{36}$.
6. $\frac{11}{12}$ of $\frac{16}{33}$? *Ans.* $\frac{4}{9}$.	14. $\frac{12}{15}$ of $\frac{35}{64}$ of $\frac{75}{63}$? *Ans.* $\frac{25}{48}$.
7. $\frac{17}{19}$ of $\frac{38}{68}$? *Ans.* $\frac{1}{2}$.	15. $\frac{25}{64}$ of $\frac{96}{75}$ of $\frac{48}{135}$? *Ans.* $\frac{8}{45}$.
8. $\frac{14}{15}$ of $\frac{42}{84}$? *Ans.* $\frac{7}{15}$.	16. $\frac{24}{35}$ of $\frac{22}{36}$ of $\frac{75}{88}$? *Ans.* $\frac{5}{14}$.
9. $\frac{45}{48}$ of $\frac{64}{75}$? *Ans.* $\frac{4}{5}$.	17. $\frac{77}{88}$ of $\frac{126}{135}$ of $\frac{117}{154}$? *Ans.* $\frac{273}{440}$.

ORAL EXERCISES.

1. Preston had $\frac{3}{4}$ of a box of oranges, and gave $\frac{1}{2}$ of them to his sister; what part of the box did she receive?

2. A newsboy earned $\frac{3}{4}$ of a dollar, and gave his mother $\frac{1}{4}$ of what he earned; how much did he give his mother?

3. Mr. Reed owned $\frac{3}{5}$ of the stock of a bank, and sold $\frac{2}{3}$ of his stock; how much did he sell?

4. A merchant owned $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of a ship; what part of the ship did he own?

5. At the rate of $\frac{3}{8}$ of a dollar a gallon, what will $\frac{3}{4}$ of a gallon of vinegar cost?

6. At the rate of $\frac{3}{4}$ of a dollar a yard, what will $\frac{7}{8}$ of a yard of velvet ribbon cost?

7. At the rate of $\frac{1}{10}$ of a dollar a yard, what will $\frac{5}{8}$ of a yard of elastic cost?

WRITTEN EXERCISES.

1. If a tailor pays \$$6\frac{1}{4}$ for a yard of velvet, what is $\frac{2}{5}$ of a yard worth?

SOLUTION.—\$$6\frac{1}{4}$ = \$$\frac{25}{4}$; $\frac{2}{5}$ of \$$\frac{25}{4}$ equals \$$\frac{10}{4}$, or \$$2\frac{1}{2}$.

2. A owns $\frac{3}{8}$ of the stock of a bank, and B owns $\frac{4}{5}$ as much as A; how much does B own? *Ans.* $\frac{3}{10}$.

3. At the rate of \$$9\frac{3}{5}$ a ton, what is the cost of $\frac{5}{8}$ of a ton of plaster? *Ans.* \$6.

4. Henry has $\frac{5}{6}$ of 4 quarts of nuts, and Mary has $\frac{4}{5}$ as many as Henry; how much has Mary? *Ans.* $2\frac{2}{3}$ quarts.

5. A barrel of fish cost \$$6\frac{3}{4}$, and a barrel of flour cost $\frac{4}{3}$ as much; what was the cost of the flour? *Ans.* \$9.

6. A dressmaker bought $\frac{7}{8}$ of a yard of velvet at \$$5\frac{3}{5}$ a yard; what did it cost? *Ans.* \$$4\frac{9}{10}$.

7. My sister went to a store with \$$15\frac{1}{2}$, and spent $\frac{2}{3}$ of $\frac{3}{4}$ of it; how much did she spend? *Ans.* \$$7\frac{3}{4}$.

8. Mary shared $\frac{15}{16}$ of a bushel of chestnuts with 9 of her schoolmates; what did each receive? *Ans.* $\frac{3}{32}$ bushel.

COMMON DENOMINATOR.

141. Fractions have a *common denominator* when they have the same number for a denominator.

142. Similar Fractions are those which have the same denominator; as, 3 *fifths* and 2 *fifths*.

143. Dissimilar Fractions are those which have different denominators; as, 3 *fourths* and 3 *fifths*.

ORAL EXERCISES.

1. Reduce $\frac{2}{3}$ and $\frac{3}{4}$ to similar fractions.

SOLUTION.—A common denominator for *thirds* and *fourths* is *twelfths*. In *one* there are $\frac{12}{12}$, and in $\frac{1}{3}$ there are $\frac{1}{3}$ of $\frac{12}{12}$, or $\frac{4}{12}$, and in $\frac{2}{3}$ there are 2 times $\frac{4}{12}$, or $\frac{8}{12}$.

Reduce to similar fractions

2. $\frac{2}{3}$ and $\frac{3}{4}$.	**5.** $\frac{5}{6}$ and $\frac{6}{7}$.	**8.** $\frac{4}{5}$ and $\frac{7}{8}$.
3. $\frac{3}{4}$ and $\frac{5}{6}$.	**6.** $\frac{3}{4}$ and $\frac{7}{8}$.	**9.** $\frac{6}{7}$ and $\frac{13}{14}$.
4. $\frac{4}{5}$ and $\frac{5}{6}$.	**7.** $\frac{7}{8}$ and $\frac{9}{10}$.	**10.** $\frac{5}{9}$ and $\frac{11}{12}$.

11. Describe the process of reducing two fractions to a common denominator.

12. How, then, shall we determine the common denominator of dissimilar fractions? Hence the following principle:

A common denominator of several dissimilar fractions is a common multiple of their denominators.

WRITTEN EXERCISES.

1. Reduce $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{7}$ to a common denominator.

SOLUTION.—Since the product of the denominators of the fractions is a common multiple of their denominators, $4 \times 5 \times 7$, which equals 140, will be the common denominator. Then multiplying both terms of $\frac{3}{4}$ by 5×7, we have $\frac{3}{4} = \frac{105}{140}$ (Prin. 1). Multiplying both terms of $\frac{4}{5}$ by 4×7, we have $\frac{4}{5} = \frac{112}{140}$, etc. Hence the following

OPERATION.

$$4 \times 5 \times 7 = 140$$
$$\frac{3 \times 5 \times 7}{4 \times 5 \times 7} = \frac{105}{140}$$
$$\frac{4 \times 4 \times 7}{5 \times 4 \times 7} = \frac{112}{140}$$
$$\frac{5 \times 4 \times 5}{7 \times 4 \times 5} = \frac{100}{140}$$

Rule.—*Multiply both terms of each fraction by the denominators of the other fractions.*

Reduce to a common denominator:

2. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$. *Ans.* $\frac{12}{24}, \frac{16}{24}, \frac{18}{24}$.
3. $\frac{2}{3}, \frac{3}{5}, \frac{1}{4}$. *Ans.* $\frac{40}{60}, \frac{36}{60}, \frac{15}{60}$.
4. $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$. *Ans.* $\frac{24}{48}, \frac{12}{48}, \frac{8}{48}$.
5. $\frac{1}{2}, \frac{2}{3}, \frac{4}{7}$. *Ans.* $\frac{21}{42}, \frac{28}{42}, \frac{24}{42}$.
6. $\frac{3}{4}, \frac{3}{5}, \frac{3}{7}$. *Ans.* $\frac{105}{140}, \frac{84}{140}, \frac{60}{140}$.
7. $\frac{3}{4}, \frac{5}{6}, \frac{7}{8}$. *Ans.* $\frac{144}{192}, \frac{160}{192}, \frac{168}{192}$.
8. $\frac{2}{5}, \frac{4}{7}, \frac{5}{6}$. *Ans.* $\frac{84}{210}, \frac{120}{210}, \frac{175}{210}$.
9. $\frac{4}{5}, \frac{3}{4}, \frac{6}{11}$. *Ans.* $\frac{176}{220}, \frac{165}{220}, \frac{120}{220}$.
10. $\frac{7}{8}, \frac{8}{9}, \frac{10}{11}$. *Ans.* $\frac{693}{792}, \frac{704}{792}, \frac{720}{792}$.
11. $\frac{3}{5}, \frac{5}{8}, \frac{11}{12}$. *Ans.* $\frac{288}{480}, \frac{300}{480}, \frac{440}{480}$.

LEAST COMMON DENOMINATOR.

144. The **Least Common Denominator** of several fractions is the smallest denominator in which all may be expressed. Hence the principle:

The least common denominator of several fractions is the least common multiple of their denominators.

WRITTEN EXERCISES.

1. Reduce $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to their least common denominator.

SOLUTION.—We find the least common multiple of the denominators to be 24, hence 24 is the least common denominator. Dividing 24 by 4, the denominator of $\frac{3}{4}$, we find we must multiply 4 by 6 to produce 24; hence, multiplying both terms of $\frac{3}{4}$ by 6, we have $\frac{3}{4}=\frac{18}{24}$ (Prin. 1). Dividing 24 by 6, the denominator of $\frac{5}{6}$, we find we must multiply 6 by 4 to produce 24; hence, multiplying both terms of $\frac{5}{6}$ by 4, we have $\frac{5}{6}=\frac{20}{24}$, etc. Hence the following

OPERATION.

L. C. M. = 24.

$$\frac{3}{4}=\frac{3\times6}{4\times6}=\frac{18}{24}$$
$$\frac{5}{6}=\frac{5\times4}{6\times4}=\frac{20}{24}$$
$$\frac{7}{8}=\frac{7\times3}{8\times3}=\frac{21}{24}$$

Rule.—I. *Find the least common multiple of the denominators for the least common denominator.*

II. *Divide the least common denominator by the denominator of each fraction, and multiply both terms by the quotient.*

Reduce compound fractions to simple ones, mixed numbers to improper fractions, and all to their lowest terms, before finding the least common denominator.

Reduce to their least common denominator:

2. $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}$. *Ans.* $\frac{8}{12}, \frac{9}{12}, \frac{10}{12}$.
3. $\frac{3}{4}, \frac{7}{10}, \frac{11}{12}$. *Ans.* $\frac{45}{60}, \frac{42}{60}, \frac{55}{60}$.
4. $\frac{3}{7}, \frac{5}{14}, \frac{13}{20}$. *Ans.* $\frac{60}{140}, \frac{50}{140}, \frac{91}{140}$.
5. $\frac{5}{6}, \frac{9}{11}, \frac{11}{12}$. *Ans.* $\frac{110}{132}, \frac{108}{132}, \frac{121}{132}$.

6. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{10}$. *Ans.* $\frac{45}{60}$, etc.	12. $\frac{7}{12}$, $\frac{5}{32}$, $\frac{11}{48}$. *Ans.* $\frac{56}{96}$, etc.
7. $\frac{5}{6}$, $\frac{6}{9}$, $\frac{21}{24}$. *Ans.* $\frac{20}{24}$, etc.	13. $\frac{2}{3}$, $\frac{12}{15}$, $\frac{15}{18}$, $\frac{33}{36}$. *Ans.* $\frac{40}{60}$, etc.
8. $\frac{7}{8}$, $\frac{8}{9}$, $\frac{11}{12}$. *Ans.* $\frac{63}{72}$, etc.	14. $\frac{18}{24}$, $\frac{12}{14}$, $\frac{15}{18}$, $\frac{21}{36}$. *Ans.* $\frac{63}{84}$, etc.
9. $\frac{2}{5}$, $\frac{4}{7}$, $\frac{18}{30}$. *Ans.* $\frac{84}{210}$, etc.	15. $\frac{9}{10}$, $\frac{8}{15}$, $\frac{12}{14}$, $\frac{20}{21}$. *Ans.* $\frac{189}{210}$, etc.
10. $\frac{3}{4}$, $\frac{4}{5}$, $\frac{6}{11}$. *Ans.* $\frac{165}{220}$, etc.	16. $\frac{11}{15}$, $\frac{18}{20}$, $\frac{20}{21}$, $\frac{23}{35}$. *Ans.* $\frac{308}{420}$, etc.
11. $\frac{12}{14}$, $\frac{15}{18}$, $\frac{28}{36}$. *Ans.* $\frac{216}{252}$, etc.	17. $\frac{17}{36}$, $\frac{4}{5}$ of $\frac{4}{9}$, $\frac{5}{8}$ of $\frac{7}{9}$.

ADDITION OF FRACTIONS.

145. Addition of Fractions is the process of finding the sum of two or more fractions.

ORAL EXERCISES.

1. How many sixths in the sum of $\frac{1}{2}$ and $\frac{2}{3}$?

SOLUTION.—$\frac{1}{2}$ is equal to $\frac{3}{6}$, and $\frac{2}{3}$ is equal to $\frac{4}{6}$; $\frac{3}{6}$ and $\frac{4}{6}$ are $\frac{7}{6}$, which is equal to $1\frac{1}{6}$.

Find the sum

2. Of $\frac{1}{4}$ and $\frac{1}{5}$.	5. Of $\frac{4}{5}$ and $\frac{5}{6}$.	8. Of $\frac{3}{5}$ and $\frac{3}{10}$.
3. Of $\frac{2}{3}$ and $\frac{2}{5}$.	6. Of $\frac{3}{4}$ and $\frac{7}{8}$.	9. Of $\frac{1}{3}$ and $\frac{2}{5}$.
4. Of $\frac{3}{4}$ and $\frac{2}{3}$.	7. Of $\frac{5}{6}$ and $\frac{3}{8}$.	10. Of $\frac{1}{5}$ and $\frac{1}{10}$.

11. How, then, shall we add two fractions whose denominators are unlike? Hence the following principles:

1. *To add fractions they must be similar.*

2. *To add dissimilar fractions they must be reduced to a common denominator.*

WRITTEN EXERCISES.

1. What is the sum of $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$?

SOLUTION.—Reducing the fractions to a common denominator that they may be similar fractions, we have $\frac{2}{3}=\frac{16}{24}$, $\frac{5}{6}=\frac{20}{24}$, $\frac{7}{8}=\frac{21}{24}$; 16 *twenty-fourths* plus 20 *twenty-fourths* plus 21 *twenty-fourths* equal 57 *twenty-fourths*, or $2\frac{3}{8}$.

OPERATION.

$$\frac{2}{3}+\frac{5}{6}+\frac{7}{8}=$$
$$\frac{16}{24}+\frac{20}{24}+\frac{21}{24}=\frac{57}{24}=2\frac{3}{8}$$

Hence the following

Rule.—*Reduce the fractions to a common denominator, add the numerators, and write the sum over the common denominator.*

1. Reduce compound fractions to simple ones, and reduce each fraction and the sum to lowest terms.

2. To add mixed numbers, add the integers and fractions separately, and then unite their sums.

Find the sum of

2. $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}$.	*Ans.* $2\frac{1}{4}$.	**16.** $\frac{3}{4}, \frac{4}{5}, \frac{5}{8}$.	*Ans.* $2\frac{7}{40}$.
3. $\frac{3}{4}, \frac{5}{6}, \frac{7}{8}$.	*Ans.* $2\frac{11}{24}$.	**17.** $\frac{4}{5}, \frac{9}{10}, \frac{11}{15}$.	*Ans.* $2\frac{13}{30}$.
4. $\frac{3}{4}, \frac{5}{6}, \frac{7}{12}$.	*Ans.* $2\frac{1}{6}$.	**18.** $\frac{5}{6}, \frac{7}{8}, \frac{11}{12}$.	*Ans.* $2\frac{5}{8}$.
5. $\frac{5}{6}, \frac{5}{8}, \frac{5}{12}$.	*Ans.* $1\frac{7}{8}$.	**19.** $\frac{5}{8}, \frac{7}{9}, \frac{11}{12}$.	*Ans.* $2\frac{23}{72}$.
6. $\frac{2}{3}, \frac{5}{6}, \frac{8}{9}$.	*Ans.* $2\frac{7}{18}$.	**20.** $\frac{7}{8}, \frac{11}{12}, \frac{15}{16}$.	*Ans.* $2\frac{35}{48}$.
7. $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}$.	*Ans.* $2\frac{23}{60}$.	**21.** $\frac{8}{9}, \frac{11}{12}, \frac{17}{18}$.	*Ans.* $2\frac{3}{4}$.
8. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$.	*Ans.* $2\frac{13}{60}$.	**22.** $\frac{7}{10}, \frac{14}{15}, \frac{19}{20}$.	*Ans.* $2\frac{7}{12}$.
9. $\frac{3}{4}, \frac{7}{8}, \frac{11}{12}$.	*Ans.* $2\frac{13}{24}$.	**23.** $\frac{21}{24}, \frac{27}{30}, \frac{39}{40}$.	*Ans.* $2\frac{3}{4}$.
10. $\frac{4}{5}, \frac{5}{6}, \frac{7}{8}$.	*Ans.* $2\frac{61}{120}$.	**24.** $\frac{7}{8}, \frac{9}{10}, \frac{11}{12}$.	*Ans.* $2\frac{83}{120}$.
11. $\frac{5}{6}, \frac{7}{12}, \frac{11}{18}$.	*Ans.* $2\frac{1}{36}$.	**25.** $8\frac{3}{4}, 6\frac{2}{3}, 2\frac{9}{10}$.	*Ans.* $18\frac{19}{60}$.
12. $\frac{4}{7}, \frac{3}{4}, \frac{11}{14}$.	*Ans.* $2\frac{3}{28}$.	**26.** $11\frac{7}{8}, 10\frac{1}{12}, 7\frac{1}{16}$.	*Ans.* $29\frac{1}{48}$.
13. $4\frac{1}{2}, 3\frac{3}{4}, \frac{11}{12}$.	*Ans.* $9\frac{1}{6}$.	**27.** $8\frac{7}{8}, 9\frac{8}{9}, 12\frac{11}{12}$.	*Ans.* $31\frac{49}{72}$.
14. $\frac{7}{8}, \frac{8}{9}, 2\frac{5}{12}$.	*Ans.* $4\frac{13}{72}$.	**28.** $7\frac{16}{55}, 8\frac{8}{15}, 9\frac{19}{33}$.	*Ans.* $25\frac{2}{5}$.
15. $\frac{4}{5}, \frac{5}{6}, \frac{7}{8}, \frac{15}{16}$.	*Ans.* $3\frac{107}{240}$.	**29.** $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}$.	*Ans.* $1\frac{201}{280}$.

PRACTICAL EXAMPLES.

1. John deposited \$$6\frac{1}{4}$ in a savings-bank, and Joseph deposited \$$7\frac{1}{2}$; how much did both deposit? *Ans.* \$$13\frac{3}{4}$.

2. A miller ground $8\frac{1}{4}$ bushels of wheat for A, and $10\frac{5}{12}$ bushels for B; how much wheat did he grind for both of these persons? *Ans.* $18\frac{2}{3}$ bushels.

3. A man has three fields, the first containing $15\frac{1}{4}$ acres, the second $16\frac{2}{15}$ acres, the third $18\frac{5}{12}$ acres; how many acres in the three fields? *Ans.* $49\frac{4}{5}$ acres.

4. I burned in my grate $251\frac{2}{3}$ pounds of coal in November, $287\frac{7}{8}$ pounds in December, and $225\frac{8}{9}$ pounds in January; how much did I burn in the three months? *Ans.* $765\frac{31}{72}$ pounds.

5. On Monday a grocer received $\$34\frac{5}{8}$; on Tuesday $\$14\frac{3}{16}$ more; on Wednesday as much as on the two previous days; what were his receipts for the three days? *Ans.* \$167.

6. A bicycler rode $27\frac{3}{4}$ miles on Monday, $33\frac{1}{4}$ miles on Tuesday, $37\frac{3}{4}$ miles on Wednesday, and $42\frac{3}{5}$ miles on Thursday; how far did he ride in the four days? *Ans.* $141\frac{7}{20}$ miles.

7. Mr. Kirk finds that his furnace consumed last winter $2\frac{3}{4}$ tons of coal in December, $2\frac{5}{8}$ tons in January, and $3\frac{1}{16}$ in February; how much coal was burned during the three winter months? *Ans.* $8\frac{3}{4}\frac{1}{8}$ tons.

8. A drygoods merchant sold a lady $18\frac{3}{4}$ yards of flannel, $21\frac{7}{8}$ yards of silk, and as many yards of calico as of both the other goods; how many yards of goods did he sell to her? *Ans.* $81\frac{1}{4}$ yards.

SUBTRACTION OF FRACTIONS.

146. Subtraction of Fractions is the process of finding the difference between two fractions.

ORAL EXERCISES.

1. How many twelfths in the difference between $\frac{3}{4}$ and $\frac{2}{3}$?

SOLUTION.—$\frac{3}{4}$ equals $\frac{9}{12}$ and $\frac{2}{3}$ equals $\frac{8}{12}$, and the difference between $\frac{9}{12}$ and $\frac{8}{12}$ is $\frac{1}{12}$.

Subtract

2. $\frac{1}{4}$ from $\frac{1}{3}$.	5. $\frac{2}{5}$ from $\frac{1}{2}$.	8. $\frac{3}{4}$ from $\frac{7}{8}$.
3. $\frac{1}{3}$ from $\frac{3}{4}$.	6. $\frac{1}{3}$ from $\frac{5}{6}$.	9. $\frac{5}{8}$ from $\frac{7}{8}$.
4. $\frac{3}{4}$ from $\frac{4}{5}$.	7. $\frac{4}{7}$ from $\frac{3}{5}$.	10. $\frac{5}{8}$ from $\frac{6}{7}$.

11. How, then, shall we subtract two fractions whose denominators are unlike? Hence the following principles:

1. *To subtract fractions they must be similar.*

2. *To subtract dissimilar fractions they must be reduced to a common denominator.*

WRITTEN EXERCISES.

1. What is the difference between $\frac{7}{8}$ and $\frac{5}{9}$?

SOLUTION.—Reducing the fractions to a common denominator that they may express similar fractional units, we have $\frac{7}{8}=\frac{63}{72}$ and $\frac{5}{9}=\frac{40}{72}$. 63 *seventy-seconds* minus 40 *seventy-seconds* equals 23 *seventy-seconds*. Hence the following

OPERATION.

$$\frac{7}{8}-\frac{5}{9}=$$
$$\frac{63}{72}-\frac{40}{72}=\frac{23}{72}$$

Rule.—*Reduce the fractions to a common denominator, take the difference of the numerators and write it over the common denominator.*

Subtract

2. $\frac{4}{5}$ from $\frac{6}{7}$. *Ans.* $\frac{2}{35}$.

3. $\frac{3}{7}$ from $\frac{5}{9}$. *Ans.* $\frac{8}{63}$.

4. $\frac{8}{9}$ from $\frac{14}{15}$. *Ans.* $\frac{2}{45}$.

5. $\frac{8}{11}$ from $\frac{11}{12}$. *Ans.* $\frac{25}{132}$.

6. $\frac{9}{16}$ from $\frac{17}{18}$. *Ans.* $\frac{55}{144}$.

7. $\frac{13}{14}$ from $\frac{15}{16}$. *Ans.* $\frac{1}{112}$.

8. $\frac{13}{15}$ from $\frac{18}{19}$. *Ans.* $\frac{23}{285}$.

9. $\frac{17}{24}$ from $\frac{29}{36}$. *Ans.* $\frac{7}{72}$.

10. $\frac{17}{30}$ from $\frac{29}{48}$. *Ans.* $\frac{3}{80}$.

11. $\frac{15}{28}$ from $\frac{31}{35}$. *Ans.* $\frac{7}{20}$.

12. $\frac{25}{36}$ from $\frac{47}{54}$. *Ans.* $\frac{19}{108}$.

13. $2\frac{4}{5}$ from $6\frac{5}{6}$. *Ans.* $4\frac{1}{30}$.

14. $5\frac{7}{8}$ from $9\frac{11}{12}$. *Ans.* $4\frac{1}{24}$.

15. $8\frac{8}{9}$ from $15\frac{14}{15}$. *Ans.* $7\frac{2}{45}$.

16. Subtract $7\frac{5}{8}$ from $11\frac{3}{8}$.

SOLUTION.—We cannot subtract $\frac{5}{8}$ from $\frac{3}{8}$, so we take 1 from 11, which added to $\frac{3}{8}$ equals $1\frac{3}{8}$, or $\frac{11}{8}$; $\frac{5}{8}$ from $\frac{11}{8}$ leaves $\frac{6}{8}$, or $\frac{3}{4}$, and 7 from 10 leaves 3; hence the difference is $3\frac{3}{4}$.

Subtract

17. $7\frac{5}{9}$ from $11\frac{2}{9}$. *Ans.* $3\frac{2}{3}$.

18. $9\frac{3}{11}$ from $14\frac{1}{11}$. *Ans.* $4\frac{9}{11}$.

19. $7\frac{5}{12}$ from $13\frac{5}{8}$. *Ans.* $6\frac{5}{24}$.

20. $18\frac{1}{11}$ from 23. *Ans.* $4\frac{10}{11}$.

21. $9\frac{1}{14}$ from $62\frac{1}{2}$. *Ans.* $53\frac{3}{7}$.

22. $24\frac{1}{2}$ from $50\frac{1}{5}$. *Ans.* $25\frac{7}{10}$.

23. $48\frac{5}{6}$ from $52\frac{3}{8}$. *Ans.* $3\frac{13}{24}$.

24. $36\frac{3}{10}$ from $49\frac{1}{12}$. *Ans.* $12\frac{47}{60}$.

25. $58\frac{19}{24}$ from $72\frac{11}{16}$. *Ans.* $13\frac{43}{48}$.

26. $63\frac{11}{20}$ from $75\frac{11}{15}$. *Ans.* $12\frac{11}{60}$.

PRACTICAL EXAMPLES.

1. A drygoods merchant bought two pieces of cambric, one containing $42\frac{5}{8}$ yards, and the other $43\frac{3}{4}$ yards; after selling $69\frac{7}{8}$ yards, how many yards remained? *Ans.* $16\frac{1}{2}$ yards.

2. From a piece of silk containing $30\frac{1}{4}$ yards there were sold $3\frac{1}{4}$ yards, $4\frac{5}{8}$ yards, and $12\frac{1}{2}$ yards; how many yards remained? *Ans.* $9\frac{7}{8}$ yards.

3. Mrs. Brown paid $\$3\frac{11}{50}$ for material for a wrapper, $\$\frac{6}{25}$ for calico for an apron, and $\$\frac{9}{100}$ for some edging; what change would she have left from a \$5 bill? *Ans.* $\$1\frac{9}{20}$.

4. Wilson received from his uncle a birthday present of \$3; he spent $\$\frac{3}{4}$ for a knife, $\$\frac{1}{4}$ for a slate, $\$1\frac{1}{4}$ for a story-book, and $\$\frac{1}{10}$ for some marbles; how much money had he left after these purchases? *Ans.* $\$\frac{13}{20}$.

5. A lady went shopping with a \$100 bill; she paid $\$18\frac{3}{4}$ for a bonnet, $\$42\frac{5}{8}$ for a dress, and $\$35\frac{1}{4}$ for a cloak; how much money did she take home with her? *Ans.* $\$3\frac{3}{8}$.

6. Four towns, A, B, C, and D, are situated in a straight line running east and west; A is $53\frac{1}{4}$ miles west of C; D is $42\frac{1}{2}$ miles east of B; B is $27\frac{3}{4}$ miles west of C; required the distance from A to D. *Ans.* 68 miles.

NOTE.—Draw a diagram to indicate the situation of the places.

MULTIPLICATION OF FRACTIONS.

147. Multiplication of Fractions is the process of finding a product where one or both factors are fractions.

148. There are **Two Cases**: 1st. The multiplier an integer; 2d. The multiplier a fraction.

CASE I.

149. To multiply a fraction by an integer.

ORAL EXERCISES.

1. How many are 5 times $\frac{2}{15}$?

SOLUTION 1ST.—5 times $\frac{2}{15}$ are $\frac{10}{15}$, which reduced to its lowest terms equals $\frac{2}{3}$; hence, 5 times $\frac{2}{15}$ equals $\frac{2}{3}$.

SOLUTION 2D.—5 times $\frac{1}{15}$ equals $\frac{5}{15}$, or $\frac{1}{3}$; if 5 times $\frac{1}{15}$ equal $\frac{1}{3}$, 5 times $\frac{2}{15}$ equals 2 times $\frac{1}{3}$, or $\frac{2}{3}$; hence, 5 times $\frac{2}{15}$ equals $\frac{2}{3}$.

How many are

2. 2 times $\frac{5}{6}$?	**5.** 3 times $5\frac{5}{6}$?	**8.** 6 times $\frac{17}{18}$?
3. 5 times $\frac{7}{15}$?	**6.** 4 times $8\frac{1}{12}$?	**9.** 9 times $\frac{19}{27}$?
4. 7 times $\frac{20}{21}$?	**7.** 8 times $7\frac{1}{4}$?	**10.** 7 times $5\frac{11}{14}$?

11. Since 5 times $\frac{2}{15}$ equals $\frac{2}{3}$, how may this result be obtained by omitting the analysis?

12. How, then, may a fraction be multiplied by an integer which is contained in the denominator?

13. From the above analyses we derive the following principle:

Principle 4.—*Multiplying the numerator or dividing the denominator of a fraction by an integer multiplies the fraction by the integer.*

WRITTEN EXERCISES.

1. Multiply $\frac{23}{24}$ by 8.

SOLUTION.—Multiplying the numerator (Prin. 4), we have $\frac{23}{24} \times 8 = \frac{23 \times 8}{24}$, which, cancelling the common factor 8, equals $\frac{23}{3}$, or $7\frac{2}{3}$.

OPERATION.

$$\frac{23}{24} \times 8 = \frac{23 \times 8}{24} = \frac{23}{3}, \text{ or } 7\frac{2}{3}$$

$$\text{Or, } \frac{23}{24} \times 8 = \frac{23}{24 \div 8} = \frac{23}{3}, \text{ or } 7\frac{2}{3}$$

Or, dividing the denominator (Prin. 4), we have 8 times $\frac{23}{24} = \frac{23}{3}$, or $7\frac{2}{3}$. Hence the following

Rule.—*To multiply a fraction by an integer, multiply the numerator or divide the denominator by the integer.*

NOTES.—1. It will often be found convenient to express the multiplication and then cancel common factors.

2. In mixed numbers multiply the integers and the fractions separately.

Multiply

2. $\frac{9}{10}$ by 5.	*Ans.* $4\frac{1}{2}$.	**8.** $\frac{19}{26}$ by 13.	*Ans.* $9\frac{1}{2}$.
3. $\frac{17}{18}$ by 6.	*Ans.* $5\frac{2}{3}$.	**9.** $\frac{15}{28}$ by 14.	*Ans.* $7\frac{1}{2}$.
4. $\frac{11}{12}$ by 9.	*Ans.* $8\frac{1}{4}$.	**10.** $\frac{105}{108}$ by 18.	*Ans.* $17\frac{1}{2}$.
5. $\frac{17}{22}$ by 11.	*Ans.* $8\frac{1}{2}$.	**11.** $\frac{105}{224}$ by 32.	*Ans.* 15.
6. $\frac{14}{15}$ by 10.	*Ans.* $9\frac{1}{3}$.	**12.** $\frac{131}{144}$ by 60.	*Ans.* $54\frac{7}{12}$.
7. $\frac{17}{18}$ by 12.	*Ans.* $11\frac{1}{3}$.	**13.** $\frac{151}{156}$ by 26.	*Ans.* $25\frac{1}{6}$.

14. $8\frac{5}{8}$ by 4.	*Ans.* $34\frac{1}{2}$.	22. $19\frac{18}{21}$ by 14.	*Ans.* 278.
15. $9\frac{2}{3}$ by 6.	*Ans.* 58.	23. $25\frac{19}{25}$ by 15.	*Ans.* $386\frac{2}{5}$.
16. $7\frac{2}{3}$ by 9.	*Ans.* 69.	24. $46\frac{15}{26}$ by 13.	*Ans.* $605\frac{1}{2}$.
17. $8\frac{9}{10}$ by 5.	*Ans.* $44\frac{1}{2}$.	25. $54\frac{25}{42}$ by 35.	*Ans.* $1910\frac{5}{6}$.
18. $18\frac{7}{8}$ by 8.	*Ans.* 151.	26. $65\frac{19}{51}$ by 68.	*Ans.* $4445\frac{1}{3}$.
19. $28\frac{15}{20}$ by 10.	*Ans.* $287\frac{1}{2}$.	27. $76\frac{28}{57}$ by 38.	*Ans.* $2906\frac{2}{3}$.
20. $\frac{121}{128}$ by 48.	*Ans.* $45\frac{3}{8}$.	28. $128\frac{19}{54}$ by 18.	*Ans.* $2310\frac{1}{3}$.
21. $\frac{229}{1728}$ by 144.	*Ans.* $19\frac{1}{12}$.	29. $428\frac{18}{121}$ by 11.	*Ans.* $4709\frac{7}{11}$.

30. Mr. Smith bought 15 tons of coal at $\$5\frac{1}{4}$ a ton; what was the cost of the coal? *Ans.* $\$78\frac{3}{4}$.

CASE II.

150. To multiply an integer or a fraction by a fraction.

ORAL EXERCISES.

1. What is the product of $\frac{5}{6}$ by $\frac{3}{4}$?

SOLUTION.—$\frac{5}{6}$ multiplied by *one* equals $\frac{5}{6}$, hence $\frac{5}{6}$ multiplied by $\frac{1}{4}$ equals $\frac{1}{4}$ of $\frac{5}{6}$, which is $\frac{5}{24}$; and $\frac{5}{6}$ multiplied by $\frac{3}{4}$ equals 3 times $\frac{5}{24}$, which is $\frac{15}{24}$, or $\frac{5}{8}$.

What is the product

2. Of $\frac{6}{7}$ by $\frac{2}{3}$?	6. Of $2\frac{1}{2}$ by $1\frac{1}{5}$?
3. Of $\frac{9}{10}$ by $\frac{5}{6}$?	7. Of $7\frac{1}{2}$ by $1\frac{2}{3}$?
4. Of $\frac{18}{20}$ by $\frac{5}{9}$?	8. Of $4\frac{4}{9}$ by $\frac{11}{20}$?
5. Of $\frac{14}{15}$ by $\frac{3}{7}$?	9. Of $5\frac{1}{3}$ by $1\frac{7}{8}$?

10. In multiplying $\frac{5}{6}$ by $\frac{3}{4}$, what do we multiply together for the numerator of the product, and what for the denominator?

11. How, then, do we multiply a fraction by a fraction?

WRITTEN EXERCISES.

1. Multiply $\frac{15}{18}$ by $\frac{9}{11}$.

SOLUTION.—$\frac{15}{18}$ multiplied by *one* is $\frac{15}{18}$, hence $\frac{15}{18}$ multiplied by $\frac{9}{11}$ equals $\frac{9}{11}$ of $\frac{15}{18}$, which equals 15×9 divided by 18×11, which, by cancelling and reducing, we find is equal to $\frac{15}{22}$.

OPERATION.

$$\frac{15}{18} \times \frac{9}{11} = \frac{15 \times \cancel{9}}{\underset{2}{\cancel{18}} \times 11} = \frac{15}{22}$$

SOLUTION 2D.—$\frac{9}{11}$ times $\frac{15}{18}$ equals $\frac{1}{11}$ of 9 times $\frac{15}{18}$; 9 times $\frac{15}{18}$ equals $\frac{15\times9}{18}$ (Prin. 4), and $\frac{1}{11}$ of $\frac{15\times9}{18}$ equals $\frac{15\times9}{18\times11}$ (Prin. 3), which, by cancelling and reducing, we find is equal to $\frac{15}{22}$.

Rule.—*Multiply the numerators together for the numerator of the product, and the denominators for the denominator of the product.*

1. Cancel common factors from numerators and denominators before multiplying.

2. Reduce mixed numbers to improper fractions before multiplying. An integer may be expressed as a fraction by writing 1 under it as a denominator.

What is the product of

2. $\frac{15}{18}$ by $\frac{9}{10}$? *Ans.* $\frac{3}{4}$.
3. $\frac{18}{25}$ by $\frac{10}{12}$? *Ans.* $\frac{3}{5}$.
4. $\frac{42}{56}$ by $\frac{14}{18}$? *Ans.* $\frac{7}{12}$.
5. $\frac{49}{99}$ by $\frac{33}{56}$? *Ans.* $\frac{7}{24}$.
6. $\frac{35}{54}$ by $\frac{36}{49}$? *Ans.* $\frac{10}{21}$.
7. 144 by $\frac{13}{24}$? *Ans.* 78.
8. 720 by $\frac{31}{63}$? *Ans.* $354\frac{2}{7}$.
9. $\frac{105}{231}$ by $\frac{65}{104}$? *Ans.* $\frac{25}{88}$.
10. $\frac{315}{1144}$ by $\frac{88}{255}$? *Ans.* $\frac{21}{221}$.
11. $\frac{9}{10}\times\frac{32}{27}\times\frac{35}{48}$? *Ans.* $\frac{7}{9}$.
12. $\frac{18}{24}\times\frac{28}{45}\times\frac{115}{116}$? *Ans.* $\frac{161}{348}$.
13. $\frac{45}{49}\times\frac{56}{65}\times\frac{91}{72}$? *Ans.* 1.
14. 284 by $\frac{35}{36}$? *Ans.* $276\frac{1}{9}$.
15. 28 by $3\frac{4}{7}$? *Ans.* 100.
16. 180 by $15\frac{4}{5}$? *Ans.* 2844.
17. 256 by $7\frac{15}{16}$? *Ans.* 2032.
18. 180 by $25\frac{17}{24}$? *Ans.* $4627\frac{1}{2}$.
19. $12\frac{1}{2}$ by $16\frac{2}{3}$? *Ans.* $208\frac{1}{3}$.
20. $17\frac{2}{5}$ by $8\frac{1}{3}$? *Ans.* 145.
21. $27\frac{3}{11}$ by $21\frac{5}{6}$? *Ans.* $595\frac{5}{11}$.
22. $39\frac{3}{5}$ by $66\frac{2}{3}$? *Ans.* 2640.
23. $\frac{9}{10}$ of $\frac{18}{35}$ by $\frac{7}{24}$ of $\frac{16}{27}$?
24. $\frac{16}{28}$ of $\frac{18}{24}$ by $\frac{28}{45}$ of $\frac{115}{116}$?
25. $\frac{19}{27}$ of $\frac{102}{95}$ by $\frac{144}{189}$ of $\frac{273}{1728}$?

SPECIAL METHOD WITH MIXED NUMBERS.

1. Multiply 28 by $7\frac{3}{5}$.

SOLUTION.—Multiplying 28 by 3, and dividing by 5, we have $28\times\frac{3}{5}$ equals $16\frac{4}{5}$; then, multiplying 28 by 7, we have 196; adding 196 to $16\frac{4}{5}$, we have $212\frac{4}{5}$. Therefore, etc.

OPERATION.

$$\begin{array}{r} 28 \\ \underline{7\frac{3}{5}} \\ 5)\underline{84} \\ 16\frac{4}{5} \\ \underline{196} \\ 212\frac{4}{5} \end{array} \quad \textit{Ans.}$$

NOTE.—This method of multiplying by a mixed number is more convenient than the one usually presented.

What is the product of

2. 17 by $4\frac{3}{4}$? *Ans.* $80\frac{3}{4}$.

3. 20 by $2\frac{11}{12}$? *Ans.* $58\frac{1}{3}$.

4. 32 by $11\frac{2}{3}$? *Ans.* $373\frac{1}{3}$.

5. 27 by $12\frac{5}{6}$? *Ans.* $346\frac{1}{2}$.

6. 120 by $16\frac{2}{3}$? *Ans.* 2000.

7. 36 by $15\frac{4}{5}$? *Ans.* $568\frac{4}{5}$.

8. 21 by $18\frac{3}{7}$? *Ans.* 387.

9. 22 by $29\frac{5}{12}$? *Ans.* $647\frac{1}{6}$.

10. 75 by $42\frac{11}{12}$? *Ans.* $3218\frac{3}{4}$.

11. 112 by $17\frac{15}{16}$? *Ans.* 2009.

ORAL EXERCISES.

1. How much will $8\frac{3}{4}$ pounds of oatmeal cost at 5 cents a pound?

2. A club bought 20 baseballs at $\$\frac{3}{4}$ each; how much did they cost?

3. A man gave to each of 8 beggars $\frac{3}{4}$ of a dollar; how much did he give away?

4. A teacher gave each of 10 girls $3\frac{3}{5}$ yards of ribbon for badges; how many yards did it take?

5. When silk is selling at 75¢ per yard, what will be the cost of $\frac{1}{3}$ of a yard?

6. If it takes $3\frac{1}{2}$ yards of cloth to make a coat, how many yards will be required to make 7 coats?

7. If tea is worth $\$\frac{3}{4}$ a pound, what will $\frac{2}{3}$ of a pound cost?

8. A lady bought $6\frac{1}{4}$ yards of cloth at $\$5\frac{1}{3}$ a yard; what did it cost her?

9. Multiplying by $\frac{1}{4}$ is the same as taking what part of a number?

WRITTEN EXERCISES.

Required the cost of

1. 45 pairs of shoes, at $\$1\frac{1}{4}$ a pair. *Ans.* $\$56\frac{1}{4}$.

2. 120 yards of ribbon, at $16\frac{3}{4}$ cts. a yard. *Ans.* \$20.10.

3. $475\frac{1}{4}$ lbs. of sugar, at 6 cts. a pound. *Ans.* $\$28.51\frac{1}{2}$.

4. 56 inkstands, at $33\frac{1}{3}$ cts. apiece. *Ans.* $\$18.66\frac{2}{3}$.

5. $8\frac{3}{4}$ tons of hay, at $\$7\frac{3}{4}$ a ton. *Ans.* $\$67\frac{13}{16}$.

6. 275 bush. of potatoes, at $62\frac{1}{2}$ cts. a bush. *Ans.* $\$171.87\frac{1}{2}$.

7. $87\frac{1}{2}$ lb. of paper, at $3\frac{2}{3}$ cents a pound. *Ans.* \$$3.20\frac{5}{6}$.

8. 465 Rochester lamps, at \$$7\frac{3}{5}$ apiece. *Ans.* \$3534.

9. $27\frac{1}{2}$ qts. of cranberries, at $18\frac{1}{2}$ cts. a qt. *Ans.* \$$5.08\frac{3}{4}$.

10. 19 barrels of vinegar, at \$$5\frac{7}{8}$ a barrel. *Ans.* \$$111\frac{5}{8}$.

11. 250 tons of coal, at \$$6\frac{3}{8}$ a ton. *Ans.* \$$1593\frac{3}{4}$.

12. 35 dozen spoons, at \$$7\frac{3}{8}$ a dozen. *Ans.* \$$258\frac{1}{8}$.

13. 18 barrels of flour, at \$$6\frac{5}{8}$ a barrel. *Ans.* \$$119\frac{1}{4}$.

14. $12\frac{7}{8}$ cords of wood, at \$$5\frac{3}{4}$ a cord. *Ans.* \$$74\frac{1}{32}$.

15. I bought a lot of crockery, of which the retail price was \$$576\frac{3}{8}$, but I got a reduction of $\frac{1}{3}$ for wholesale and $\frac{3}{50}$ for cash; what was the amount paid? *Ans.* \$$361\frac{39}{200}$.

16. Mr. Winslow rented a house at \$$58\frac{1}{3}$ a month, and remained in it $4\frac{1}{4}$ years; what did his rent amount to during the time? *Ans.* \$2975.

17. Mr. Alden bought $52\frac{1}{2}$ bushels of wheat at $62\frac{1}{2}$ cts. per bushel, and sold $\frac{1}{3}$ of it at $69\frac{1}{2}$ cts., and the remainder at $71\frac{2}{3}$ cts. per bushel; what did he clear by the transaction? *Ans.* \$$4.43\frac{1}{3}$.

DIVISION OF FRACTIONS.

151. Division of Fractions is the process of dividing when one or both of the terms are fractional.

152. There are **Two Cases**: 1st. The divisor an integer; 2d. The divisor a fraction.

CASE I.

153. To divide a fraction by an integer.

ORAL EXERCISES.

1. How many times is 4 contained in $\frac{8}{9}$?

Solution.—*One* is contained in $\frac{8}{9}$, $\frac{8}{9}$ times; and if 1 is contained in $\frac{8}{9}$, $\frac{8}{9}$ times, 4 is contained in $\frac{8}{9}$, $\frac{1}{4}$ of $\frac{8}{9}$ times, or $\frac{2}{9}$. Therefore, etc,

How many times is

2. 3 contained in $\frac{6}{7}$?	**6.** 4 contained in $8\frac{4}{5}$?
3. 5 contained in $\frac{10}{11}$?	**7.** 6 contained in $12\frac{3}{5}$?
4. 6 contained in $\frac{6}{9}$?	**8.** 7 contained in $16\frac{1}{3}$?
5. 7 contained in $\frac{11}{12}$?	**9.** 9 contained in $19\frac{1}{10}$?

10. Since $\frac{8}{9} \div 4 = \frac{2}{9}$, as shown by the analysis, how may we divide a fraction by a number that will divide the numerator?

11. Since $\frac{7}{9} \div 4 = \frac{7}{36}$, as shown by the analysis, how may we divide a fraction by a number that will not divide the numerator?

12. From the above analysis we derive the following principle:

Principle 5.—*Dividing the numerator or multiplying the denominator of a fraction by an integer, divides the fraction by the integer.*

WRITTEN EXERCISES.

1. Divide $\frac{12}{13}$ by 6, and also by 8.

SOLUTION.—Dividing the numerator of $\frac{12}{13}$ by 6, we have $\frac{12}{13} \div 6$ equals $\frac{2}{13}$ (Prin. 5).

OPERATION.

$$\frac{12}{13} \div 6 = \frac{2}{13}$$

SOLUTION 2D.—Multiplying the denominator of $\frac{12}{13}$ by 8 (Prin. 3), we have $\frac{12}{13} \div 8 = \frac{12}{104}$, or $\frac{3}{26}$. Hence the following

OPERATION.

$$\frac{12}{13} \div 8 = \frac{12}{104} = \frac{3}{26}$$

Rule.—*To divide a fraction by an integer, divide the numerator, or multiply the denominator, by the integer.*

1. It is often convenient to express the division with the reciprocal of the divisor, and cancel common factors.
Thus, $\frac{12}{13} \div 8 = \frac{12}{13} \times \frac{1}{8} = \frac{3}{26}$.

2. Reduce a mixed number to a fraction, or divide the integer, unite the remainder with the fraction, and divide the result.

Divide

2. $\frac{28}{41}$ by 4.	*Ans.* $\frac{7}{41}$.	**7.** $\frac{125}{131} \div 5$.	*Ans.* $\frac{25}{131}$.
3. $\frac{75}{86}$ by 3.	*Ans.* $\frac{25}{86}$.	**8.** $\frac{162}{231} \div 9$.	*Ans.* $\frac{6}{77}$.
4. $\frac{96}{115}$ by 6.	*Ans.* $\frac{16}{115}$.	**9.** $\frac{154}{161} \div 11$.	*Ans.* $\frac{2}{23}$.
5. $\frac{91}{114} \div 7$.	*Ans.* $\frac{13}{114}$.	**10.** $\frac{168}{231} \div 12$.	*Ans.* $\frac{2}{33}$.
6. $\frac{104}{105} \div 8$.	*Ans.* $\frac{13}{105}$.	**11.** $\frac{182}{245} \div 13$.	*Ans.* $\frac{2}{35}$.

12. $\frac{225}{231} \div 15$. *Ans.* $\frac{5}{77}$.

13. $\frac{252}{291} \div 14$. *Ans.* $\frac{6}{97}$.

14. $\frac{315}{336} \div 18$. *Ans.* $\frac{5}{96}$.

15. $\frac{528}{561} \div 16$. *Ans.* $\frac{11}{187}$.

16. $\frac{308}{385} \div 21$. *Ans.* $\frac{4}{105}$.

17. $11\frac{1}{4} \div 5$. *Ans.* $2\frac{1}{4}$.

18. $14\frac{2}{5} \div 6$. *Ans.* $2\frac{2}{5}$.

19. $13\frac{5}{7} \div 8$. *Ans.* $1\frac{5}{7}$.

20. $61\frac{5}{6}$ by 7. *Ans.* $8\frac{5}{6}$.

21. $41\frac{5}{8}$ by 9. *Ans.* $4\frac{5}{8}$.

22. $33\frac{3}{5}$ by 12. *Ans.* $2\frac{4}{5}$.

23. $174\frac{2}{9}$ by 14. *Ans.* $12\frac{4}{9}$.

24. $878\frac{2}{11}$ by 15. *Ans.* $58\frac{6}{11}$.

25. $1284\frac{8}{13}$ by 25. *Ans.* $51\frac{5}{13}$.

26. $1347\frac{1}{17}$ by 30. *Ans.* $44\frac{46}{51}$.

27. $2890\frac{7}{20}$ by 36. *Ans.* $80\frac{23}{80}$.

CASE II.

154. To divide an integer or a fraction by a fraction.

ORAL EXERCISES.

1. How many times is $\frac{3}{4}$ contained in $\frac{5}{6}$?

SOLUTION.—$\frac{1}{4}$ is contained in *one* 4 times; hence $\frac{3}{4}$ is contained in 1, $\frac{1}{3}$ of 4 times, or $\frac{4}{3}$ times. If $\frac{3}{4}$ is contained in 1, $\frac{4}{3}$ times, $\frac{3}{4}$ is contained in $\frac{5}{6}$, $\frac{4}{3}$ times $\frac{5}{6}$, or $\frac{20}{18}$, or $\frac{10}{9}$ times.

How many times is

2. $\frac{4}{5}$ contained in $\frac{6}{7}$?

3. $\frac{2}{3}$ contained in $\frac{3}{4}$?

4. $\frac{3}{8}$ contained in $\frac{5}{6}$?

5. $\frac{3}{4}$ contained in $1\frac{4}{5}$?

6. $\frac{5}{9}$ contained in $2\frac{2}{3}$?

7. $\frac{7}{11}$ contained in $8\frac{3}{11}$?

8. In dividing $\frac{5}{6}$ by $\frac{3}{4}$, what operation does the analysis require us to perform?

9. How, then, may we divide a fraction by a fraction without the analysis?

WRITTEN EXERCISES.

1. Divide $\frac{9}{10}$ by $\frac{7}{8}$.

SOLUTION.—$\frac{1}{8}$ is contained in 1, 8 times, and $\frac{7}{8}$ is contained in 1, $\frac{1}{7}$ of 8 times, or $\frac{8}{7}$ times. If $\frac{7}{8}$ is contained in 1, $\frac{8}{7}$ times, $\frac{7}{8}$ is contained in $\frac{9}{10}$, $\frac{8}{7}$ times $\frac{9}{10}$, or $\frac{72}{70}$, or $\frac{36}{35}$ times. Hence the following

OPERATION.

$$\frac{9}{10} \div \frac{7}{8} =$$
$$\frac{9}{10} \times \frac{8}{7} = \frac{72}{70} = \frac{36}{35}$$

Rule.—*Multiply the dividend by the divisor inverted.*

Reduce mixed numbers to simple fractions. When the divisor is a compound fraction, invert each term, cancel, and multiply.

Find the quotients of

2. $\frac{20}{49} \div \frac{4}{7}$. *Ans.* $\frac{5}{7}$.
3. $\frac{24}{29} \div \frac{8}{9}$. *Ans.* $\frac{27}{29}$.
4. $\frac{35}{36} \div \frac{5}{6}$. *Ans.* $1\frac{1}{6}$.
5. $\frac{32}{49} \div \frac{4}{7}$. *Ans.* $1\frac{1}{7}$.
6. $\frac{33}{50} \div \frac{11}{14}$. *Ans.* $\frac{21}{25}$.
7. $\frac{54}{65} \div \frac{27}{52}$. *Ans.* $1\frac{3}{5}$.
8. $3\frac{1}{5} \div \frac{8}{9}$. *Ans.* $3\frac{3}{5}$.
9. $6\frac{3}{4} \div \frac{9}{11}$. *Ans.* $8\frac{1}{4}$.
10. $7\frac{1}{3} \div \frac{11}{12}$. *Ans.* 8.
11. $8\frac{4}{9} \div 1\frac{1}{3}$. *Ans.* $6\frac{1}{3}$.
12. $8\frac{3}{4} \div 2\frac{1}{2}$. *Ans.* $3\frac{1}{2}$.
13. $\frac{96}{35} \div \frac{72}{105}$. *Ans.* 4.
14. $\frac{105}{144} \div \frac{245}{432}$. *Ans.* $1\frac{2}{7}$.
15. $\frac{108}{121} \div \frac{144}{198}$. *Ans.* $1\frac{5}{22}$.
16. $8\frac{4}{5} \div 3\frac{3}{10}$. *Ans.* $2\frac{2}{3}$.
17. $8\frac{2}{3} \div 2\frac{1}{6}$. *Ans.* 4.
18. $25 \div 3\frac{3}{4}$. *Ans.* $6\frac{2}{3}$.
19. $15 \div 8\frac{3}{4}$. *Ans.* $1\frac{5}{7}$.
20. $558 \div 8\frac{6}{7}$. *Ans.* 63.
21. $804 \div 22\frac{1}{3}$. *Ans.* 36.
22. $729 \div 22\frac{1}{2}$. *Ans.* $32\frac{2}{5}$.
23. $5\frac{7}{8} \div 12\frac{8}{15}$. *Ans.* $\frac{15}{32}$.
24. $15\frac{15}{16} \div 7\frac{8}{11}$. *Ans.* $2\frac{1}{16}$.
25. $17\frac{3}{20} \div 10\frac{8}{9}$. *Ans.* $1\frac{23}{40}$.
26. $21\frac{3}{7} \div 12\frac{8}{21}$. *Ans.* $1\frac{19}{26}$.
27. $37\frac{3}{13} \div 18\frac{2}{39}$. *Ans.* $2\frac{1}{16}$.
28. $33\frac{1}{3} \div 19\frac{1}{21}$. *Ans.* $1\frac{3}{4}$.
29. $\frac{9}{16} \div \frac{9}{20}$ of $\frac{36}{39}$. *Ans.* $1\frac{17}{48}$.
30. $\frac{28}{42}$ of $7\frac{9}{10} \div \frac{72}{84}$. *Ans.* $6\frac{13}{90}$.
31. $(45 \div \frac{90}{112}) \div (\frac{27}{40} \times \frac{112}{135})$.

SPECIAL METHOD WITH MIXED NUMBERS.

1. Divide $66\frac{2}{3}$ by 4.

SOLUTION.—Dividing 66 by 4, we have 16 and a remainder of 2; 2 equals $\frac{6}{3}$, which, added to $\frac{2}{3}$, equals $\frac{8}{3}$; $\frac{8}{3}$ divided by 4, equals $\frac{2}{3}$; hence, the quotient is $16\frac{2}{3}$.

OPERATION.

$$4\overline{)66\frac{2}{3}}$$
$$16\frac{2}{3}$$

2. Divide 32 by $4\frac{1}{3}$.

SOLUTION.—We reduce both numbers to a common denominator. $4\frac{1}{3}$ equals $\frac{13}{3}$, and 32 equals $\frac{96}{3}$; $\frac{13}{3}$ is contained in $\frac{96}{3}$, $7\frac{5}{13}$ times; hence the quotient is $7\frac{5}{13}$.

OPERATION.

$4\frac{1}{3}$	32
3	3
13	96
	$7\frac{5}{13}$

Divide

3. $822\frac{5}{6}$ by 4. *Ans.* $205\frac{17}{24}$.
4. $785\frac{3}{5}$ by 7. *Ans.* $112\frac{8}{35}$.
5. $654\frac{3}{4}$ by 8. *Ans.* $81\frac{27}{32}$.
6. $287\frac{5}{8}$ by 12. *Ans.* $23\frac{31}{32}$.
7. $129\frac{3}{5}$ by 16. *Ans.* $8\frac{1}{10}$.
8. 29 by $4\frac{2}{3}$. *Ans.* $6\frac{3}{14}$.
9. 81 by $5\frac{3}{4}$. *Ans.* $14\frac{2}{23}$.
10. 165 by $7\frac{5}{8}$. *Ans.* $21\frac{39}{61}$.
11. 189 by $6\frac{1}{10}$. *Ans.* $30\frac{60}{61}$.
12. 254 by $5\frac{1}{12}$. *Ans.* $49\frac{59}{61}$.

ORAL EXERCISES.

1. If it takes $\frac{3}{4}$ of a yard of silk to make a shopping-bag, how many can be made from 24 yards of silk?

2. If a yard of serge cost $\frac{3}{5}$ of a dollar, how many yards can I buy for 9 dollars?

3. A dressmaker bought some silk at $\$1\frac{1}{2}$ a yard, paying $\$7\frac{1}{2}$ for it; how many yards did she buy?

4. Henry, having $\frac{12}{16}$ of a quart of nuts, divided them among 8 of his schoolmates; how many did each receive?

5. At $\frac{1}{2}$ a dollar a day for board, how many days' board can a lady get for $10.50?

6. If one-half a peck of apples cost 15 cents, what will a bushel and a half cost?

7. A man divides 14 acres into building lots of $\frac{7}{8}$ of an acre each; how many lots did he make?

8. A lady distributed $35 among some poor children, giving them $\$4\frac{3}{8}$ apiece; how many children were there?

9. At the rate of 4 oranges for 6 cents, what will be the cost of 5 dozen oranges?

10. A school-boy shared 18 apples equally with his companions, giving to each $4\frac{1}{2}$ apples; required the number of his companions.

11. A grocer sold 8 bushels of potatoes, worth $\$\frac{5}{8}$ a bushel, and received for them eggs at the rate of $\$\frac{2}{3}$ a dozen; how many eggs did he get?

PRACTICAL EXAMPLES.

1. If a yard of silk costs $\$2\frac{1}{4}$, how many yards can I buy for $\$16\frac{7}{8}$?

Solution.—$\$16\frac{7}{8}=\$\frac{135}{8}$; $\$2\frac{1}{4}=\$\frac{9}{4}$; I can buy as many yards as $\frac{135}{8}\div\frac{9}{4}$, or $\frac{135}{8}\times\frac{4}{9}$, which, by cancelling and multiplying, we find equals $\frac{15}{2}$, or $7\frac{1}{2}$.

Note.—Express the operation, and then cancel common factors.

2. If $7\frac{1}{2}$ yards of ribbon cost $\$4.68\frac{3}{4}$, what is the price per yard? *Ans.* $\$0.62\frac{1}{2}$.

3. If $16\frac{2}{3}$ pounds of paper cost \$6.25, what is the price per pound? *Ans.* \$0.37$\frac{1}{2}$.

4. If $8\frac{1}{4}$ yards of silk-warp Henrietta cost \$12$\frac{3}{8}$, what is the price per yard? *Ans.* \$1$\frac{1}{2}$.

5. If $8\frac{7}{8}$ barrels of flour cost \$56$\frac{4}{5}$, what is the cost per barrel? *Ans.* \$6$\frac{2}{5}$.

6. If $6\frac{3}{4}$ tons of hay cost \$46$\frac{13}{32}$, what is the price per ton? *Ans.* \$6$\frac{7}{8}$.

7. If 15 barrels of vinegar cost \$71$\frac{1}{4}$, what is the cost of a barrel? *Ans.* \$4$\frac{3}{4}$.

8. A grocer paid \$33$\frac{3}{4}$ for 15 Christmas turkeys; what was the average price of each? *Ans.* \$2$\frac{1}{4}$.

9. A charitable society divided \$250$\frac{1}{4}$ equally among 7 poor women; how much did each receive? *Ans.* \$35$\frac{3}{4}$.

10. I paid \$38$\frac{1}{2}$ for $6\frac{7}{8}$ yards of cloth; what was the price per yard? *Ans.* \$5$\frac{3}{5}$.

11. John shared \$75 with his sisters, giving to each \$7$\frac{1}{2}$; required the number of sisters. *Ans.* 9.

12. A lady distributed \$349$\frac{1}{4}$ among some poor people, giving each person \$31$\frac{3}{4}$; how many were there? *Ans.* 11.

13. William shared an estate of \$7500 equally with his 4 brothers and 3 sisters; how much did each receive? *Ans.* \$937$\frac{1}{2}$.

14. The product of two numbers, diminished by $112\frac{3}{4}$, is $127\frac{2}{3}$; one number is 15, what is the other? *Ans.* $16\frac{1}{36}$.

15. A seamstress bought a sewing-machine for \$60, paying \$25 down; how long will it take to pay for it if she pays \$6$\frac{1}{4}$ a month? *Ans.* $5\frac{3}{5}$ months.

16. A man's wages are \$2$\frac{3}{4}$ a day, and his daily expenses \$1$\frac{7}{8}$; how many days must he labor to save enough to buy a suit of clothes worth \$35$\frac{7}{8}$? *Ans.* 41 days.

REDUCTION OF COMPLEX FRACTIONS.

155. A Complex Fraction is one whose numerator or denominator, or both, are fractional: as $\frac{5}{\frac{4}{5}}$, $\frac{\frac{2}{3} \text{ of } \frac{4}{5}}{\frac{3}{4} \text{ of } \frac{5}{8}}$.

A complex fraction is not really a fraction, but an expression of one fraction divided by another.

WRITTEN EXERCISES.

1. Reduce $\frac{\frac{5}{6}}{\frac{7}{8}}$ to a simple fraction.

SOLUTION.—This expression means that $\frac{5}{6}$ is to be divided by $\frac{7}{8}$, and, inverting the divisor and multiplying, we have $\frac{5}{6} \times \frac{8}{7}$, which equals $\frac{20}{21}$.

OPERATION.

$$\frac{\frac{5}{6}}{\frac{7}{8}} = \frac{5}{6} \div \frac{7}{8} = \frac{5}{6} \times \frac{8}{7} = \frac{20}{21}$$

Rule.—*Multiply the numerator of the complex fraction by its denominator inverted.*

2. $\frac{\frac{3}{4}}{\frac{5}{6}}$. *Ans.* $\frac{9}{10}$.

3. $\frac{\frac{4}{9}}{\frac{7}{10}}$. *Ans.* $\frac{40}{63}$.

4. $\frac{\frac{15}{19}}{\frac{45}{76}}$. *Ans.* $1\frac{1}{3}$.

5. $\frac{3\frac{1}{3}}{4\frac{3}{4}}$. *Ans.* $\frac{40}{57}$.

6. $\frac{7\frac{5}{8}}{11\frac{1}{11}}$. *Ans.* $\frac{11}{16}$.

7. $\frac{5\frac{5}{12}}{9\frac{8}{15}}$. *Ans.* $\frac{25}{44}$.

8. $\frac{6\frac{3}{5}}{4\frac{1}{8}}$. *Ans.* $1\frac{3}{5}$.

9. $\frac{5\frac{1}{4}}{4\frac{1}{8}}$. *Ans.* $1\frac{3}{11}$.

10. $\frac{\frac{1}{9} + \frac{5}{6}}{\frac{5}{11} + \frac{3}{7}}$. *Ans.* $1\frac{5}{72}$.

11. $\frac{\frac{9}{10} \text{ of } \frac{25}{27}}{\frac{21}{25} \text{ of } \frac{17}{18}}$. *Ans.* $1\frac{6}{119}$.

12. $\frac{5\frac{1}{2} - 3\frac{1}{3}}{7\frac{1}{8} - 3\frac{1}{5}}$. *Ans.* $\frac{260}{471}$.

13. $\frac{18\frac{3}{4}}{62\frac{1}{2}} \times \frac{4\frac{3}{5}}{7\frac{3}{16}}$. *Ans.* $\frac{24}{125}$.

14. $\frac{\frac{21}{25}}{\frac{16}{19}} \times \frac{\frac{45}{49}}{\frac{27}{64}}$. *Ans.* $2\frac{6}{85}$.

15. $\frac{\frac{17}{35} \div \frac{85}{95}}{\frac{152}{105} \div \frac{19}{28}}$. *Ans.* $\frac{57}{224}$.

16. If a horse eats $\frac{3}{4}$ of a bushel of oats in a day, in how many days will he eat $10\frac{1}{2}$ bushels? *Ans.* 14 days.

17. What is the value of $\frac{9}{10}$ of $\frac{25}{27}$ divided by $\frac{15}{16}$ of $\frac{17}{18}$? *Ans.* $\frac{16}{17}$.

RELATION OF NUMBERS.

156. The **Relation of Numbers** is their relative value as compared with one another.

This subject is equivalent to Ratio, but is presented here as affording an excellent illustration of the analysis of numbers.

CASE I.

157. To find what part one number is of another.

ORAL EXERCISES.

1. What part of 15 is 6?

SOLUTION.—*One* is $\frac{1}{15}$ of 15, and if 1 is $\frac{1}{15}$ of 15, 6 is 6 times $\frac{1}{15}$ of 15, or $\frac{6}{15}$, or $\frac{2}{5}$ of 15. Therefore 6 is $\frac{2}{5}$ of 15.

2. 6 is what part of 36? 10 is what part of 40? 32 is what part of 72? 90 is what part of 108?

3. What part of 3 is $\frac{3}{5}$? Of 4 is $\frac{2}{5}$? Of 6 is $\frac{3}{4}$? Of 7 is $\frac{5}{8}$? Of 9 is $\frac{3}{7}$? Of 5 is $\frac{1}{2}$ of $\frac{3}{4}$?

4. What part of $\frac{3}{4}$ is $\frac{2}{5}$?

SOLUTION.—$\frac{1}{4}$ is $\frac{1}{3}$ of $\frac{3}{4}$, and $\frac{4}{4}$, or *one*, is 4 times $\frac{1}{3}$, or $\frac{4}{3}$ of $\frac{3}{4}$. If 1 equals $\frac{4}{3}$ of $\frac{3}{4}$, $\frac{2}{5}$ equals $\frac{2}{5}$ of $\frac{4}{3}$, which equals $\frac{8}{15}$, of $\frac{3}{4}$.

5. What part of $\frac{5}{8}$ is $\frac{3}{4}$? Of $\frac{6}{7}$ is $\frac{3}{8}$? Of $\frac{4}{5}$ is $\frac{5}{8}$? Of $\frac{5}{7}$ is $2\frac{1}{3}$? Of $\frac{6}{7}$ is $\frac{2}{3}$ of $\frac{6}{7}$?

WRITTEN EXERCISES.

What part is

1. 35 of 70?	*Ans.* $\frac{1}{2}$.	**5.** $\frac{17}{21}$ of $\frac{85}{42}$?	*Ans.* $\frac{2}{5}$.
2. 16 of 48?	*Ans.* $\frac{1}{3}$.	**6.** $\frac{28}{37}$ of $\frac{54}{74}$?	*Ans.* $\frac{28}{27}$.
3. 78 of 24?	*Ans.* $\frac{13}{4}$.	**7.** $9\frac{1}{15}$ of $11\frac{2}{3}$?	*Ans.* $\frac{136}{175}$.
4. 84 of 98?	*Ans.* $\frac{6}{7}$.	**8.** $11\frac{12}{13}$ of $7\frac{6}{7}$?	*Ans.* $\frac{217}{143}$.

9. A boy bought 15 apricots for 12 cents; what will 60 apricots cost at the same rate?

SOLUTION.—If 15 apricots cost 12 cents, 60 apricots, which are $\frac{60}{15}$, or 4 times 15 apricots, will cost 4 times 12 cents, or 48 cents.

OPERATION.
15 cost 12 cts.
60 cost 12 cts. × 4
= 48 cts.

10. A girl bought 15 lemons for 20 cents; what will 45 lemons cost at the same rate? *Ans.* 60 cents.

11. A farmer bought 25 sheep for \$137; what will 125 sheep cost at the same rate? *Ans.* \$685.

12. A drover paid \$1000 for 28 horses; what will 35 horses cost at the same rate? *Ans.* \$1250.

13. I purchase 98 pencils for 84 cents; what will 175 pencils cost at the same rate? *Ans.* \$1.50.

14. A careless boy lost $\frac{4}{5}$ of $\frac{5}{6}$ of a certain sum of money; how many fifths of $\frac{5}{6}$ of the sum remained? *Ans.* $\frac{1}{5}$.

15. Two boys each picked $\frac{7}{8}$ of a bushel of chestnuts, and the first boy sold $\frac{2}{3}$ of his to the second; what part of what the second has equals what the first had? *Ans.* $\frac{1}{5}$.

CASE II.

158. When a number and its relation to another number are given, to find that number.

ORAL EXERCISES.

1. If $\frac{5}{8}$ of a number equals 21, what is the number?

SOLUTION.—If $\frac{5}{8}$ of a number is 21, $\frac{1}{8}$ of the number is $\frac{1}{5}$ of 21, which is 7, and $\frac{8}{8}$ of the number is 5 times 7, or 35.

2. Nine is $\frac{3}{4}$ of what number? 15 is $\frac{3}{5}$ of what number? 20 is $\frac{4}{5}$ of what number? 36 is $\frac{6}{7}$ of what number? 42 is $\frac{7}{8}$ of what number?

3. $\frac{4}{5}$ of the money a merchant has in bank equals \$400; how much has he in bank?

4. Mr. Wilson drew out of bank $\frac{1}{5}$ of his deposit, and then had \$240 in bank; what was his deposit?

5. A man willed $\frac{1}{3}$ of his property to his wife, and the remainder, \$3600, to his son; what was his property?

6. Mary gave $\frac{1}{3}$ of her money for a silk dress and $\frac{1}{4}$ for a cloak, and had \$50 remaining; how much had she at first?

7. Edith, after spending $\frac{3}{5}$ of her money, found that $16 was $\frac{2}{5}$ of what she had remaining; how much had she at first?

8. If $\frac{1}{4}$ of an army were killed and wounded, $\frac{2}{3}$ taken prisoners, and 800 men escaped, of how many men did the army consist?

WRITTEN EXERCISES.

Find the number of which

1. 180 is $\frac{3}{4}$ *Ans.* 240.
2. 168 is $\frac{4}{5}$. *Ans.* 210.
3. 220 is $\frac{5}{6}$. *Ans.* 264.
4. 217 is $\frac{7}{8}$. *Ans.* 248.
5. 450 is $\frac{5}{11}$. *Ans.* 990.
6. 480 is $\frac{6}{13}$. *Ans.* 1040.
7. 549 is $\frac{9}{8}$. *Ans.* 488.
8. 343 is $\frac{7}{4}$. *Ans.* 196.

9. A has $240, which is $\frac{3}{4}$ of B's money; how much has B?

SOLUTION.—If $240 is $\frac{3}{4}$ of B's money, $\frac{1}{4}$ of B's money equals $\frac{1}{3}$ of $240, which is $80, and $\frac{4}{4}$ of B's money equals 4 times $80, or $320.

OPERATION.

$\frac{3}{4}$ B's = $240

$\frac{1}{4}$ B's = $80

$\frac{4}{4}$ B's = $320

10. A farmer sold 360 sheep, which is $\frac{4}{5}$ of what he has left; how many has he left? *Ans.* 450.

11. A drover sold $\frac{2}{7}$ of his cows, and then had 320 remaining; how many had he before the sale? *Ans.* 448 cows.

12. A newsboy sold 1240 papers this week, which was $\frac{8}{9}$ as many as he sold last week; how many did he sell last week? *Ans.* 1395.

13. A lady invested $\frac{4}{5}$ of her money in bank stock, and had $7840 remaining; how much money had she before her investment? *Ans.* $39,200.

14. A postman delivered 840 letters one week, which was $\frac{6}{7}$ of the number delivered the previous week; how many did he deliver both weeks? *Ans.* 1820.

ARITHMETICAL ANALYSIS.

159. **Arithmetical Analysis** with fractions includes the following four cases:

CASE I.

160. To pass from one integer to another integer.

ORAL EXERCISES.

1. If 5 girls earn $\$3\frac{3}{4}$ in a day, how much will 7 girls earn at the same rate?

Solution.—If 5 girls earn $\$3\frac{3}{4}$ or $\$\frac{15}{4}$ in a day, one girl will earn $\frac{1}{5}$ of $\$\frac{15}{4}$, or $\$\frac{3}{4}$; and 7 girls will earn 7 times $\$\frac{3}{4}$, which are $\$\frac{21}{4}$, or $\$5\frac{1}{4}$.

2. If 3 boys spend $\$7\frac{1}{2}$ on a holiday, what will 8 boys spend at the same rate?

3. If 4 "trolleys" take in $\$9\frac{3}{8}$ in a trip, how much will 10 trolleys take in at the same rate?

4. How long will it take a farmer to plow 10 furrows, if it takes him $8\frac{1}{4}$ minutes to plow 3 furrows?

5. How much will a newsboy earn in 7 days, if he earns $\$20\frac{4}{5}$ in 4 days?

6. If 3 felt hats can be bought for $\$6\frac{3}{4}$, how much will 10 such hats cost?

7. How far will a sail-boat "run" in 8 hours, if it "runs" $21\frac{1}{2}$ miles in 4 hours?

WRITTEN EXERCISES.

1. If 5 cows cost $\$93\frac{3}{4}$, what will 8 cows cost at the same rate?

Solution.—If 5 cows cost $\$93\frac{3}{4}$, or $\$\frac{375}{4}$, one cow costs $\frac{1}{5}$ of $\$\frac{375}{4}$, or $\$\frac{75}{4}$; and 8 cows will cost 8 times $\$\frac{75}{4}$, or $150.

OPERATION.

5 cows cost $\$93\frac{3}{4} = \$\frac{375}{4}$
1 cow costs $\frac{1}{5}$ of $\$\frac{375}{4} = \$\frac{75}{4}$
8 cows cost $\$\frac{75}{4} \times 8 = \$150$

2. Amos bought 4 rabbits for $2.36; what will 6 rabbits cost at the same rate? *Ans.* $3.54.

3. Mrs. Smith bought 7 turkeys for $\$8\frac{3}{4}$; what will 15 turkeys cost at the same rate? *Ans.* $18.75.

4. How much must I pay for 18 yards of lace, at the rate of 5 yards for $\$18\frac{3}{4}$? *Ans.* $\$67\frac{1}{2}$.

5. What did it cost a teacher to supply his class with 25 atlases, at the rate of 4 atlases for $\$6\frac{6}{7}$? *Ans.* $\$42\frac{6}{7}$.

6. What must the school-board pay for 22 Readers, at the rate of 8 Readers for $3.80? *Ans.* $10.45.

7. A farmer sold 5 tons of hay for $\$78\frac{8}{9}$; how much will he get for 20 tons at the same rate? *Ans.* $\$315\frac{5}{9}$.

8. Required the cost of making 48 rods of fence, at the rate of $35 for 12 rods. *Ans.* $140.

9. A butcher paid $62.50 for calves, at the rate of $6.25 for 2 calves; how many calves did he buy? *Ans.* 20 calves.

10. A freight-train ran $112\frac{1}{2}$ miles in 9 hours; how far at this rate would it run in 81 hours? *Ans.* $1012\frac{1}{2}$ miles.

CASE II.

161. To pass from a fraction to an integer.

ORAL EXERCISES.

1. A tailor bought $\frac{3}{4}$ of a piece of cloth for $36; how many dollars are 4 pieces of the cloth worth?

SOLUTION.—If $\frac{3}{4}$ of a piece cost $36, $\frac{1}{4}$ of a piece costs $\frac{1}{3}$ of $36, or $12, and $\frac{4}{4}$ of a piece, or 1 piece, costs 4 times $12, or $48; and 4 pieces will cost 4 times $48, or $192.

2. If $\frac{5}{8}$ of the earnings of a typewriter in a week are $10, what will she earn in 7 weeks?

3. A dressmaker bought 8 yards of velvet at the rate of $8 for $2\frac{2}{3}$ yards; what did she pay for it?

4. A boy sold 12 dozen daily papers at the rate of $\frac{1}{3}$ of a dozen for $\frac{1}{2}$ a dime; what did he receive for his papers?

5. A stationer bought 25 reams of paper at the rate of $\frac{5}{8}$ of a ream for $\frac{4}{5}$ of a dollar; required the cost of the paper.

6. Two-fifths of the distance I rode on my bicycle was 20 miles; how far did my brother ride if he rode 6 times as far as I did?

WRITTEN EXERCISES.

1. If $\frac{2}{3}$ of an acre of land cost $96, what will one acre cost?

	OPERATION.
SOLUTION.—If $\frac{2}{3}$ of an acre cost $96, $\frac{1}{3}$ of an acre cost $\frac{1}{2}$ of $96, which is $48; and $\frac{3}{3}$ of an acre, or 1 acre, will cost 3 times $48, or $144.	$\frac{2}{3}$ cost $96 $\frac{1}{3}$ cost $48 $\frac{3}{3}$ cost $144

2. If $\frac{3}{4}$ of what a mechanic paid for his house is $2580, what did the house cost him? *Ans.* $3440.

3. If $\frac{5}{6}$ of a box of oranges cost $5.50, what will 8 boxes cost at the same rate? *Ans.* $52.80.

4. What will a dress pattern of 15 yards of silk cost, at the rate of $2\frac{2}{3}$ yards for $4? *Ans.* $22.50.

5. If $\frac{7}{8}$ of a cask of sugar cost $17.43, what will be the cost of 20 similar casks of sugar? *Ans.* $398.40.

6. What must a manufacturer pay for 18 tons of iron ore, at the rate of $11.64 for $\frac{6}{7}$ of a ton? *Ans.* $244.44.

7. If $100 is $\frac{4}{5}$ of the cost of a yoke of oxen, what would be the cost of 6 yoke of oxen at the same rate? *Ans.* $750.

8. I bought $3\frac{1}{2}$ acres of land for $294.35; how much must I pay for 25 acres at the same rate? *Ans.* $2102.50.

9. A dealer bought 30 barrels of cranberries at the rate of $7\frac{1}{2}$ barrels for $60; what did they cost him? *Ans.* $240.

10. A dairyman sold 66 gallons of milk at the rate of $2\frac{3}{4}$ gallons for $62\frac{1}{2}$ cents; required the cost. *Ans.* $15.

11. If a typewriter could write 75 pages in $7\frac{1}{2}$ hours, how much would she write in a week, 6 days, working 8 hours a day? *Ans.* 480 pages.

12. How much will it cost to transport 75 tons of freight from Boston to Albany at $\$1\frac{1}{4}$ a hundredweight? *Ans.* $1875.

CASE III.

162. To pass from an integer to a fraction.

ORAL EXERCISES.

1. What cost $\frac{3}{8}$ of a bushel of English walnuts if 3 bushels cost $6?

SOLUTION.—If 3 bushels of walnuts cost $6, 1 bushel costs $\frac{1}{3}$ of $6, or $2; then $\frac{1}{8}$ of a bushel costs $\frac{1}{8}$ of $2, or $\$\frac{1}{4}$, and $\frac{3}{8}$ of a bushel costs 3 times $\$\frac{1}{4}$, or $\$\frac{3}{4}$.

2. If an errand-boy can earn $\$3\frac{3}{4}$ in 5 days, how much will he earn in $\frac{1}{2}$ of a day?

3. If a postman walks $6\frac{3}{4}$ miles in 3 hours, how far will he walk at this rate in $5\frac{1}{3}$ hours?

4. If 5 bushels of oats cost $\$1\frac{7}{8}$, for what can I sell $\frac{3}{8}$ of a bushel at the rate of cost?

5. If 6 pounds of tea cost $2.40, for what can I sell $6\frac{1}{4}$ pounds and neither gain nor lose?

6. Mr. Drexel's yacht sailed 40 miles in 8 hours; how far at this rate did it sail in $5\frac{1}{5}$ hours?

WRITTEN EXERCISES.

1. If 1 cord of wood cost $4.50, what will $\frac{2}{3}$ of a cord cost?

SOLUTION.—If 1 cord of wood cost $4.50, $\frac{1}{3}$ of a cord costs $\frac{1}{3}$ of $4.50, which is $1.50; and if $\frac{1}{3}$ of a cord cost $1.50, $\frac{2}{3}$ of a cord will cost 2 times $1.50, or $3.

OPERATION.

1 cord cost $4.50
$\frac{1}{3}$ cord cost $1.50
$\frac{2}{3}$ cord cost $3.00

2. A lady paid $35.50 for a colt; how much would she have paid had she given $\frac{3}{5}$ as much? *Ans.* $21.30.

3. A grocer bought 6 dozen eggs for $1.50; what will $2\frac{3}{4}$ dozen cost at this rate? *Ans.* $68\frac{3}{4}$ cents.

4. Bought 15 boxes of oranges for $\$22\frac{1}{2}$, and sold $\frac{4}{5}$ of a box at the same rate; what was received for them? *Ans.* $1.20.

5. A manufacturer bought $16\frac{2}{5}$ tons of lead ore at the rate of $\$22\frac{3}{4}$ for 7 tons; what did it cost? *Ans.* $\$53\frac{3}{10}$.

6. A farmer bought $10\frac{4}{9}$ acres of land at the rate of $2247 for 7 acres; required the cost. *Ans.* $\$3352\frac{1}{3}$.

7. Mr. Smith bought $22\frac{7}{12}$ cords of wood at the rate of $\$18\frac{3}{4}$ for 5 cords; what did it cost him? *Ans.* $\$84\frac{11}{16}$.

8. When hay is selling at the rate of 6 tons for $\$67\frac{1}{5}$, what must I pay for $10\frac{1}{14}$ tons? *Ans.* $\$112\frac{4}{5}$.

9. When peanuts are selling at the rate of 8 bushels for $\$7\frac{1}{2}$, how much will $18\frac{11}{15}$ bushels cost? *Ans.* $\$17\frac{9}{16}$.

10. How much must a foundry-man pay for $29\frac{3}{5}$ tons of iron ore, at the rate of 12 tons for $\$33\frac{3}{4}$? *Ans.* $83.25.

CASE IV.

163. To pass from a fraction to a fraction.

ORAL EXERCISES.

1. What will $\frac{5}{6}$ of a yard of silk cost, if $\frac{3}{4}$ of a yard cost $\$\frac{9}{10}$?

SOLUTION.—If $\frac{3}{4}$ of a yard of silk cost $\$\frac{9}{10}$, $\frac{1}{4}$ of a yard will cost $\frac{1}{3}$ of $\$\frac{9}{10}$, or $\$\frac{3}{10}$, and $\frac{4}{4}$ of a yard will cost 4 times $\$\frac{3}{10}$, or $\$\frac{6}{5}$. If 1 yard cost $\$\frac{6}{5}$, $\frac{5}{6}$ of a yard will cost $\frac{5}{6}$ of $\$\frac{6}{5}$, or $1.

2. What will $\frac{7}{8}$ of a yard of cloth cost, if $\frac{5}{8}$ of a yard cost $\$6\frac{2}{3}$?

3. If $\frac{4}{5}$ of a barrel of flour is worth $\$5\frac{1}{3}$, what is $\frac{7}{8}$ of a barrel worth?

4. What is $\frac{5}{8}$ of an acre of land worth, if $\frac{7}{8}$ of an acre is worth $126?

5. How much must I pay for $\frac{10}{21}$ of a cord of wood, if $\frac{5}{7}$ of a cord is worth $1.50?

6. What must I pay for $\frac{5}{6}$ of a bushel of berries, if $\frac{3}{4}$ of a bushel is worth $6?

7. I paid $1400 for $4\frac{2}{3}$ acres of land; how much will $5\frac{3}{4}$ acres cost at the same rate?

WRITTEN EXERCISES.

1. If $\frac{3}{4}$ of an acre of land cost $210, what will $\frac{6}{7}$ of an acre cost?

SOLUTION. If $\frac{3}{4}$ of an acre cost $210, $\frac{1}{4}$ of an acre will cost $\frac{1}{3}$ of $210, and $\frac{4}{4}$ of an acre will cost $\frac{4}{3}\times\$210$, and $\frac{6}{7}$ of an acre will cost $\frac{6}{7}\times\frac{4}{3}\times\$210$, which, by cancellation, we find is $240.

OPERATION.

$$\frac{6}{7}\times\frac{4}{3}\times\$210=\$240. \quad \textit{Ans.}$$

NOTE.—The method by cancellation is preferred with the more difficult problems.

2. If $7\frac{2}{5}$ yards of cloth are given for $16\frac{1}{4}$ cords of wood, how many cords could be got for $10\frac{3}{5}$ yards? *Ans.* $23\frac{17}{20}$.

3. How much will $7\frac{3}{4}$ tons of hay cost at the rate of $4\frac{8}{9}$ tons for \$$58\frac{2}{3}$? *Ans.* \$93.

4. A bought $7\frac{3}{5}$ yards of cloth for \$$10\frac{6}{7}$; what would $8\frac{3}{4}$ yards cost at the same rate? *Ans.* \$$12\frac{1}{2}$.

5. How many barrels of sugar can you get for \$$87\frac{3}{4}$, if $2\frac{1}{2}$ barrels are worth \$$24\frac{3}{8}$? *Ans.* 9 barrels.

6. If 9 cows eat $7\frac{1}{5}$ bushels of turnips in a day, how many cows would eat $33\frac{3}{5}$ bushels in the same time? *Ans.* 42 cows.

7. If 7 sheep cost \$$22\frac{3}{4}$, how many sheep can you buy for \$$487\frac{1}{2}$? *Ans.* 150 sheep.

8. If I give $7\frac{2}{3}$ tons of hay for $40\frac{8}{9}$ tons of coal, how much coal would I get for $8\frac{1}{4}$ tons of hay? *Ans.* 44 tons.

9. A paid \$$189\frac{1}{5}$ for $8\frac{3}{5}$ acres of land; how much must he pay at the same rate for $25\frac{1}{4}$ acres? *Ans.* \$$555\frac{1}{2}$.

10. If $5\frac{3}{8}$ tons of hay are given for $28\frac{2}{3}$ tons of coal, how much coal could be obtained for $7\frac{3}{4}$ tons of hay? *Ans.* $41\frac{1}{3}$ tons.

11. How many bushels of wheat can I buy for \$$114\frac{2}{5}$, at the rate of $29\frac{2}{3}$ bushels for \$$28\frac{3}{5}$? *Ans.* $118\frac{2}{3}$.

12. If $35\frac{3}{7}$ yards of cloth cost \$$39\frac{6}{7}$, how much will $5\frac{7}{9}$ yards cost at the same rate? *Ans.* \$$6\frac{1}{2}$.

13. How much will $57\frac{3}{4}$ tons of hay cost at the rate of $41\frac{8}{9}$ tons for \$$502\frac{2}{3}$? *Ans.* \$693.

14. If I receive $42\frac{11}{16}$ barrels of flour for $15\frac{1}{4}$ tons of iron, how many barrels of flour should I receive for $30\frac{1}{2}$ tons? *Ans.* $85\frac{3}{8}$ barrels.

ORAL EXERCISES.

MISCELLANEOUS EXAMPLES.

1. If 4 boys earn \$$6\frac{2}{5}$ in a week, how much does each boy earn?

2. If a barrel of apples cost \$$5\frac{1}{4}$, how many "quarters" will 2 barrels cost?

3. $\frac{3}{5}$ of \$80 is 2 times what a lady gave for a bureau; what was the cost of the bureau?

4. A man owning $\frac{2}{3}$ of the stock of a bank sold $\frac{3}{4}$ of his share; how much did he sell?

5. Mary bought a slate for $\frac{1}{4}$ of a dollar, and an arithmetic for $\frac{5}{8}$ of a dollar; what was the cost of both?

6. If $\frac{1}{2}$ of a number, increased by $\frac{1}{3}$ of the number, equals 50, what is the number?

7. A lady owes a store bill of \$$\frac{3}{8}$; if she hands the clerk \$$\frac{3}{4}$, how much change would she receive?

8. If $\frac{1}{2}$ of a pole is in the air, $\frac{1}{3}$ in the water, and the rest in the mud, how much is in the mud?

9. How much will 14 pounds of cloves cost, if $3\frac{1}{3}$ pounds can be bought for 50 cents?

10. How far will a man drive in $5\frac{1}{3}$ hours, at the rate of 21 miles in $3\frac{1}{2}$ hours?

11. Harry found 60 cents, which is $\frac{3}{4}$ of $\frac{1}{2}$ of what he then had; how much had he at first?

12. Mrs. Smith exchanged 10 pounds of butter, at 15 cents a pound, for calico worth $6\frac{1}{4}$ cents a yard; how many yards did she receive?

13. William gave his sister 20 cents, which is $\frac{1}{5}$ of what he had at first, and $\frac{1}{2}$ of what his sister now has; how much had each at first?

REVIEW OF FRACTIONS.

Reduce to an improper fraction:

1. $26\frac{12}{13}$.	5. $68\frac{18}{19}$.	9. $463\frac{1}{11}$.
2. $38\frac{13}{14}$.	6. $75\frac{21}{25}$.	10. $407\frac{21}{37}$.
3. $47\frac{15}{16}$.	7. $86\frac{18}{31}$.	11. $528\frac{35}{36}$.
4. $76\frac{13}{18}$.	8. $94\frac{11}{35}$.	12. $685\frac{16}{25}$.

Reduce to a mixed number:

13. $\frac{350}{13}$.	17. $\frac{1310}{19}$.	21. $\frac{5094}{11}$.
14. $\frac{545}{14}$.	18. $\frac{1896}{25}$.	22. $\frac{15080}{37}$.
15. $\frac{767}{18}$.	19. $\frac{2684}{31}$.	23. $\frac{19048}{36}$.
16. $\frac{1381}{18}$.	20. $\frac{3301}{35}$.	24. $\frac{17141}{25}$.

Reduce to lowest terms:

25. $\frac{570}{630}$. *Ans.* $\frac{19}{21}$.
26. $\frac{756}{966}$. *Ans.* $\frac{18}{23}$.
27. $\frac{1785}{3675}$. *Ans.* $\frac{17}{35}$.
28. $\frac{2450}{3290}$. *Ans.* $\frac{35}{47}$.
29. $\frac{4305}{5355}$. *Ans.* $\frac{41}{51}$.
30. $\frac{11055}{13365}$. *Ans.* $\frac{67}{81}$.
31. $\frac{19635}{22407}$. *Ans.* $\frac{85}{97}$.
32. $\frac{28952}{30492}$. *Ans.* $\frac{94}{99}$.
33. $\frac{8580}{10296}$. *Ans.* $\frac{5}{6}$.
34. $\frac{19998}{60192}$. *Ans.* $\frac{101}{304}$.
35. $\frac{47916}{48708}$. *Ans.* $\frac{121}{123}$.
36. $\frac{25300}{25520}$. *Ans.* $\frac{115}{116}$.

Reduce to simple fractions:

37. $\frac{15}{16}$ of $\frac{24}{35}$ of $\frac{48}{45}$. *Ans.* $\frac{48}{70}$.
38. $\frac{3}{7}$ of $\frac{42}{75}$ of $\frac{65}{84}$. *Ans.* $\frac{13}{70}$.
39. $\frac{13}{17}$ of $\frac{84}{125}$ of $\frac{85}{91}$. *Ans.* $\frac{12}{25}$.
40. $\frac{28}{45}$ of $\frac{31}{38}$ of $\frac{57}{62}$. *Ans.* $\frac{7}{15}$.
41. $\frac{18}{25}$ of $\frac{35}{27}$ of $\frac{117}{140}$. *Ans.* $\frac{39}{50}$.
42. $\frac{13}{15}$ of $\frac{12}{35}$ of $\frac{21}{30}$. *Ans.* $\frac{26}{125}$.
43. $\frac{3}{7}$ of $6\frac{3}{10}$ of $4\frac{6}{11}$. *Ans.* $12\frac{3}{11}$.
44. $\frac{3\frac{1}{2}}{7}$ of $41\frac{1}{4}$ of $3\frac{1}{15}$. *Ans.* $63\frac{1}{4}$.

REVIEW OF FRACTIONS.

Find the sum of

1. $1\frac{7}{12}+3\frac{11}{15}+4\frac{19}{20}$. *Ans.* $10\frac{4}{15}$.
2. $8\frac{4}{7}+5\frac{1}{4}+3\frac{7}{8}$. *Ans.* $17\frac{39}{56}$.
3. $8\frac{1}{5}+45\frac{5}{8}+2\frac{17}{20}$. *Ans.* $56\frac{27}{40}$.
4. $\frac{4\frac{2}{9}}{8\frac{4}{9}}+\frac{1\frac{1}{3}}{3\frac{1}{3}}$. *Ans.* $\frac{9}{10}$.
5. $37\frac{3}{4}+28\frac{4}{5}+15\frac{7}{20}$. *Ans.* $81\frac{9}{10}$.
6. $16\frac{5}{7}+19\frac{7}{12}+73\frac{2}{3}$.
7. $\frac{3\frac{1}{2}+6\frac{1}{4}}{3}+\frac{24\frac{1}{3}+16\frac{1}{6}}{8}$.
8. $\frac{6\frac{2}{3}}{13\frac{1}{3}}+\frac{2\frac{5}{6}}{11\frac{1}{3}}+\frac{8\frac{1}{3}}{25}$. *Ans.* $1\frac{1}{12}$.

Find the difference of

9. $700-\frac{7}{100}$. *Ans.* $699\frac{93}{100}$.
10. $36\frac{3}{8}-18\frac{3}{7}$. *Ans.* $17\frac{53}{56}$.
11. $58\frac{7}{40}-20\frac{13}{60}$. *Ans.* $37\frac{23}{24}$.
12. $25\frac{8}{21}-16\frac{17}{35}$. *Ans.* $8\frac{94}{105}$.
13. $69\frac{7}{10}-33\frac{12}{25}$. *Ans.* $36\frac{11}{50}$.
14. $84\frac{7}{36}-15\frac{7}{27}$. *Ans.* $68\frac{101}{108}$.
15. $123\frac{13}{24}-56\frac{13}{32}$. *Ans.* $67\frac{13}{96}$.
16. $\frac{8\frac{11}{14}}{23\frac{3}{7}}-\frac{25}{100}$. *Ans.* $\frac{1}{8}$.

Find the product of

17. $\frac{3}{4}\times1\frac{2}{5}\times3\frac{4}{21}$. *Ans.* $3\frac{7}{20}$.
18. $16\frac{2}{3}\times\frac{3}{50}\times63\frac{7}{8}$. *Ans.* $63\frac{7}{8}$.
19. $13\frac{1}{9}\times3\frac{2}{3}\times4\frac{1}{2}$. *Ans.* $216\frac{1}{3}$.
20. $3\frac{5}{12}\times24\times6\frac{2}{7}$. *Ans.* $515\frac{3}{7}$.
21. $45\frac{3}{5}\times8\frac{3}{4}\times9\frac{1}{7}$. *Ans.* 3648.
22. $8\frac{12}{25}\times3\frac{1}{8}\times2\frac{17}{20}$. *Ans.* $75\frac{21}{40}$.
23. $5\frac{1}{2}\times5\frac{1}{2}\times5\frac{1}{2}$. *Ans.* $166\frac{3}{8}$.
24. $5\frac{1}{4}\times5\frac{5}{21}\times6\frac{6}{11}$. *Ans.* 180.

Find the quotient of

25. $15\frac{3}{4} \div 4\frac{1}{5}$. *Ans.* $3\frac{3}{4}$.

26. $18\frac{3}{4} \div 4\frac{1}{6}$. *Ans.* $4\frac{1}{2}$.

27. $93\frac{3}{4} \div 8\frac{1}{3}$. *Ans.* $11\frac{1}{4}$.

28. $133\frac{1}{3} \div 4\frac{1}{6}$. *Ans.* 32.

29. $450\frac{1}{3} \div 21$. *Ans.* $21\frac{4}{9}$.

30. $16\frac{3}{7} \times 9 \div \frac{81}{91}$. *Ans.* $166\frac{1}{9}$.

31. $\frac{44}{87} \times 732\frac{1}{4} \div 14\frac{2}{3}$. *Ans.* $25\frac{1}{4}$.

32. $20\frac{4}{7} \div \frac{2}{11}$ of $8\frac{8}{35}$. *Ans.* $13\frac{3}{4}$.

Find the value of *Ans.*

33. $\frac{2}{3} + \frac{4}{5} + \frac{5}{6} - \frac{14}{15}$. $1\frac{11}{30}$.

34. $\frac{8}{9} + \frac{5}{8} - \frac{7}{18} + \frac{1}{36}$. $1\frac{11}{72}$.

35. $\frac{7}{11} + \frac{3}{7} - \frac{5}{22} - \frac{5}{44}$. $\frac{223}{308}$.

36. $\frac{9}{15} - \frac{3}{10} - \frac{1}{40} + \frac{4}{11}$. $\frac{281}{440}$.

Ans.

37. $5 + 6\frac{4}{7} - 7\frac{7}{10} + \frac{23}{30}$. $4\frac{67}{105}$.

38. $8\frac{1}{9} - 3\frac{5}{7} + 8\frac{11}{35} - 5\frac{3}{5}$. $7\frac{1}{9}$.

39. $5\frac{3}{5} + 2\frac{1}{3} - 4\frac{5}{21} - \frac{12}{35}$. $3\frac{37}{105}$.

40. $7\frac{1}{4} + 3\frac{4}{15} - 3\frac{3}{4} + 9$. $15\frac{23}{30}$.

Find the value of

41. $\dfrac{12\frac{3}{5}}{7}$. *Ans.* $1\frac{4}{5}$.

42. $\dfrac{17\frac{16}{21}}{2\frac{1}{3}}$. *Ans.* $7\frac{30}{49}$.

43. $\dfrac{37\frac{3}{5}}{9\frac{2}{5}}$. *Ans.* 4.

44. $\dfrac{4\frac{1}{6}}{\dfrac{42\frac{2}{3}}{4}}$. *Ans.* $\frac{25}{64}$.

45. $\dfrac{6\frac{1}{4}}{\dfrac{4\frac{1}{4}}{12}}$. *Ans.* $17\frac{11}{17}$.

46. $\dfrac{\frac{2}{3} \text{ of } \frac{3}{4}}{15}$. *Ans.* $\frac{1}{30}$.

47. $\dfrac{5\frac{1}{2}}{\frac{1}{8} \text{ of } 1\frac{1}{7}}$. *Ans.* $38\frac{1}{2}$.

48. $\dfrac{8\frac{1}{3} - 2\frac{1}{2}}{6\frac{1}{4}}$. *Ans.* $\frac{14}{15}$.

49. $\dfrac{3\frac{1}{2} + 4\frac{1}{3} - 7\frac{15}{24}}{6\frac{1}{4}}$. *Ans.* $\frac{1}{30}$.

50. $\dfrac{2 + \frac{1}{3}}{2 - \frac{1}{3}}$ of $\dfrac{\frac{3}{2}}{2\frac{1}{3}}$. *Ans.* $\frac{9}{10}$.

Find the value of *Ans.*

51. $\frac{4}{7}$ of $\frac{14}{25}$ of $\frac{15}{22}$. $\frac{12}{55}$.

52. $\frac{9}{10}$ of $\frac{24}{27}$ of $\frac{81}{90}$. $\frac{18}{25}$.

53. $\frac{14}{27}$ of $\frac{20}{21}$ of $\frac{15}{35}$ of $\frac{28}{40}$. $\frac{4}{27}$.

54. $3 \times 2\frac{3}{4} - \frac{5}{9}$ of $4\frac{11}{40}$. $5\frac{7}{8}$.

55. $\frac{3}{5}$ of $7\frac{1}{2} \times \frac{3}{7}$ of $\frac{49}{90}$. $1\frac{1}{20}$.

56. $\frac{4}{11}$ of $\frac{33}{38} \times \frac{5}{16}$ of $15\dfrac{3\frac{1}{5}}{16}$. $1\frac{1}{2}$.

Ans.

57. $20\frac{4}{7} \div \frac{4}{7}$ of $\frac{12}{14}$ of $\frac{16}{35}$. $91\frac{7}{8}$.

58. $\frac{14}{27}$ of $\frac{13}{36}$ of $7\frac{1}{3} \div 4\frac{1}{3} \times \frac{44}{81}$. $\frac{7}{12}$.

59. $\frac{40}{63}$ of $5\frac{3}{7} \div \frac{19}{27}$ of $\frac{24}{49}$. 10.

60. $\dfrac{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}}{1 + \frac{1}{2} - \frac{1}{3} - \frac{1}{4}} \times \frac{2}{3}$ of $\dfrac{1}{1\frac{1}{2}}$. $\frac{28}{99}$.

61. $\frac{21}{25} \times \frac{75}{84} \times \frac{4}{45} \times \frac{225}{256}$. $\frac{15}{256}$.

62. $\frac{49}{72} \times \frac{27}{32} \times \frac{144}{343} \times \frac{54}{729}$. $\frac{1}{56}$.

63. $\frac{576}{961} \times \frac{992}{1089} \times \frac{231}{1024} \times \frac{93}{77}$. *Ans.* $\frac{18}{121}$.

64. $\frac{39}{72} \times \frac{218}{425} \times \frac{75}{117} \div \frac{327}{289}$. *Ans.* $\frac{17}{108}$.

65. $(\frac{28}{81} \times \frac{45}{56}) \div (\frac{44}{54} \times \frac{42}{32})$. *Ans.* $\frac{20}{77}$.

66. $(\frac{1}{3} + \frac{8}{15}) \times \frac{3}{11} + (\frac{4}{15} + \frac{5}{6}) \times 3$. *Ans.* $3\frac{59}{110}$.

67. $(\frac{5}{6} + \frac{10}{11}) \times (\frac{3}{4} + \frac{7}{15}) \times \frac{72}{73} \times \frac{363}{230}$. *Ans.* $3\frac{3}{10}$.

68. $7\frac{7}{10} \times 4\frac{6}{11} + (4\frac{1}{2} - 3\frac{4}{5}) - 12\frac{3}{35}$. *Ans.* $23\frac{43}{70}$.

69. $(77\frac{20}{21} - 44\frac{3}{7}) \times (6\frac{5}{8} - 5\frac{3}{4}) \div (7\frac{3}{4} - 3\frac{7}{12})$. *Ans.* $7\frac{1}{25}$.

WRITTEN EXERCISES.

What cost

1. $56\frac{7}{8}$ tons of hay, at $\$10\frac{1}{4}$ per ton? *Ans.* $\$582\frac{31}{32}$.
2. $45\frac{3}{4}$ yards of muslin, at $12\frac{1}{2}$ cts. a yard? *Ans.* $\$5.71\frac{7}{8}$.
3. $216\frac{2}{3}$ lbs. of mutton, at $6\frac{3}{4}$ cts. a pound? *Ans.* $\$14.62\frac{1}{2}$.
4. $319\frac{1}{3}$ yards of silk, at $62\frac{1}{2}$ cts. a yard? *Ans.* $\$199.58\frac{1}{3}$.
5. $\frac{5}{6}$ of a barrel of flour, if 1 barrel cost $\$5\frac{1}{4}$? *Ans.* $\$4\frac{3}{8}$.
6. $\frac{15}{16}$ of a yard of lace, if 2 yards cost $\$18\frac{2}{5}$? *Ans.* $\$8\frac{5}{8}$.
7. $2\frac{3}{8}$ tons of coal, if $5\frac{1}{3}$ tons cost \$32? *Ans.* $\$14\frac{1}{4}$.
8. $5\frac{7}{9}$ yards of cashmere, if 1 yard cost $\$1\frac{1}{8}$? *Ans.* $\$6\frac{1}{2}$.
9. $57\frac{3}{4}$ tons of pig iron, if $2\frac{1}{2}$ tons cost \$30? *Ans.* \$693.
10. $28\frac{3}{4}$ yards of cloth, if 5 yards cost $\$13\frac{3}{5}$? *Ans.* $\$78\frac{1}{5}$.
11. John has $\frac{4}{5}$ of \$4975, which is $2\frac{1}{2}$ times his wife's money; how much money has his wife? *Ans.* \$1592.
12. Mary owns 520 acres of land, and $2\frac{3}{5}$ times Mary's equals $3\frac{5}{7}$ times Henry's; how much has Henry? *Ans.* 364 acres.
13. A shoe-dealer paid \$75 for a case of overshoes, at $\$\frac{3}{4}$ a pair; how many pairs were in the case? *Ans.* 100.
14. A merchant paid \$48 for cuffs, at $18\frac{3}{4}$ cents a pair; how man dozen pairs did he buy? *Ans.* $21\frac{1}{3}$ dozen.
15. A grocer bought 80 jars of currant jelly, at $12\frac{1}{2}$ cents a jar, and 15 gallons of syrup, at $33\frac{1}{3}$ cents a gallon; what did he pay for both? *Ans.* \$15.

16. A man has $10\frac{3}{16}$ bushels of peanuts, and puts them into paper-bags holding $\frac{1}{64}$ of a bushel; how many bags does he fill? *Ans.* 652.

17. A drygoods clerk spends every year $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{6}$ of his salary; what part of it remains? *Ans.* $\frac{1}{12}$.

18. How many times will $\frac{1}{3}$ of a barrel of $31\frac{1}{2}$ gallons fill a vessel holding $\frac{5}{6}$ of a gallon? *Ans.* $12\frac{3}{5}$.

19. If 3 be added to both terms of the fraction $\frac{7}{8}$, will the value of the fraction be increased or diminished?

20. If 3 be subtracted from both terms of the fraction $\frac{7}{8}$, will the value of the fraction be increased or diminished?

21. Will the value of the fraction $\frac{9}{7}$ be increased or diminished if 3 be added to both terms? If 3 be subtracted from both terms?

22. Mary, in attempting to multiply a fraction by $\frac{15}{16}$, got the terms of the multiplicand inverted, and obtained a product of $\frac{75}{76}$; what was the given fraction? *Ans.* $\frac{19}{20}$.

23. Maria, in attempting to divide a fraction by $\frac{19}{18}$, inverted the dividend instead of the divisor, and obtained a quotient of $\frac{41}{38}$; what was the given fraction? *Ans.* $\frac{36}{41}$.

PRACTICAL EXAMPLES.

1. If a boy lives $20\frac{1}{2}$ rods from school, how far does he walk in a week to attend two sessions of the school? *Ans.* 410 rods.

2. I bought a dozen oranges at the rate of 4 for 3 cents, and sold them at the rate of 3 for 4 cents; what was my profit? *Ans.* 7 cents.

3. A man carried to a store $4\frac{2}{3}$ dozen eggs, which he sold at the rate of 9 for $18\frac{3}{4}$ cents; how much did he receive for his eggs? *Ans.* $1.16\frac{2}{3}$.

4. If a train travels 35 miles an hour, what distance will it go between half-past 8 in the morning and a quarter past 5 in the afternoon? *Ans.* $306\frac{1}{4}$ miles.

5. A miller buys 8640 pounds of prime Minnesota wheat at 90 cents per bushel of 60 pounds; what does he pay for the wheat? *Ans.* $129.60.

6. It requires 3 yards of cloth $1\frac{1}{2}$ yards wide to make a cloak; how much cloth $\frac{3}{4}$ yards wide does it require to line it? *Ans.* 6 yards.

7. A lady buys 5 lb. of tea and 48 lb. of sugar; she pays 60¢ per lb. for the tea, and her whole bill is $5.40; what is the price per lb. of the sugar? *Ans.* 5 cents.

8. A chemist in making gunpowder mixes $8\frac{1}{3}$ lb. of saltpetre, $1\frac{2}{3}$ lb. of sulphur, and $1\frac{2}{3}$ lb. of charcoal; how many pounds of gunpowder will there be? *Ans.* $11\frac{2}{3}$ pounds.

9. A man, having done $\frac{7}{8}$ of a piece of work, requires 2 days more to finish it; how many hours does he require to do the whole, working 8 hours a day? *Ans.* 128 hours.

10. In 1892 James was 32 years old, and his father was $1\frac{3}{4}$ times as old; the father was how many times as old as the son in 1884? *Ans.* Father twice as old.

11. A mechanic works 300 days in a year at $$3\frac{1}{2}$ a day; his expenses average $$2\frac{1}{4}$ a day for 365 days; how much does he save in a year? *Ans.* $$228\frac{3}{4}$.

12. A grocer buys 50 dozen eggs at 15 cents a dozen; he sells them at the rate of 8 for 12 cents; what is his profit on the transaction? *Ans.* $1.50.

13. An errand-boy has to walk from the store to a house $1\frac{3}{4}$ miles east of the store, and then to a house $2\frac{1}{4}$ miles west of the store; how far had he walked when he returned to the store? *Ans.* 8 miles.

14. From a piece of cloth measuring $28\frac{1}{2}$ yards there have been sold $2\frac{1}{2}$ yards, $7\frac{3}{4}$ yards, $12\frac{5}{8}$ yards; the remainder is worth $$14.06\frac{1}{4}$; what was the value of the whole piece? *Ans.* $$71\frac{1}{4}$.

15. If lemons are $37\frac{1}{2}$ cents a dozen, how many boxes, each containing 240 lemons, can be bought for $120? *Ans.* 16.

16. How many cords of wood, at $\$4\frac{1}{4}$ a cord, must I give for $85\frac{2}{3}$ bushels of wheat, at 75 cents a bushel, and 75 bushels of rye, at 56 cents a bushel? *Ans.* 25.

17. Two trains are $97\frac{3}{4}$ miles apart, and running toward each other, one at the rate of $48\frac{1}{2}$ miles an hour, the other at the rate of $25\frac{1}{3}$ miles an hour; how far apart will they be in half an hour? *Ans.* $60\frac{5}{6}$ miles.

18. A lady bought a tract of land for \$5000, and sold $\frac{3}{5}$ of it to Mr. Jones, and $\frac{2}{3}$ of the remainder to Mr. Allen; what was the cost of what remained? *Ans.* $\$666\frac{2}{3}$.

19. Mr. Mason worked $22\frac{1}{2}$ days, and after paying his board and other expenses with $\frac{2}{5}$ of his earnings, had $\$16.87\frac{1}{2}$ left; what were his daily wages? *Ans.* \$1.25.

20. A truck-raiser gave $12\frac{3}{4}$ bushels of potatoes, at 25 cents a bushel, for butter worth $18\frac{3}{4}$ cents a pound; how much butter did he get? *Ans.* 17 pounds.

21. A real-estate agent bought land for \$9750, and sold it so as to gain $\frac{1}{13}$ of the cost, the gain being \$5 an acre; how many acres did he buy? *Ans.* 150 acres.

22. A merchant bought 100 yards of double-width sheeting; he sold $\frac{1}{4}$ of it to Mr. Brown, and $\frac{2}{5}$ of the remainder to Mr. Black; what was the value of the remainder at $31\frac{1}{4}$ cents a yard? *Ans.* $\$14.06\frac{1}{4}$.

THE PRINCIPLES OF FRACTIONS.

164. The **Principles of Fractions** were derived from analysis and inference; we now present a more general demonstration of them.

1. *Multiplying the numerator of a fraction by any number multiplies the value of the fraction by that number.*

If we multiply the numerator of a fraction by any number, as 5, the resulting fraction will express 5 times as many fractional units, each of the same size as before; hence the value of the fraction is 5 times as great.

2. *Dividing the numerator of a fraction by any number divides the value of the fraction by that number.*

If we divide the numerator of a fraction by any number, as 4, the resulting fraction will express $\frac{1}{4}$ as many fractional units, each of the same size as before; hence the value of the fraction is divided by 4.

3. *Multiplying the denominator of a fraction by any number divides the value of the fraction by that number.*

Since the denominator denotes the number of equal parts into which the unit is divided, if we multiply the denominator of a fraction by any number, as 5, the unit will be divided into 5 times as many equal parts; hence each fractional unit will be $\frac{1}{5}$ as large as before, and, the same number of fractional units being taken, the value of the fraction is $\frac{1}{5}$ as great.

4. *Dividing the denominator of a fraction by any number multiplies the value of the fraction by that number.*

Since the denominator denotes the number of equal parts into which the unit is divided, if we divide it by any number, as 4, the unit will be divided into $\frac{1}{4}$ as many equal parts; hence each fractional unit will be 4 times as large as before, and, the same number of fractional units being taken, the value of the fraction will be 4 times as great.

5. *Multiplying both numerator and denominator of a fraction by the same number does not change the value of the fraction.*

Since multiplying the numerator multiplies the value of the fraction, and multiplying the denominator divides the value of the fraction, multiplying both numerator and denominator both multiplies and divides the value of the fraction by the same number, and hence does not change its value.

6. *Dividing both numerator and denominator of a fraction by the same number does not change its value.*

Since dividing the numerator divides the value of the fraction, and dividing the denominator multiplies the value, dividing both numerator and denominator both divides and multiplies the value of the fraction, and hence does not change its value.

165. These principles may all be embodied in one general law as follows:

General Principle.—*A change in the* NUMERATOR *by multiplication or division produces a* SIMILAR *change in the value of the fraction, but such a change in the* DENOMINATOR *produces an* OPPOSITE *change in the value of the fraction.*

INTRODUCTION TO DECIMALS.

ORAL EXERCISES.

1. If a unit is divided into 10 equal parts, what is 1 of these parts called?

2. If one-tenth is divided into 10 equal parts, what is 1 part called? 2 of these parts? 3 of these parts?

3. If $\frac{1}{100}$ is divided into 10 equal parts, what is 1 part called? 2 of these parts? 5 of these parts?

4. What part of 5 is 5 tenths? of 6 is 6 tenths? of 5 tenths is 5 hundredths? of 6 tenths is 6 hundredths?

5. In the number 5555 the 5 units is what part of the 5 tens? the 5 tens is what part of the 5 hundreds?

6. A term in units place denotes what part of the value which it does in tens place?

7. A term in tens place denotes what part of the value which it does in hundreds place?

8. By the same law, a term to the right of units place would denote what part of its value in units place? *Ans.* One-tenth.

9. How may we indicate that a term is at the right of units place? *Ans.* By placing a dot (.) at the right of units place.

10. How will you express $2\frac{5}{10}$ in this manner? *Ans.* 2.5.

11. What does the dot between the 2 and the 5 denote?

Ans. That 2 is in units place and 5 in tenths place.

12. In the expression 11.1, the 1 at the right of units denotes what part of a unit?

13. In the expression 11.11, the second term at the right of the period denotes what part of a tenth? what part of a unit?

14. What shall we call the first place at the right of units? Second place? Third? Fourth? Fifth?

15. Write without a denominator 2 tenths; 3 tenths; 4 tenths; 3 hundredths; 5 hundredths; 1 thousandth; 3 thousandths.

16. Read the following expressions: 4.6; 25.45; 26.34; 18.05; 25.235; 36.205; 46.008.

17. These fractions arising from the successive division by 10 are called *decimal fractions.* The term decimal is derived from *decem,* meaning *ten.*

SECTION V.

DECIMAL FRACTIONS.

166. A **Decimal Fraction** is a number of *tenths, hundredths, thousandths*, etc.

167. A **Decimal Fraction** is usually expressed by placing a point before the numerator and omitting the denominator; thus, .5 expresses $\frac{5}{10}$; .05 expresses $\frac{5}{100}$; .005, $\frac{5}{1000}$, etc.

168. The **Symbol** of a decimal is the *period*, called the *decimal point* or *separatrix*. It indicates the decimal, and *separates* decimals and integers.

169. The places at the right of the decimal point are called *decimal places*. The first place to the right of the decimal point is *tenths*, the second place is *hundredths*, etc.

170. The method of expressing decimal fractions is a continuation of the method of notation for integers. This beautiful law, as applied to integers and fractions, is exhibited in the following

NOTATION AND NUMERATION TABLE.

Hund.-millions.	Ten-millions.	Millions.	Hund.-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	*Decimal Point.*	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hund.-thousandths.	Millionths.	Ten-millionths.	Hund.-millionths.	Billionths.
6	6	6	6	6	6	6	6	6	.	6	6	6	6	6	6	6	6	6
9th.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.		1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.
Integers.										Decimals.								

171. A **Decimal** is a decimal fraction expressed by the method of decimal notation; as, .5, .25, etc.

172. **A Pure Decimal** is one which consists of decimal figures only; as, .345.

173. **A Mixed Decimal** is one which consists of an integer and a decimal; as, 4.35.

174. **A Complex Decimal** is one which contains a common fraction at the right of the decimal; as, .45$\frac{1}{3}$.

1. The first treatise upon decimals was written by Stevinus, and published in 1585.
2. The decimal point, Dr. Peacock thinks, was introduced by Napier, the inventor of logarithms, in 1617.

EXERCISES IN NUMERATION.

1. Read the decimal .35.

SOLUTION.—This expresses 3 tenths 5 hundredths, or, since 3 tenths equals 30 hundredths, and 30 hundredths plus 5 hundredths equal 35 hundredths, it may also be read 35 hundredths. Hence the following

Rule.—*Begin at tenths, and read the terms in order toward the right, giving each term its proper denomination.*

Or, read the decimal as a whole number, and give it the denomination of the right-hand term.

In the second method we may determine the denominator by numerating *from* the decimal point, and the numerator by numerating *toward* the decimal point.

Read the following decimals:

2. .45	**7.** .409	**12.** 6.0509
3. .57	**8.** .703	**13.** 7.25008
4. .95	**9.** .8407	**14.** 54.20035
5. .047	**10.** .0674	**15.** 872.00341
6. .156	**11.** .00213	**16.** 456.000027

EXERCISES IN NOTATION.

1. Express 54 thousandths in the form of a decimal.

SOLUTION.—54 thousandths equal 50 thousandths plus 4 thousandths, or 5 hundredths and 4 thousandths; hence, we write the 4 in the third or thousandths place, 5 in the second or hundredths place, and fill the vacant tenths place with a cipher, and we have .054. Hence the following

Rule.—*Place the decimal point, and then write each term so*

that it may express its proper denomination, prefixing ciphers when necessary.

Or, write the numerator, and then place the decimal point so that the right-hand term shall express the denomination of the decimal, prefixing ciphers when necessary.

To avoid ambiguity, the word *and* should be used only between the integral and decimal parts of a number. Thus, three hundred seven ten-thousandths should be written .0307, but three hundred and seven ten-thousandths, 300.0007.

Express the following in decimal form:

2. Thirty-seven hundredths.

3. 3 tenths 5 hundredths.

4. 8 tenths 9 hundredths.

5. Forty-five thousandths.

6. 7 tenths 4 thousandths.

7. 8 tenths 9 thousandths.

8. Six hundred fifteen thousandths.

9. Four tenths 8 ten-thousandths.

10. Nine hundredths 7 ten-thousandths.

11. Two hundred and 69 ten-thousandths.

12. Four tenths 5 hundredths 6 hundred thousandths.

13. Three hundredths 9 ten-thousandths 8 hundred-thousandths.

14. Nine hundred sixty-nine hundred-thousandths.

15. Four tenths 4 millionths.

16. Seven hundredths five thousandths four millionths.

17. Seventy-five thousand and seven millionths.

18. Sixty-three hundred and nine ten-millionths.

19. Fourteen thousand and five hundred-millionths.

20. Four million 7 thousand and 8 hundred-millionths.

Express the following as decimals:

21. $\frac{8}{10}$.	**24.** $\frac{407}{1000}$.	**27.** $96\frac{105}{1000}$.
22. $\frac{75}{100}$.	**25.** $150\frac{3}{100}$.	**28.** $563\frac{285}{10000}$.
23. $\frac{87}{1000}$.	**26.** $167\frac{104}{1000}$.	**29.** $875\frac{6018}{100000}$.

PRINCIPLES.

1. *Moving the decimal point one place to the right multiplies the decimal by* 10; *two places, multiplies by* 100, *etc.*

For, if the point be moved one place to the right, each figure will express ten times as much as before; hence the whole decimal will be ten times as great, etc.

2. *Moving the decimal point one place to the left divides the decimal by* 10; *two places, divides by* 100, *etc.*

For, if the point be moved one place to the left, each figure will express 1 tenth of its previous value; hence the whole decimal will be only 1 tenth as great, etc.

3. *Prefixing a decimal cipher to a decimal divides the decimal by* 10.

For this moves each figure one place to the right in the scale, in which case each figure expresses 1 tenth as much as before, and hence the decimal is only 1 tenth as great.

4. *Annexing a decimal cipher to a decimal does not change its value.*

For each figure retains the same place as before, and hence expresses the same value as before, and consequently the value of the decimal is unchanged.

ORAL EXERCISES.

1. How many tenths in 1? in $\frac{1}{2}$? in $\frac{1}{5}$? in $\frac{2}{5}$?

2. How many hundredths in $\frac{1}{2}$? in $\frac{2}{5}$? in $\frac{3}{5}$? in $\frac{9}{10}$? in $\frac{13}{50}$?

3. How many hundredths in $\frac{1}{4}$? in $\frac{3}{4}$? in $\frac{1}{25}$? in $\frac{3}{25}$? in $\frac{7}{50}$?

4. How many halves in .5? in .50? 5ths in .2? in .4?

5. How many 4ths in .25? in .75? eighths in .125? in .375?

6. Express as a common fraction .5; .8; .25; .50; .75; .80; .125.

REDUCTION OF DECIMALS.

175. There are two cases of the reduction of decimals:

1st. To reduce decimals to common fractions.

2d. To reduce common fractions to decimals.

CASE I.

176. To reduce a decimal to a common fraction.

1. Reduce .25 to a common fraction.

SOLUTION.—.25 expressed in the form of a common fraction is $\frac{25}{100}$, which reduced to its lowest terms equals $\frac{1}{4}$. Hence the following

OPERATION.

$.25 = \frac{25}{100} = \frac{1}{4}$. *Ans.*

Rule.—*Write the denominator under the decimal, omitting the decimal point, and reduce the common fraction to its lowest terms.*

WRITTEN EXERCISES.

Reduce the following decimals to common fractions:

2. .45.	*Ans.* $\frac{9}{20}$.	**7.** 10.25.	*Ans.* $10\frac{1}{4}$.
3. .72.	*Ans.* $\frac{18}{25}$.	**8.** 15.725.	*Ans.* $15\frac{29}{40}$.
4. .375.	*Ans.* $\frac{3}{8}$.	**9.** 23.075.	*Ans.* $23\frac{3}{40}$.
5. .625.	*Ans.* $\frac{5}{8}$.	**10.** 15.064.	*Ans.* $15\frac{8}{125}$.
6. 4.75.	*Ans.* $4\frac{3}{4}$.	**11.** 47.0125.	*Ans.* $47\frac{1}{80}$.

Reduce the following complex decimals to common fractions:

12. $.9\frac{1}{3}$.	*Ans.* $\frac{14}{15}$.	**17.** $5.9\frac{2}{7}$.	*Ans.* $5\frac{13}{14}$.
13. $.16\frac{2}{3}$.	*Ans.* $\frac{1}{6}$.	**18.** $12.18\frac{3}{4}$.	*Ans.* $12\frac{3}{16}$.
14. $.3\frac{3}{4}$.	*Ans.* $\frac{3}{8}$.	**19.** $33.03\frac{1}{3}$.	*Ans.* $33\frac{1}{30}$.
15. $.56\frac{1}{4}$.	*Ans.* $\frac{9}{16}$.	**20.** $55.83\frac{1}{3}$.	*Ans.* $55\frac{5}{6}$.
16. $\$0.33\frac{1}{3}$.	*Ans.* $\$\frac{1}{3}$.	**21.** $\$66.66\frac{2}{3}$.	*Ans.* $\$66\frac{2}{3}$.

CASE II.

177. To reduce a common fraction to a decimal.

1. Reduce $\frac{5}{8}$ to a decimal.

OPERATION.

$$\frac{5}{8} = \frac{1}{8} \text{ of } 5 = 8\overline{)5.000} \quad .625$$

SOLUTION.—$\frac{5}{8}$ equals $\frac{1}{8}$ of 5. 5 equals 50 tenths; $\frac{1}{8}$ of 50 tenths is 6 tenths and 2 tenths remaining; 2 tenths equal 20 hundredths; $\frac{1}{8}$ of 20 hundredths equals 2 hundredths and 4 hundredths remaining; 4 hundredths equal 40 thousandths; $\frac{1}{8}$ of 40 thousandths is 5 thousandths. Therefore, $\frac{5}{8}$ equals .625.

Rule.—I. *Annex ciphers to the numerator, and divide by the denominator.*

II. *Point off as many decimal places in the quotient as there are ciphers annexed.*

1. When the division will not terminate, the common fraction cannot be exactly expressed by a decimal. Such decimals are called *interminate* or *infinite* decimals.

2. The symbol + annexed to a decimal indicates that it contains other decimal terms. The symbol – annexed to a decimal indicates that the last decimal term is increased by 1.

WRITTEN EXERCISES.

Reduce the following common fractions to decimals:

2. $\frac{1}{4}$.	*Ans.* .25.	**8.** $\frac{14}{15}$.	*Ans.* .933+.
3. $\frac{3}{4}$.	*Ans.* .75.	**9.** $\frac{15}{16}$.	*Ans.* .9375.
4. $\frac{5}{8}$.	*Ans.* .625.	**10.** $\frac{17}{32}$.	*Ans.* .53125.
5. $\frac{7}{8}$.	*Ans.* .875.	**11.** $\frac{25}{36}$.	*Ans.* .6944+.
6. $\frac{5}{16}$.	*Ans.* .3125.	**12.** $\frac{37}{45}$.	*Ans.* .8222+.
7. $\frac{7}{16}$.	*Ans.* .4375.	**13.** $\frac{113}{256}$.	*Ans.* .44140625.

ADDITION OF DECIMALS.

178. Addition of Decimals is the process of finding the sum of two or more decimals.

ORAL EXERCISES.

1. Add $\frac{4}{10}$ and $\frac{6}{10}$.	**5.** Add 25 and .5.
2. Add .4 and .6.	**6.** Add .07 and .08.
3. Add .04 and .05.	**7.** Add .05 and .136.
4. Add .4 and .06.	**8.** Add 2.5 and 4.25.

WRITTEN EXERCISES.

1. What is the sum of 4.47, 5.758, 25.475?

OPERATION.

$$\begin{array}{r} 4.47 \\ 5.758 \\ \underline{25.475} \\ 35.703 \end{array}$$

SOLUTION.—We write the numbers so that terms of the same order shall stand in the same column, and begin at the right to add. 5 thousandths plus 8 thousandths are 13 thousandths, which equals 1 hundredth and 3 thousandths; we write the 3 thousandths, and add the 1 hundredth to the next column. 7 hundredths plus 5 hundredths plus 7 hundredths equal 19 hundredths, and the 1 hundred added is 20 hundredths, which equals 2 tenths and no hundredths; we write 0 hundredths, etc.

Rule.—I. *Write the numbers so that terms of the same order stand in the same column.*

II. *Add as in whole numbers, and place the decimal point between the units and tenths of the sum.*

Find the value of

2. 87.79 + 47.05 + 245.406. *Ans.* 380.246.

3. 77.07 + 421.437 + 88.35. *Ans.* 586.857.

4. 127.081 + 658.079 + 581.7618. *Ans.* 1366.9218.

5. $\$35\frac{3}{8} + \$18.12\frac{1}{2} + \$49.56\frac{1}{4}$. *Ans.* \$103.0625.

6. 24.33 + 46.43 + 63.175. *Ans.* 133.935.

7. 65.789 + 594.035 + 876.324. *Ans.* 1536.148.

8. 679.906 + 739.57047 + 443.421. *Ans.* 1862.89747.

9. $\$45\frac{5}{8} + \$16.87\frac{1}{2} + \$15\frac{7}{8}$. *Ans.* \$78.375.

10. $23.04 + 8.6796 + .0005 + 7.00019 + 8\frac{999}{1000} + \frac{3}{25}$.

11. $9.084 + 96.875 + 7.2832 + 250\frac{1}{80} + .576 + \frac{33}{100000}$.

12. $.81074 + .009157 + \frac{3}{4} + 30\frac{1}{4} + 2.06 + 7.91$.

13. Add 897 and 9 ten-thousandths, 17 millionths, 18 thousandths and 98 ten-millionths, 167 hundred-thousandths, and 195 ten-millionths. *Ans.* 897.0206163.

14. Add 7 and 9 tenths, 54 and 8 hundredths, 67 and 59 hundredths, 228 and 387 thousandths. *Ans.* 357.957.

15. Add 487 and 5 thousandths, 65 and 11 hundred-thousandths, 1876 and 564 millionths, 189 ten-thousandths, and 999 ten-millionths. *Ans.* 2428.0246739.

16. Find the sum of 3 tenths, $6\frac{1}{2}$ hundredths, $4\frac{1}{4}$ thousandths, $3\frac{1}{8}$ ten-thousandths, and $6\frac{1}{5}$ hundred-thousandths. *Ans.* .3696245.

SUBTRACTION OF DECIMALS.

179. Subtraction of Decimals is the process of finding the difference between two decimals.

ORAL EXERCISES.

1. Subtract $\frac{4}{10}$ from $\frac{9}{10}$.
2. Subtract .4 from .9.
3. Subtract .06 from .08.
4. Subtract .05 from .55.
5. Subtract $\frac{6}{10}$ from .9.
6. Subtract .05 from .5.
7. Subtract .25 from .63.
8. Subtract .75 from 1.08.

WRITTEN EXERCISES.

1. From 729.362 take 658.235.

OPERATION.
729.362
658.235
71.127

SOLUTION.—We write the numbers so that terms of the same order stand in the same column, and begin at the right to subtract. We cannot subtract 5 thousandths from 2 thousandths, hence we add 10 thousandths to 2 thousandths, which equals 12 thousandths; 5 thousandths from 12 thousandths leaves 7 thousandths, which we write in the order of thousandths; since we have added 10 thousandths or 1 hundredth to the minuend, we must add 1 hundredth to the subtrahend; 1 hundredth and 3 hundredths are 4 hundredths; 4 hundredths from 6 hundredths leaves 2 hundredths, etc.

Rule.—I. *Write the subtrahend under the minuend, so that terms of the same order stand in the same column.*

II. *Subtract as in whole numbers, and place the decimal point between the units and tenths of the remainder.*

Find the value of

2. 804.432 — 523.451.

3. 906.448 — 744.916.

4. 378.3057 — 129.084.

5. 230.207 — 184.1231.

6. 12.07 — 11.432765.

7. 384⅞ — 54.8749.

8. From 5 take 5 hundred-millionths. *Ans.* 4.99999995.

9. From 2 and .002 take .02 and .000002. *Ans.* 1.981998.

10. From 2½ take 2½ thousandths and 2½ billionths.
Ans. 2.4974999975.

MULTIPLICATION OF DECIMALS.

180. Multiplication of Decimals is the process of finding the product when one or both factors are decimals.

ORAL EXERCISES.

What is the value of

1. 2 times $\frac{3}{10}$? $3\times.5$?

2. 3 times .6? $5\times.8$?

3. $\frac{5}{10}$ times $\frac{7}{10}$? $.5\times.7$?

4. $\frac{4}{10}$ times $\frac{8}{10}$? $.4\times.8$?

5. $\frac{3}{10}\times.6$? $.6\times.7$?

6. $5\times\frac{8}{100}$? $5\times.08$?

7. $\frac{3}{10}\times\frac{12}{100}$? $.3\times.12$?

8. $.6\times.15$? $.12\times.12$?

9. How many decimal places in the product of *units* multiplied by *tenths?* *Tenths* by *tenths?* *Units* by *hundredths?* *Tenths* by *hundredths?* *Hundredths* by *hundredths?*

10. How many decimal places in the product if there are two in the multiplicand and one in the multiplier? If there are two in each factor? If three in multiplicand and two in multiplier?

Principle.—*The product will contain as many decimal places as both factors.*

WRITTEN EXERCISES.

1. Multiply 6.48 by .74.

SOLUTION 1ST.—Multiplying as in whole numbers, the product is 47952. Since each factor contains 2 decimal places, the product should contain 2 plus 2, or 4, decimal places (Prin.); therefore the product is 4.7952.

SOLUTION 2D.—$6.48 \times .74 = \frac{648}{100} \times \frac{74}{100} = \frac{47952}{10000} = \frac{1}{10000} \times 47952 = 4.7952$. From either of these solutions we derive the following

OPERATION.

$$\begin{array}{r} 6.48 \\ \underline{.74} \\ 2592 \\ \underline{4536} \\ 4.7952 \end{array}$$

Rule.—*Multiply as in whole numbers, and from the right of the product point off as many decimal places as there are in both factors, prefixing ciphers when necessary.*

NOTE.—For the answers, see corresponding examples in the Division of Decimals.

Multiply

2. .64 by .34.
3. .75 by .67.
4. .128 by 4.6.
5. 5.36 by 5.8.
6. .057 by 9.7.
7. 8.75 by .082.
8. 42.8 by 6.37.
9. 73.21 by 3.85.
10. 79.52 by .019.
11. 321.701 by 1.52.
12. 61.544 by 20.074.
13. 32.185 by 6.0708.
14. 45.0079 by 6.072.
15. 1.08096 by 3.5702.
16. .03507 by .005873.
17. 2.0709 by .000246.
18. $25\frac{3}{32} \times .00046$.
19. $12\frac{7}{16} \times .000484$.
20. .8 of $\frac{7}{9} \times .56\frac{1}{4}$.
21. $10\frac{1}{2} \times .06\frac{3}{4} \times 20$.

22. Multiply 4 hundredths by 4 thousandths, and add 4 tenths to the product. *Ans.* .40016.

23. What is the product of one-tenth by one-tenth? one hundred by one-hundredth? one million by one-millionth?

DIVISION OF DECIMALS.

181. **Division of Decimals** is the process of finding the quotient when one or both terms are decimals.

ORAL EXERCISES.

What is the value of

1. $\frac{6}{10} \div 2$? $.6 \div 2$?

2. $\frac{36}{100} \div 6$? $.36 \div 6$?

3. $.50 \div 2$? $.05 \div 2$?

4. $.150 \div 2$? $.15 \div 2$?

5. $5 \div 4$? $.5 \div 4$? $.05 \div 4$?

6. $6 \div \frac{3}{10}$? $6 \div .3$?

7. $.8 \div 4$? $8 \div .4$?

8. $7.2 \div .6$? $.72 \div .6$?

9. $8.4 \div .7$? $.84 \div .7$?

10. $96 \div .8$? $9.6 \div .8$?

11. What is the product of *tenths* by *tenths*? What, then, is the quotient of *hundredths* by *tenths*? What is the product of *tenths* by *hundredths*? What is the quotient of *thousandths* divided by *tenths*?

12. If there are three decimal places in the multiplicand and two in the multiplier, how many in the product? If there are five in the dividend and three in the divisor, how many in the quotient?

Principle.—*The quotient will contain as many decimal places as the number of decimal places in the dividend exceeds those in the divisor.*

WRITTEN EXERCISES.

1. Divide 24.6168 by 4.68.

OPERATION.

```
4.68)24.6168(5.26
     2340
     ----
     1216
      936
     ----
     2808
     2808
```

SOLUTION 1ST.—Dividing as in whole numbers, the quotient is 526. Since the dividend contains 4 decimal places and the divisor 2, the quotient should contain 4 minus 2, or 2 decimal places (Prin.); therefore the quotient is 5.26.

SOLUTION 2D.—$24.6168 \div 4.68 = \frac{246168}{10000} \div \frac{468}{100} = \frac{246168}{10000} \times \frac{100}{468} = \frac{246168}{100 \times 468} = \frac{1}{100} \times \frac{246168}{468} = \frac{1}{100}$ of $526 = 5.26$. From either of these solutions we derive the following

Rule.—I. *Divide as in whole numbers, annexing ciphers to the dividend when needed to continue the division.*

II. *Point off as many decimals in the quotient as the number of decimal places in the dividend exceeds the number in the divisor.*

1. Before beginning the division, annex ciphers when necessary to make the number of decimal places in the dividend equal to the number of decimal places in the divisor.

2. When there are ciphers at the right of the divisor, cut them off, divide by the significant part, and then point off as many decimal places as before, plus the number of ciphers cut off.

NOTE.—For the answers, see corresponding examples in Multiplication of Decimals.

Divide

2. .2176 by .34.
3. .5025 by .67.
4. .5888 by 4.6.
5. 31.088 by 5.8.
6. .5529 by 9.7.
7. .7175 by .082.
8. 272.636 by 6.37.
9. 281.8585 by 3.85.
10. 1.51088 by .019.
11. 488.98552 by 1.52.
12. 1235.434256 by 20.074.
13. 195.388698 by 6.0708.
14. 273.2879688 by 6.072.
15. 3.859243392 by 3.5702.
16. .00020596611 by .03507.
17. .0005094414 by .000246.
18. .011543125 by .00046.
19. .00601975 by .000484.
20. .35 by .8 of $\frac{7}{8}$.
21. 14.175 by .06$\frac{3}{4}$ × 20.

What is the value of

22. .075 ÷ .025? *Ans.* 3.
23. .625 ÷ 1.25? *Ans.* .5.
24. .008 ÷ .04? *Ans.* .2.
25. .018 ÷ .06? *Ans.* .3.
26. .008 ÷ 2.5? *Ans.* .0032.
27. 2 ÷ .125? *Ans.* 16.
28. .51 ÷ .015? *Ans.* 34.
29. 310 ÷ .0625? *Ans.* 4960.
30. .00625 ÷ 25? *Ans.* .00025.
31. 25 ÷ .000625? *Ans.* 40000.

32. Divide 7 hundredths by 35 thousandths. *Ans.* 2.

33. Divide 625 ten-thousandths by 25 millionths. *Ans.* 2500.

34. Divide 40016 millionths by 4 hundredths.

Ans. 1.0004.

35. Divide the sum of $2\frac{1}{2}$ thousandths and $2\frac{1}{2}$ billionths by 125 millionths, and to the quotient add 25 tenths.

Ans. 22.50002.

MISCELLANEOUS EXAMPLES.

Reduce the following common fractions to decimals:

1. $\frac{13}{25}$.	6. $\frac{35}{64}$.	11. $\frac{42}{49}$.
2. $\frac{17}{25}$.	7. $\frac{29}{36}$.	12. $\frac{16}{49}$.
3. $\frac{19}{32}$.	8. $\frac{35}{36}$.	13. $\frac{15}{77}$.
4. $\frac{21}{32}$.	9. $\frac{57}{72}$.	14. $\frac{73}{75}$.
5. $\frac{29}{64}$.	10. $\frac{111}{144}$.	15. $\frac{92}{99}$.

Reduce the following decimals to common fractions:

16. .52.	21. .546875.	26. $.857142\frac{6}{7}$.
17. .68.	22. $.80\frac{5}{9}$.	27. $.32653\frac{3}{49}$.
18. .59375.	23. $.97\frac{2}{9}$.	28. $.194805\frac{15}{77}$.
19. .65625.	24. $.791\frac{2}{3}$.	29. $.97\frac{1}{3}$.
20. .453125.	25. $.7708\frac{1}{3}$.	30. $.92\frac{92}{99}$.

Find the value of the following expressions:

31. $.21$ of $\frac{2}{3} \times 50 \times .011\frac{3}{7}$. *Ans.* .08.

32. $(2.04 \div 17 + .235 \times 5000) - \frac{7}{8}$. *Ans.* 1174.245.

33. $(789 - .789) \div (.675 \times .75$ of $8)$. *Ans.* 194.62.

34. $2.15 \div (3.5 - \frac{1}{7}$ of $3.43) + 45 \times .181$. *Ans.* $8.859\frac{2}{7}$.

35. $\$25.6\frac{7}{8} \times .05\frac{1}{3} + \$15 \times 6.03\frac{1}{4}$. *Ans.* $\$91.85\frac{3}{4}$.

36. $(\$347.84 \div \$10.87) \times (.0025 + .01\frac{3}{4}) \times 50$. *Ans.* 32.

37. $(\$1080 \times 3.27) \div \$10.90 - (\$790 \div \$3.95)$. *Ans.* 124.

38. $(200 + .02 - \frac{1}{2}) \times .8\frac{1}{4} + (.8\frac{1}{4} - .12\frac{7}{10} \times 5\frac{2}{5})$.

Ans. 164.7432.

MISCELLANEOUS PROBLEMS.

1. The product of two numbers is $\frac{4}{9}$, and one of them is $\frac{5}{24}$ of 2; what is the other? *Ans.* $1\frac{1}{15}$.

2. Add 2.5 cwt., 1.25 cwt., 5 cwt., 4.375 cwt., 1.875 cwt., 2.5 cwt., 1.225 cwt., and 1.275 cwt. *Ans.* 20 cwt.

3. What number multiplied by $\frac{52}{153}$ of $5\frac{1}{2}$ is equal to $\frac{8}{11}$ of $27\frac{1}{2}$? *Ans.* $8\frac{7}{286}$.

4. Divide the sum of six thousandths and six millionths by their difference to 6 decimal places. *Ans.* 1.002002+.

5. From the sum of $\frac{2}{11}$, $\frac{9}{5}$, and $\frac{1}{8}$ take the remainder obtained by subtracting $\frac{7}{35}$ from $\frac{4}{13}$ of $\frac{65}{56}$. *Ans.* $1\frac{585}{616}$.

6. What number must be divided by $\frac{3}{7}$ of $2\frac{1}{3}$ of 4 to make a quotient equal to the value of $6\frac{2}{3}$ times $\frac{3}{20}$ of 6? *Ans.* 24.

7. How many dress patterns of 12.50 yards each can be cut from four pieces of French muslin containing 25 yards each? *Ans.* 8 patterns.

8. A ton of iron ore from the mines of Cornwall yields .65 of a ton of pure iron; how much iron will 675.4 tons of ore yield? *Ans.* 439.01 tons.

9. An oil-refiner has on hand 18,445 gallons of coal oil; how many casks, each containing 42.5 gallons, can be filled with it? *Ans.* 434 casks.

10. How many lengths of 6-inch stove-pipe can be made from 81 pounds of Norway iron, if one length requires 3.24 pounds? *Ans.* 25.

11. An engine pumped 39.75 barrels of 31.5 gallons each from a tank containing 1500 gallons; how many gallons remained? *Ans.* 247.875 gal.

12. Mr. Michener bought a lot of wheat and sold .15 of it to one man, and .25 of it to another, and kept 572.85 bushels; how much did he buy? *Ans.* 954.75 bu.

13. A clerk had $\$225\frac{3}{8}$, and earned $\frac{1}{8}$ of $3\frac{1}{3}$ times $\$88\frac{1}{8}$

more; then losing part of his money, found that he had $186⅜ remaining; what amount did he lose? *Ans.* $97⅜.

14. Two speculators bought 3240 acres of Western land, which they divide so that one has .37½ and the other .62½ of it; how many acres has each? *Ans.* 1215; 2025.

15. A farmer paid $4000 for a truck farm, giving $76.25 an acre for 27.25 acres, and $85.75 for the remainder; how many acres did he buy? *Ans.* 49.66+ acres.

16. I bought 3 loads of wood, the first containing 1.05 cords, the second 1.02 cords, the third .945 cords; what did it cost at $4.50 a cord? *Ans.* $13.56¾.

17. A dealer bought 345.75 tons of hay, at $13.25 a ton, ⅓ of which he sold at $15.75 a ton, and the rest at cost; how much was the gain? *Ans.* $288.12½.

18. A dealer bought 1200 bu. of wheat; how many bins, each holding 20.25 bu., will it fill, and how many bushel-bags can be filled with what remains? *Ans.* 59 bins.

19. How many cords of wood, at $4.12½ a cord, must I give for 91.25 bushels of wheat, at $0.65 a bushel, and 85 bushels of rye, at $0.75 a bushel? *Ans.* 29.83⅓ cords.

20. A lady devotes .10 of her income to charity, .25 for educating her children, .55 for her living expenses, and saves the remainder, which is $127.50; required the lady's entire income. *Ans.* $1275.

21. The circumference of the fore wheel of a wagon is 13.75 feet, and of the hind wheel 15.25 feet; how much oftener does one turn than the other in going 5280 feet, or one mile? *Ans.* 37.77+ times.

22. A grain-dealer paid $4230 for a quantity of grain, ⅓ of it being for wheat, at $0.75 a bushel, ¼ of it for corn, at $0.55 a bushel, and the remainder for rye, at $0.87½ a bushel; how many bushels of each kind did he purchase?

Ans. Wheat, 1880 bu.; corn, 1922 $\frac{8}{11}$ bu.; rye, 2014 $\frac{2}{7}$ bu.

UNITED STATES MONEY.

182. United States Money, or the currency of the United States, is expressed in the decimal system.

183. The several **denominations** and their relation to one another are presented in the following table:

TABLE.

10 mills equal 1 cent.	10 dimes equal 1 dollar.
10 cents " 1 dime.	10 dollars " 1 eagle.

$\frac{1}{4}$ of a dollar = 25 cents; $\frac{1}{2}$ of a dollar = 50 cents; $\frac{3}{4}$ of a dollar = 75 cents; $\frac{1}{2}$ of a cent = 5 mills.

184. The **dollar** is the unit, and is indicated by the symbol $; the eagle and dollar are read as a number of dollars. Thus, $245 is read 245 dollars, instead of 24 eagles, 5 dollars.

185. Dimes and cents are usually read as a number of cents. Thus, $6.75 is read 6 dollars and 75 cents.

186. When the number of cents is less than 10, a cipher must be written in tenths place. Thus, 3 dollars and 4 cents are written $3.04.

187. The **mill** is one-tenth of a cent or one thousandth of a dollar, and is written in thousandths place. Thus, $8.375 is read 8 dollars 37 cents and 5 mills.

1. In checks, notes, drafts, etc. cents are usually written as hundredths of a dollar in the form of a common fraction, as 12\frac{75}{100}$.

2. When the final result of a business computation contains mills, if 5 or more they are reckoned 1 cent, and if less than 5 they are rejected. Thus, $7.187 would be reckoned as $7.19, and $3.162 as $3.16.

EXERCISES IN NUMERATION.

Read the following

1. $14.75.	**5.** $25.04.	**9.** $200.002.
2. $15.854.	**6.** $11.25.	**10.** $404.404.
3. $24.244.	**7.** $210.210.	**11.** $560.075.
4. $144.125.	**8.** $145.005.	**12.** $2045.001.

EXERCISES IN NOTATION.

Write on the slate or board:

1. Six dollars and twenty-five cents.
2. Ten dollars, thirty cents, and seven mills.
3. Thirty-five dollars, forty-four cents, and five mills.
4. Fifty-four dollars, eight cents, and eight mills.
5. Six eagles, six dollars, six cents, and six mills.
6. One hundred dollars, five and one-half cents.
7. Seventy-nine dollars, seven dimes, and $7\frac{1}{2}$ cents.
8. Nine hundred forty-eight dollars, $82\frac{1}{2}$ cents.
9. Seven thousand sixty-nine dollars, 75 cents.

FUNDAMENTAL OPERATIONS.

188. Since **United States Money** is expressed in the decimal scale, all the operations may be performed as in decimals.

WRITTEN EXERCISES.

1. Find the sum of \$$18\frac{3}{4}$ and $18\frac{3}{4}$ cents. *Ans.* \$$18.93\frac{3}{4}$.

2. From \$25 take 25 cents and 25 mills. *Ans.* \$$24.72\frac{1}{2}$.

3. Find the sum of \$$37\frac{1}{2}$, $37\frac{1}{2}$ dimes, $37\frac{1}{2}$ cents, and 375 mills. *Ans.* \$42.

4. From 2 dollars take 2 mills; from \$404 take 404 cents. *Ans.* \$1.998; \$399.96.

5. Subtract $8\frac{1}{2}$ cents from $8\frac{1}{2}$ dollars, and add the remainder to $8\frac{1}{2}$ dimes. *Ans.* \$$9.26\frac{1}{2}$.

6. A stock-dealer bought 125 calves at \$$3\frac{3}{4}$ apiece; what did they cost? *Ans.* \$468.75.

7. A drover sold the government 33 horses at the rate of \$$87\frac{1}{2}$ apiece; what did he receive? *Ans.* \$2887.50.

8. If a cattle-feeder received \$256.25 for the sale of 25 cows, what did they bring him a head? *Ans.* \$10.25.

9. If a carpenter's wages are $16.50 a week, how many days must he labor to earn $264? *Ans.* 96 days.

10. I paid $26.25 for 5 barrels of flour; what would 17 barrels have cost me at the same rate? *Ans.* $89.25.

11. A country merchant paid $95 for a lot of calicoes, at $12\frac{1}{2}$ cents a yard; how many yards did he buy? *Ans.* 760.

12. At the rate of $4.75 per cord, how many cords of wood can be bought for $1553.25? *Ans.* 327 cords.

13. Divide \$$16\frac{3}{4}$ by $12\frac{1}{2}$ cents; divide $48 by 6 dimes; divide $18\frac{3}{4}$ dimes by 15 mills. *Ans.* 134; 80; 125.

14. A lady bought a watch for \$$75\frac{1}{2}$, a chain for \$$22\frac{3}{4}$, a brooch for \$$6\frac{1}{4}$, and sold them all at a gain of \$$6\frac{3}{4}$; what did she receive for them? *Ans.* $111.25.

15. A teacher bought a scrap-book for $1.25, an inkstand for $62\frac{1}{2}$ cents, an atlas for $3.50, a globe for $4.25, and handed the clerk a ten-dollar bill; how much change should she receive? *Ans.* \$$0.37\frac{1}{2}$.

16. A merchant bought 12 hogsheads of molasses of 63 gallons each, at the rate of $32\frac{1}{2}$ cents a gallon, and sold them at 40 cents a gallon; what was the gain? *Ans.* $56.70.

17. A dressmaker bought 3 yards of muslin at $6\frac{1}{4}$ cents a yard, 7 yards of linen at 87 cents a yard, and handed the clerk a $10 bill; what was her change? *Ans.* $3.72.

18. A dealer sold one day, in Chicago, Low Grade extra flour, amounting to $15,000, at $6.25 per barrel; how many barrels were sold? *Ans.* 2400 barrels.

19. A lady in furnishing her house bought 3 sets of chairs at $5.25 a set, 2 tables at $3.75 apiece, 3 rocking-chairs at $3.25 apiece, and 45 yards of carpet at $1.25 a yard; what was the amount of the bill? *Ans.* $89.25.

20. A druggist bought 8 barrels of turpentine, each containing 31.5 gallons, at \$$0.28\frac{1}{2}$ a gallon, and sold it for \$$0.37\frac{1}{2}$ a gallon; what did he gain? *Ans.* $22.68.

21. A coal-dealer bought a boat-load of coal for $250, and by retailing it at $5.75 a ton he gained $37.50; how many tons in the load? *Ans.* 50 tons.

22. I paid for sending a telegram from New York to Philadelphia at the rate of $0.25 for 10 words, and 2 cents for each additional word; what did my dispatch of 24 words cost me? *Ans.* $0.53.

23. Atwood & Wilson sold 420 bushels of new ungraded corn at 60 cents per bushel, and received in exchange 120 bushels of oats at 36¼ cents per bushel, and No. 1 Minnesota wheat, at $0.69½ per bushel; how many bushels did they receive? *Ans.* 300 bushels.

COMMERCIAL TRANSACTIONS.

189. In **Commercial Transactions** there are ordinarily three distinct things considered—the *quantity*, the *price*, and the *cost*.

190. The **Quantity** is the amount bought or sold, expressed by the number of times it contains the *unit of measure*.

191. The **Price** is the value of one of the units of measure of any commodity. The *Cost* is the value of the whole quantity.

192. An **Aliquot Part** of a number is the whole or mixed number which will exactly divide that number.

ALIQUOT PARTS OF $1.

5 cents = $$\frac{1}{20}$.	20 cents = $$\frac{1}{5}$.
6¼ cents = $$\frac{1}{16}$.	25 cents = $$\frac{1}{4}$.
8⅓ cents = $$\frac{1}{12}$.	50 cents = $$\frac{1}{2}$.
10 cents = $$\frac{1}{10}$.	16⅔ cents = $$\frac{1}{6}$.
12½ cents = $$\frac{1}{8}$.	33⅓ cents = $$\frac{1}{3}$.

193. A few special cases of finding price, cost, and quantity will be presented.

CASE I.

194. To find the cost of a quantity, the price being an aliquot part of $1.

1. What cost 56 yards of calico at $12\frac{1}{2}$ cents a yard?

SOLUTION.—At $1 a yard, the cost would be $56; hence at $12\frac{1}{2}$ cents, which is $\frac{1}{8}$ of $1, the cost will be $\frac{1}{8}$ of $56, or $7. Hence the

OPERATION.

$$\begin{array}{r} 8\underline{)56} \\ 7 \end{array}$$

Rule.—*Take such a fractional part of the given quantity as the price is of* $1.

WRITTEN EXERCISES.

2. Required the cost of 10 pieces of muslin, each containing 44 yards, at $6\frac{1}{4}$ cents a yard. *Ans.* $$27\frac{1}{2}$.

3. What cost 4 bales of South Carolina cotton, each containing 300 lb., at $8\frac{1}{3}$ cents a pound? *Ans.* $100.

4. A lady bought a bag of Rio coffee, containing 40 lb., at $16\frac{2}{3}$ cents a lb., and 10 lb. of crushed sugar, at $6\frac{1}{4}$ cents a lb.; what was the cost of both? *Ans.* $$7.29\frac{1}{6}$.

5. A lady bought 20 yards of black cashmere, at $$1.12\frac{1}{2}$ a yard, 20 yards of paper muslin, at 10 cents a yard, and 3 dozen crocheted buttons, at 25 cents a dozen; what was her bill? *Ans.* $25.25.

CASE II.

195. To find the cost, the quantity and the price of 100 or 1000 being given.

1. What is the cost of 9760 tiles, at $$9.87\frac{1}{2}$ a thousand?

SOLUTION.—If 1000 tiles cost $$9.87\frac{1}{2}$, 1 tile will cost $\frac{1}{1000}$ of $$9.87\frac{1}{2}$, and 9760 tiles will cost 9760 times $\frac{1}{1000}$ of $$9.87\frac{1}{2}$, which is the same as $\frac{1}{1000}$ of 9760 times $$9.87\frac{1}{2}$, which, by multiplying and cutting off three places in the product, we find is $96.38.

OPERATION.

$$\begin{array}{r} 9.875 \\ \underline{9760} \\ 96.380000 \end{array}$$

Ans. $96.38.

Rule.—*Multiply the price by the quantity, and point off in the product two places for price per hundred, or three places for price per thousand.*

Price per hundred is expressed thus, ⅌ C; per thousand, ⅌ M.

WRITTEN EXERCISES.

Required the cost

2. Of 12,500 shingles at \$5.75 ⅌ M. *Ans.* \$71.87½.

3. Of 88745 laths, at \$0.37½ ⅌ C. *Ans.* \$332.79⅜.

4. What is the cost of 217,225 bricks, at \$7.75 ⅌ M, and 7560 lb. of sheet lead, at \$10.12½ ⅌ C? *Ans.* \$2448.94⅜.

5. Required the cost of 5794 paving-stones for a village sidewalk, at \$9.25 ⅌ C. *Ans.* \$535.94½.

6. If 62,560 cubic feet of gas are burned in a certain store in a year, at \$1.25 ⅌ M, what is the gas bill of the store for a year? *Ans.* \$78.20.

7. If a compositor is paid 45 cents per thousand ems, what will he receive for setting up a book of 500 pages of 1120 ems each? *Ans.* \$252.

8. Mr. Hull receives for making cigars \$6.75 ⅌ M; what will be his profit on 45,780 cigars, if he pays \$4.25 for the amount of tobacco required to make a thousand? *Ans.* \$114.45.

CASE III.

196. To find the price of 100 or 1000, the quantity and cost being given.

1. If I sell 7356 feet of boards for \$112.179, what is the price ⅌ M?

OPERATION.

$\$112.179 \div 7356 = \$15.25$

SOLUTION.—If 7356 feet cost \$112.179, 1 foot will cost $\frac{1}{7356}$ of \$112.179, and 1000 feet will cost 1000 times $\frac{1}{7356}$ of \$112.179, which, dividing and removing the decimal point three places to the right, we find is \$15.25. Hence the

Rule.—*Divide the cost by the quantity, and remove the decimal point in the quotient two places to the right for price per hundred, or three places for price per thousand.*

WRITTEN EXERCISES.

2. I paid \$6.56¼ for 875 pickets for the fence around my garden; what was the price ℔ C? *Ans.* \$0.75.

3. A lumber-dealer sold 15,780 cedar rails for \$512.85; what was the price ℔ C? *Ans.* \$3.25.

4. I retail envelopes at 12¢ a pack, gaining 3 cents on each pack of 25; what did they cost me ℔ M? *Ans.* \$3.60.

5. The cost of stereotyping a book of 320 pages of 850 ems each is \$340; what is the cost per 1000 ems? *Ans.* \$1.25.

6. If I can send wheat from Chicago to Buffalo at 8 cents per bushel (60 pounds), what does it cost ℔ C? *Ans.* 13⅓ cts.

CASE IV.

197. To find the cost, the quantity and the price of a ton of 2000 pounds being given.

1. At \$9.25 a ton, what will be the cost of 5784 lb. of plaster?

SOLUTION.—Dividing \$9.25, the price of a ton, by 2, we have \$4.62½, the price of 1000 lb.; and proceeding as in Case II., we have \$26.751 as the price of 5784 lb.

OPERATION.
```
2)9.25
  4.62½
  5784
26.75100
```

Rule.—*Multiply half the price of a ton by the quantity, and remove the decimal point three places to the left.*

To find the price of a ton, *divide the cost by the quantity, multiply the quotient by 2, and remove the decimal point three places to the right.*

WRITTEN EXERCISES.

2. A peddler sold to a paper-mill 2648 pounds of rags at \$12 per ton; what did he receive? *Ans.* \$15.89.

3. A porcelain-manufacturer bought 6745 pounds of clay at \$15.25 a ton; what did it cost him? *Ans.* \$51.43.

4. If 6248 pounds of bone-dust cost \$69.50, what is the price per ton? *Ans.* \$22.25 —.

5. Shipped from Phœnixville, on the Reading R. R., 54,760 pounds of pig iron @ \$3.25 per ton, and 87,584 pounds

of steel rails at $2.75 per ton; what was the charge for freight? *Ans.* $209.41+.

6. A merchant bought 2 loads of hay, weighing 3475 pounds and 4750 pounds respectively, the wagon weighing 750 pounds; what did the hay cost at $13.25 a ton? *Ans.* $44.55+.

7. If a coal-dealer paid $2.25 a ton at the mines, and $0.65 a ton for freight, what will he gain on 8768 pounds of coal by selling at $5.25 a ton? *Ans.* $10.30+.

BILLS AND ACCOUNTS.

198. A **Bill** is a written statement of indebtedness, giving the price, quantity, and cost of each item, and the entire amount.

199. The party owing a debt is called the *Debtor;* the party to whom the debt is due is called the *Creditor.*

200. An **Account** is a record of the business transactions between two parties for a given time.

201. The **Balance** of an account is the difference between the amounts of the debit and credit items.

202. The **Footing** of a bill is the sum of its items. To *extend* an item is to write its cost in the proper column.

203. A **Bill** is receipted when the words "Rec'd payment" or "Paid" are written at the bottom, followed by the signature of the creditor or his agent.

204. The abbreviations commonly used in bills are as follows:

@,	At.	¢,	Cents.	Pay't,	Payment.
℅,	Account.	Cr.,	Creditor.	Per,	By.
Acc't,	Account.	Dr.,	Debtor.	Pcs.,	Pieces.
Bal.,	Balance.	No. or #,	Number.	Rec'd,	Received.

NOTE.—Deductions are often made in bills, sometimes from the retail price of the items and sometimes from the amount of the bills. The symbol %, meaning *hundredths*, is frequently used in expressing deductions; thus, less 6% means 6 hundredths deducted.

Rule.—I. *In making out a* BILL, *extend the several items and take their sum.*

II. *In an* ACCOUNT *find the difference between the debit and credit amounts.*

Required the footings of the following bills:

PHILADELPHIA, *Nov.* 10, 1894.

MRS. SAMUEL WHEELER,

Bought of TAYLOR & SNYDER.

4 lb. Black Tea,	@ 50¢	2	00
3 lb. Coffee,	@ 40¢	1	20
5 lb. Crackers,	@ 25¢	1	25
25 lb. Oat Meal,	@ 5¢	1	25
		5	70

NEW YORK, *Nov.* 8, 1894.

MRS. JAMES O'NEILL,

Bought of HILTON, HUGHES & CO.

15 yd. Black China Silk,	@ 65¢		
7 yd. Green Henrietta Cloth,	@ 50¢		
6 yd. Navy Blue Serge,	@ 80¢		
2 Felt Hats,	@ 67¢		
2 English Velour Capes,	@ $14.75		
		48	89

Rec'd payment,

HILTON, HUGHES & CO.

BROOKLYN, *Dec.* 10, 1894.

MRS. WILLIAM MINER,

Bo't of WARNER & BURNHAM.

6 pr. Merino Hose,	@ 37½¢		
12 yds. Flannel,	@ $0.65		
6 pr. Kid Gloves,	@ $1.50		
3 pr. Cashmere Gloves,	@ 25¢		
6 Huckaback Towels,	@ 25¢		
3 Turkish Towels,	@ 18¾¢		
		21	86

Rec'd payment,

WARNER & BURNHAM.

BOSTON, *Jan.* 10, 1895.

MR. THOMAS WILSON,

Bought of MITCHELL & WALTON.

25 lb. Rio Coffee,	@ 16¢		
4 gal. N. O. Molasses,	@ 20¢		
20 lb. White Sugar,		1	00
8 lb. Oolong Tea,	@ 65¢		
6 lb. Butter,	@ 30¢		
6 lb. Cheese,	@ 15¢		
Charged in %.		13	70

NEW YORK, *July* 15, 1894.

A. T. JACKSON & CO.,

Bought of TAYLOR, WILSON & CO.

24 Prs. Men's French Calf Boots,	@ $10		
65 " Women's Enameled "	1.87½		
75 " " Fox'd Gaiters,	1.50		
88 " Boys' Kip Boots,	2.50		
110 " " Brogans,	1.25		
90 " Misses' Rubber Shoes,	.75		
60 " Women's Arctics,	2.25		
Less ⅕.			
Rec'd payment,		827	50

TAYLOR, WILSON & CO.

BALTIMORE, *May* 27, 1894.

MR. EDWARD FERRERS,

To A. F. BENNETT, DR.

For Setting Boiler and Excavating,		100	00
" 13½ Days' Work of Bricklayer,	@ $4		
" 5½ " " " Laborer,	1.12½		
" 1 Load Sand,		2	50
" 15 Bushels Lime,	.38		
" 500 Dark Stretchers,	@ $14 ℔ M		
" 500 Salmon Bricks,	" 8 " "		
		179	39

BOSTON, *June* 1, 1895.

MR. JOHN WALKER,

Bought of HOUGHTON, MIFFLIN & CO.

1895							
Apr.	4	1 Set Waverley Novels,	$\frac{3}{10}$	12			
	7	4 Holmes's Poems,	$\frac{1}{4}$	2	00		
May	2	3 Whittier's Poems,	$\frac{1}{4}$	2	00		
	6	2 Bampton Lectures,	$\frac{1}{4}$	1	75		
	"	4 Browning's Works,	$\frac{1}{5}$	12	00		
	29	3 Rambles through Japan,	$\frac{3}{10}$	1	50		
	"	2 Asiatic Breezes,	$\frac{1}{3}$	1	25		
						58	34

Rec'd payment,

JOHN B. ALLEN, for H., M. & Co.

NOTE.—The fractions denote the deductions made on the different books named. Make the deduction from each item before extending it.

PHILADELPHIA, *Oct.* 1, 1894.

MRS. HENRY BACON,

Terms: 30 days. *Bought of* TYNDALE & CO.

2	Pr. W. G. Ewers and Basins,	1.40				
2	" Cov'd Soaps	50				
2	" Brush Trays	50				
½	Doz. " Bowls	1.75				
1	" " Mugs		1	50		
2	" Stem Wines	1.00				
2	" Glass Peppers	1.25				
2	" Individual Salts	50				
3	" Oval Glass Dishes $\frac{\text{1-3 in}}{100}$ $\frac{\text{1-6 in}}{150}$ $\frac{\text{1-7 in}}{200}$					
	Crate and Porterage		1	25	18	43

NOTE.—$\frac{\text{1-3 in}}{100}$ $\frac{\text{1-6 in}}{150}$ $\frac{\text{1-7 in}}{200}$ means that there are 1 dozen 3-inch dishes @ \$1, 1 dozen 6-inch @ \$1.50, and 1 dozen 7-inch @ \$2.

WRITTEN EXERCISES.

Let pupils make out bills in proper form from the following statements:

1. Mrs. Evans bought of H. K. Thurber & Co. 25 lb. Whole Pepper, @ 16¢, 10 lb. Nutmegs, @ \$1, and 1 box O. K. Mustard, 12 lb., @ 25¢. What was the amount of the bill? *Ans.* \$17.

2. James Morrow & Co. bought of Wm. Waters & Son 4 doz. cans of Peaches, @ \$2.35; 2 doz. cans of Salmon, @ \$3.85; 4 doz. cans Tomatoes, @ \$1.80. What was the amount of the bill? *Ans.* \$24.30.

3. John Russell bought of Wm. Talcott, 6 Oak Churns, @ \$1.60; 2 doz. 1-bu. Corn Baskets, @ \$4; $\frac{1}{2}$ doz. Potato Mashers, @ \$1; 1 doz. 6-ft. Ladders, @ \$4. What was the amount of the bill? *Ans.* \$22.10.

4. James Oliver bought of Biddle Hardware Co. $\frac{1}{2}$ gro. Table Knives and Forks, @ \$8.40; $\frac{1}{2}$ doz. Razors, @ \$9; $\frac{1}{2}$ doz. Pocket Knives, @ \$6; $\frac{1}{12}$ doz. Cheese Knives, @ \$9.60. Required the amount of the bill. *Ans.* \$12.50.

5. Copeland's restaurant ordered Oct. 10, 1894, from James Williams, the following: 67 lb. Ribs of Beef, @ 20¢; 3 Lambs, @ \$5; 3 Calves' Heads, @ 75¢; 2 Calves' Livers, @ 70¢; 37 lb. Veal, @ 17¢. Make out the bill. *Ans.* \$38.34.

6. Charles Fletcher delivered the following, May 12, 1894, at the Colonnade Hotel: 15 Salmon, @ 30¢; 37 Lobsters, @ 10¢; 5 Fresh Mackerel, @ 12¢; 2 Shad, @ 40¢; 9 Sea-bass, @ 10¢; 3 doz. Soft Crabs, @ 50¢; $1\frac{1}{2}$ doz. Frogs, @ \$1.75. Extend the items and find the footing of the bill. *Ans.* \$14.63.

7. Lewis G. Summer bought of Blanding & Co., Jan. 20, 1893, 1 lb. Citric Acid, \$1.55; 5 lb. Chloride Lime, @ 8¢; jar, 20¢; 10 lb. Epsom Salts, @ 5¢; 3 lb. Gum Shellac, @ 60¢; 1 doz. Hood's Sarsaparilla, @ \$7; 2 lb. Carbolic Acid Crystals, @ \$1.75. What was the amount of the bill? *Ans.* \$14.95.

8. Mrs. Mary Taylor of Phila. presents the following bill to Mr. John Wallace, Feb. 1, 1895: Board for 4 weeks, @ \$10; fuel and light 4 weeks, @ \$1. Mr. Wallace presented the following bill to Mrs. Taylor at the same date: Jan. 11, 10 lb. Tea, @ 60¢; Jan. 15, 16 lb. Sugar, \$1; 1 Barrel No. 1 Mackerel, \$25; Jan. 26, 5 lb. Butter, @ 45¢, and 2 doz. Eggs, @ 35¢. Make out both bills, receipting the smaller and crediting the amount upon the other. *Ans.* Cr. Mrs. T., \$9.05.

SECTION VI.

DENOMINATE NUMBERS.

205. A **Denominate Number** is a concrete number in which the unit is a *measure* established by custom or law; as, 3 *feet*, 4 *pounds*, etc.

206. A **Compound Denominate Number** is a number which expresses several different units of the same kind of quantity; as, 4 yd. 2 ft. 11 in.

207. The **Terms** of a compound number are the *numbers of its different units.* Thus, the *terms* in the example given are 4 yd., 2 ft., and 11 in.

208. **Similar Compound Numbers** are compound numbers which express the same kind of quantity.

209. **Denominate Numbers** may be embraced under eight distinct classes, as follows:

1. Weight.	4. Volume.	7. Angles.
2. Length.	5. Capacity.	8. Value.
3. Surface.	6. Time.	

NOTE.—For the answers to the problems under the Written Exercises compare the problems in the opposite columns.

MEASURES OF WEIGHT.

210. **Weight** is the measure of the force by which bodies are attracted towards the earth.

211. There are three kinds of weight in common use: *Troy Weight, Apothecaries' Weight,* and *Avoirdupois Weight.*

TROY WEIGHT.

212. **Troy Weight** is used in weighing gold, silver, jewels, liquors in philosophical experiments, etc.

TABLE.

24 grains (gr.)	= 1 pennyweight . . .	pwt.
20 pennyweights	= 1 ounce	oz.
12 ounces	= 1 pound	lb.

The *standard unit* of weight is the *Troy pound*. It is equal to the weight of 22.794377 cubic inches of distilled water at the temperature of 39.83° Fahrenheit, barometer at 30 inches.

ORAL EXERCISES.

How many

1. Gr. in 4 pwt.? In 5 pwt.?
2. Pwt. in 48 gr.? In 120 gr.?
3. Pwt. in 5 oz.? In 8 oz.?
4. Ounces in 7 lb.? In 80 pwt.?
5. Ounces in 12 lb.? In 120 pwt.?
6. Pounds in 60 oz.? In 108 oz.?

WRITTEN EXERCISES.

1. How many grains in 5 oz. 14 pwt. 13 gr.?

SOLUTION.—In one ounce there are 20 pennyweights, and in 5 oz. there are 5 times 20 pennyweights, and 5 times 20 pennyweights plus 14 pennyweights are 114 pennyweights. In 1 pennyweight there are 24 grains, and in 114 pennyweights there are 114 times 24 grains, and 114 times 24 grains plus 13 grains equal 2749 grains.

OPERATION.

```
oz.  pwt.  gr.
5    14    13
20
114
 24
2749 gr.   Ans.
```

2. How many lb., oz., and pwt. in 758 gr.?

SOLUTION.—There are 20 pwt. in 1 ounce; hence in 758 pwt. there are as many ounces as 20 is contained times in 758, which are 37 ounces and 18 pwt. remaining. There are 12 oz. in 1 pound; hence in 37 oz. there are as many pounds as 12 is contained times in 37, which are 3 lb. and 1 oz. remaining. Hence, in 758 pwt. there are 3 lb. 1 oz. 18 pwt.

OPERATION.

```
      pwt.
2|0)75|8
 12)37 - 18 pwt.
    3 lb. 1 oz.
```

How many

3. Grains in 8 pwt. 12 gr.?
4. Gr. in 6 oz. 18 pwt. 15 gr.?
5. Gr. in 15 oz. 1 pwt. 21 gr.?
6. Gr. in 4 lb. 11 oz. 10 pwt. 9 gr.?
7. Pwt. in 204 gr.?
8. Ounces in 3327 gr.?
9. Ounces in 7245 gr.?
10. Pounds in 28569 gr.?

11. How many grains in a pound Troy? How many grains in an ounce Troy? *Ans.* 5760; 480.

AVOIRDUPOIS WEIGHT.

213. Avoirdupois Weight is used for weighing everything except jewels, precious metals, etc.

TABLE.

16 ounces	= 1 pound	lb.
100 pounds	= 1 hundredweight	cwt.
20 hundredweight	= 1 ton	T.

The *unit* is the *pound*. It is derived from the Troy pound, and contains 7000 grains Troy. It is equal to the weight of 27.7015 cubic inches of water at 39.83° Fahr., the barometer being at 30 inches.

ORAL EXERCISES.

How many

1. Ounces in 7 lb.? In 10 lb.?
2. Lb. in 80 oz.? In 7 cwt.?
3. Cwt. in 3200 lb.? In 20 T.?
4. T. in 1200 cwt.? In 160 cwt.?
5. How many cwt. in $\frac{1}{2}$ of a ton? In $\frac{1}{4}$ of a ton?
6. How many pounds in 15 cwt.? In 20 cwt.? In 50 cwt.?

WRITTEN EXERCISES.

How many

1. Lb. in 6 T. 18 cwt. 45 lb.?
2. Lb. in 4 T. 17 cwt. 97 lb.?
3. Oz. in 19 cwt. 53 lb. 15 oz.?
4. Oz. in 8 T. 73 lb. 14 oz.?
5. Tons in 13845 lb.?
6. Tons in 9797 lb.?
7. Cwt. in 31263 oz.?
8. Tons in 257182 oz.?

9. How many grains in a pound Avoirdupois? How many grains in an ounce Avoirdupois? *Ans.* 7000; $437\frac{1}{2}$.

10. Which is heavier, and how much, a pound of lead or a pound of silver?

11. Which is heavier, and how much, an ounce of feathers or an ounce of silver?

12. How many Avoirdupois pounds are there in 175 Troy pounds? *Ans.* 144.

APOTHECARIES' WEIGHT.

214. Apothecaries' Weight is used in prescribing and mixing dry medicines. Medicines are bought and sold by Avoirdupois Weight.

TABLE.

20 grains (gr.)	= 1 scruple	℈.
3 scruples	= 1 dram	ʒ.
8 drams	= 1 ounce	℥.
12 ounces	= 1 pound	℔.

The *unit* is the *pound*, and is identical with the Troy pound, as are also the ounce and grain, the ounce being differently divided.

ORAL EXERCISES.

How many

1. Grains in 5 ℈? In 9 ℈?
2. Scruples in 7 ʒ? In 14 ʒ?
3. Drams in 12 ℈? In 96 ℈?
4. Drams in 6 ℥? In 8 ℥?
5. Ounces in 80 ʒ? In 104 ʒ?
6. Pounds in 96 ℥? In 144 ℥?
7. How many doses of 2 grains each can be made from a dram of quinine?

WRITTEN EXERCISES.

How many

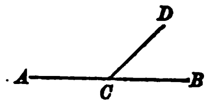
1. Drams in 8 lb. 6℥?
2. Ounces in 39 lb. 8℥?
3. Scru. in 18 lb. 4℥ 3ʒ 2℈?
4. Grs. in 16 lb. 4ʒ 2℈ 16 gr.?
5. Pounds in 816ʒ?
6. Pounds in 476℥?
7. Pounds in 5291℈?
8. Pounds in 92456 gr.?
9. How many pills of 4 grains each can be made from 5ʒ 1℈ of quinine? *Ans.* 80.

MEASURES OF LENGTH.

215. Measures of Length are used in measuring length, breadth, height, distance, etc.

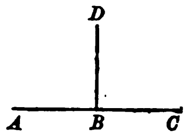
216. A Line is that which has length without breadth or thickness.

A line is estimated by ascertaining how many times it contains some definite length regarded as a unit of measure.



217. An **Angle** is the opening between two lines which diverge from a common point. Thus, *ACD* and *DCB* are angles.

218. The **Vertex** of an angle is the point from which the two lines diverge; thus, *C* is the vertex of the angle *BCD*.



219. A **Right Angle** is formed by one line perpendicular to another; as, *ABD* or *CBD*. One line is *perpendicular* to another when it makes the two adjacent angles equal.

LONG MEASURE.

220. **Long Measure** is used for the general purposes of measuring length and distances.

TABLE.

12 inches (in.)	= 1 foot	ft.
3 feet	= 1 yard	yd.
5½ yards, or 16½ feet . .	= 1 rod	rd.
320 rods	= 1 statute mile	mi.
3 miles	= 1 league	lea.
69.16 miles (69⅙ nearly) .	= 1 degree of latitude, or of longitude at the equator.	deg. or °,

1. The *standard unit* of length is the *yard*, from which all other measures of length, and also those of capacity, weight, etc., are derived.

2. The *geographic* or *nautical mile* is equal to 1 minute of one of the great circles of the earth; hence it equals $\frac{1}{60}$ of $\frac{1}{360}$ of the circumference of the earth, which equals about 1.15 statute miles.

ORAL EXERCISES.

How many

1. Feet in 96 in.? In 13 yd.?
2. Yards in 33 ft.? In 10 rd.?
3. Rods in 22 yd.? In 2 mi.?
4. Inches in 3 ft.? In 3 yd.?
5. Miles in 320 rd.? In 5 lea.?
6. Yards in 36 in.? In 108 in.?
7. What part of 1 yard are 2 ft.? Of 2 feet are 4 inches?
8. How many feet high is a horse which measures 15¼ "hands" in height, the hand being 4 inches?

WRITTEN EXERCISES.

1. How many feet in 12 rd. 3 yd. 2 ft.?

OPERATION.

```
rd.  yd.  ft.
12   3    2
5½
----
63
 6
----
69 yd.
 3
----
209 ft.  Ans.
```

NOTE.—We multiply by 5, and add to the product the 3 yds., and then, multiplying by ½, we have 69 yd.

2. How many rods in 209 ft.?

OPERATION.

```
  feet.
 3)209
5½)69 - 2 ft.
 2   2
11)138
    12 - 6 halves = 3 yd.
   Ans. 12 rd. 3 yd. 2 ft.
```

NOTE.—To divide by 5½ we reduce both to halves; then the remainder is *halves*, which we reduce to *wholes* by dividing by 2.

How many

3. Inches in 29 ft. 6 in.?
4. Feet in 5 rd. 3 yd. 1 ft.?
5. Inches in 344 rd. 8 in.?
6. In. in 2 rd. 5 yd. 2 ft. 9 in.?
7. Feet in 5 mi. 208 rd. 2 yd. 2 ft.?
8. Feet in 354 in.?
9. Rods in 92½ ft.?
10. Rods in 68120 in.?
11. Rods in 609 in.?
12. Miles in 29840 ft.?

SURVEYORS' LINEAR MEASURE.

221. Surveyors' Linear Measure is used by surveyors and engineers in measuring the dimensions of land, distances, etc.

TABLE.

7.92 inches (in.)	= 1 link	li.
25 links	= 1 rod	rd.
100 links	= 1 chain	ch.
80 chains	= 1 mile	mi.

The *unit* is a chain called *Gunter's Chain*, which consists of 100 links, and is 4 rods, or 66 feet, long.

MARINERS' MEASURE.

222. Mariners' Measure is used by seamen in measuring distances, the depth of the sea, etc.

TABLE.

6 feet = 1 fathom. 120 fathoms = 1 cable-length.
880 fathoms = 1 mile.

ORAL EXERCISES.

How many

1. Inches in a chain?
2. Feet in a chain?
3. Feet in 5 fathoms?
4. Feet in a cable-length?
5. The "sounding" taken on a vessel was 20 fathoms; how deep was the water?

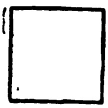
WRITTEN EXERCISES.

Reduce

1. 4 miles to links.
2. 8 mi. 43 ch. to links.
3. 10 mi. 57 fath. to faths.
4. 32000 li. to miles.
5. 68300 li. to miles.
6. 8857 fath. to miles.
7. If the Atlantic Ocean is 3000 miles wide, how many feet is that, and how many steps 3 ft. long would a person take to walk the distance? *Ans.* 5280000 steps.

MEASURES OF SURFACE.

223. A **Surface** is that which has length and breadth without thickness.

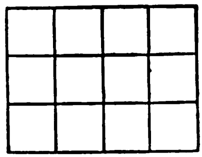


224. A **Square** is a plane surface which has four equal sides and four right angles.

The **Area** of a surface is the number of square units in the surface.

225. All **Surfaces** are *measured* by ascertaining the number of times they contain a small square regarded as the unit of measure.

Thus, in the surface in the margin there are three rows of squares, each row containing 4 squares; hence, there are 3 times 4, or 12, squares in all; and since these make up the entire surface, the measure of the surface, called its *area*, is 12 square units.



SURFACE OR SQUARE MEASURE.

226. Surface or Square Measure is used in measuring surfaces, as land, boards, amount of painting, papering, plastering, paving, etc.

TABLE.

144 square inches (sq. in.) .	= 1 square foot . . .	sq. ft.
9 square feet	= 1 square yard . .	sq. yd.
$30\frac{1}{4}$ square yards	= 1 perch or sq. rod . .	P.
160 perches	= 1 acre	A.
640 acres	= 1 square mile . .	sq. mi.

1. The *unit* for land is the *acre:* for other surfaces it is usually the *square yard.*

2. The *perch* is a surface equal to a *square rod.* The *rood* is found now only in old title-deeds and surveys; it is equal to 40 perches.

3. A square piece of land, measuring 209 feet, or about 70 paces on each side, equals very nearly one acre.

ORAL EXERCISES.

How many

1. Square inches in 4 sq. ft.?
2. Square feet in 288 sq. in.?
3. Square yards in 108 sq. ft.?
4. Perches in 121 sq. yd.?
5. Roods in 80 perches?
6. Acres in 2 sq. mi.?
7. What is the difference between 4 feet square and 4 square feet?

WRITTEN EXERCISES.

Reduce

1. 43 A. 10 P. to perches.
2. 513 P. to square yards.
3. 4 A. 134 P. to sq. ft.
4. 149 P. 27 sq. yd. to sq. in.
5. 1 A. 68 P. 29 sq. yd. to sq. in.
6. 6890 P. to acres.
7. $15518\frac{1}{4}$ sq. yd. to perches.
8. $210721\frac{1}{2}$ sq. ft. to acres.
9. 5876388 sq. in. to perches.
10. 8976096 sq. in. to acres.

SURVEYORS' SQUARE MEASURE.

227. Surveyors' Square Measure is used by surveyors in computing the area of land.

TABLE.

625 square links (sq. li.) .	= 1 square rod	. . .	P.
10,000 square links (sq. li.) .	= 1 square chain	.	sq. ch.
10 square chains	= 1 acre	. . . . . .	A.
640 acres	= 1 square mile	.	sq. mi.
36 sq. mi. (6 miles square)	= 1 township	. . .	Tp.

Also, 16 perches = 1 sq. chain; 10 sq. ch. = 1 acre; or 40 perches = 1 rood; 4 roods = 1 acre. The *perch* and *rood* are not so much used as formerly, the contents of land being commonly estimated in square miles, acres, and hundredths.

ORAL EXERCISES.

How many

1. Square links in 5 sq. ch.?
2. Square chains in 40000 sq. li.?
3. Acres in 70 sq. ch.? In 3 sq. mi.?
4. Sq. ch. in 75 acres?
5. Square miles in 1920 A.?
6. Links in 2 sq. rd.?

WRITTEN EXERCISES.

1. Reduce 21 sq. mi. 47 A. to sq. ch. *Ans.* 134870 sq. ch.
2. Reduce 1,000,000 sq. li. to acres. *Ans.* 10 A.
3. 9700 sq. chains to sq. miles. *Ans.* 1 sq. mi. 330 A.
4. 250 A. 9 sq. ch. to sq. links. *Ans.* 25090000.
5. 18 sq. mi. 75 A. 8 sq. ch. to sq. li. *Ans.* 1159580000.

MEASURES OF VOLUME.

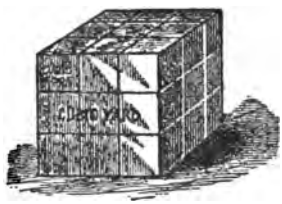
228. A Volume is that which has length, breadth, and thickness or height. A volume is also called a *solid.*

229. A Cube is a volume bounded by six equal squares.

230. All Volumes are *measured* by ascertaining the number of times they contain a *small cube* regarded as a *unit of measure.*

Thus, in the cube in the margin it will be seen that there are 3 times 3, or 9, cubes upon one surface, and since there are three such layers, there are 3 times 9, or 27, little cubes in all; and since these make up the entire volume, the measure of the cube is 27 cubic units. Such a measure is called the *contents* of the volume.

3 feet high.



3 feet wide.

CUBIC, OR SOLID MEASURE.

231. Cubic, or Solid Measure, is used in measuring things which have length, breadth, and thickness.

TABLE.

1728 cubic inches (cu. in.)	=	1 cubic foot	cu. ft.
27 cubic feet	=	1 cubic yard	cu. yd.
16 cubic feet	=	1 cord foot	cd. ft.
8 cord feet, or 128 cubic feet	=	1 cord of wood	cd.

1. A *cord of wood*, so named from being originally measured by a *cord* or *string*, is a pile 8 ft. long, 4 ft. wide, and 4 ft. high. A *cord foot* is a part of this pile 1 ft. long; it equals 16 cubic feet. See *Art.* 326.

2. The *ton* of 40 ft. for *round*, or 50 ft. for *hewn*, timber is seldom used.

ORAL EXERCISES.

How many

1. Cubic inches in 2 cu. ft.?

2. Cubic feet in 4 cu. yd.?

3. Cubic yards in 81 cu. ft.?

4. Cubic feet in 5 cd.?

5. Cords in 256 cu. ft.?

6. Cubic yards in 135 cu. ft.?

7. What is the difference between a 2-inch cube and 2 cubic inches?

WRITTEN EXERCISES.

Reduce

1. 4 cd. 6 cd. ft. to cu. ft.

2. 1220 cd. 4 cd. ft. to cd. ft.

3. 26 cd. 10 cu. ft. to cu. ft.

4. 233 cd. 62 cu. ft. to cu. ft.

5. 608 cu. ft. to cords.

6. 9764 cd. ft. to cords.

7. 3338 cu. ft. to cords.

8. 29886 cu. ft. to cords.

MEASURES OF CAPACITY.

232. Measures of Capacity are volumes used to determine the quantity of fluids and many dry substances.

233. Measures of capacity are therefore of two kinds—*Measures of Liquids* and *Measures of Dry Substances.*

234. Liquid Measures are of two kinds—*Liquid* or *Wine Measure* and *Apothecaries' Fluid Measure.*

LIQUID OR WINE MEASURE.

235. Liquid or Wine Measure is used for measuring all kinds of liquids.

TABLE.

4 gills (gi.)	= 1 pint	pt.
2 pints	= 1 quart	qt.
4 quarts	= 1 gallon	gal.

1. The *standard unit* of wine measure is the *gallon*, which contains 231 cubic inches, and will hold a little more than 8⅓ lb. Av. of distilled water. This is called the *Winchester gallon*, from the standard having been formerly kept at Winchester, England.

2. *Barrels* and *hogsheads* are of variable capacity. The barrel of 31½ gallons and hogshead of 63 gallons are used in estimating the capacity of wells, cisterns, vats, etc.

APOTHECARIES' FLUID MEASURE.

236. Apothecaries' Fluid Measure is used for measuring liquids in preparing medical prescriptions.

TABLE.

60 minims (♏)	= 1 fluidrachm	fʒ.
8 fluidrachms	= 1 fluidounce	f℥.
16 fluidounces	= 1 pint	O.
8 pints	= 1 gallon	Cong.

In estimating the quantity of fluids, a *common teaspoon* holds about 1 fluidrachm; a common *tablespoon*, about ½ a fluidounce; a *wineglass*, about 1½ fluidounces; a common *teacup*, about 4 fluidounces. The *minim* is equivalent to a drop of water, but the drops of different fluids vary in size according to the tenacity of the liquid.

ORAL EXERCISES.

How many

1. Gills in 4 pints?
2. Pints in 8 quarts?
3. Pints in 1 gallon?
4. Quarts in 12 pints?
5. Quarts in 24 gills?
6. Gallons in 36 quarts?
7. Gallons in 48 pints?
8. Pints in 48 fluidounces?

WRITTEN EXERCISES.

Reduce

1. 12 gal. 3 qt. to quarts.
2. 15 gal. 2 qt. 1 pt. to pints.
3. 23 gal. 3 qt. 1 pt. 3 gi. to gills.
4. 15 Cong. 7 O. 6 f℥ to f℥.
5. 51 qt. to gallons.
6. 125 pt. to gallons.
7. 767 gi. to gallons.
8. 16304 f℥ to Cong.
9. What cost 6 gal. 3 qt. 1 pt. of vinegar @ 3¢ a pint? *Ans.* 1.65.

DRY MEASURE.

237. Dry Measure is used in measuring dry substances, such as grain, fruit, salt, coal, etc.

TABLE.

2 pints (pt.)	= 1 quart	qt.
8 quarts	= 1 peck	pk.
4 pecks	= 1 bushel	bu.

1. The *unit* of dry measure is the Winchester bushel, formerly used in England, and named from the place where the standard was preserved. Its form is a cylinder 18½ in. in diameter and 8 in. deep. Its volume is 2150.42 cu. in., and it contains 77.627413 lb. Av. of distilled water. The New York bushel is nearly identical with the Imperial bushel of Great Britain, containing 2218.192 cu. in.

2. The *cental* of 100 lb. is a standard recently recommended by the Boards of Trade in New York, Cincinnati, Chicago, and other large cities, for estimating grain, seeds, etc.

ORAL EXERCISES.

How many

1. Pints in 6 qt.? In 8 qt.?
2. Quarts in 5 pk.? In 7 pk.?
3. Pecks in 5 bu.? In 8 bu.?
4. Quarts in 16 pt.? In 24 pt.?
5. Bushels in 12 pk.? In 36 pk.?
6. Bushels in 32 qt.? In 64 qt.?

WRITTEN EXERCISES.

Reduce

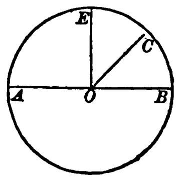
1. 15 qt. 1 pt. to pints.	5. 31 pt. to quarts.
2. 18 pk. 6 qt. 1 pt. to pints.	6. 301 pt. to pecks.
3. 24 bu. 3 pk. 7 qt. to quarts.	7. 799 qt. to bushels.
4. 37 bu. 5 qt. 1 pt. to pints.	8. 2379 pt. to bushels.

9. What cost 16 bu. 2 pk. 7 qt. of clover-seed at 18¾ cents a quart? *Ans.* $100.31¼.

CIRCULAR MEASURE.

238. Circular Measure is used to measure angles and directions, latitude and longitude, difference of time, etc.

239. A **Circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within, called the *centre.*



240. The **Circumference** of a circle is the bounding line; any part of the circumference, as *BC*, is an *arc*. An arc of one-fourth of the circumference is called a *quadrant.*

241. For the purpose of measuring angles the circumference is divided into 360 equal parts, called *degrees*; each degree into 60 equal parts, called *minutes;* each minute into 60 equal parts, called *seconds.*

242. Any angle having its vertex at the centre is measured by the arc included between its sides; thus, the angle *BOC* is measured by the arc *BC*. A right angle is measured by 90 degrees, or a quadrant; half a right angle, by 45 degrees, etc.

TABLE.

60 seconds (″)	= 1 minute	′
60 minutes	= 1 degree	°
30 degrees	= 1 sign	S.
12 signs, or 360°	= 1 circumference . . .	C.

1. The *unit* is the *degree*, which is $\frac{1}{360}$ of the circumference of a circle. A *quadrant* is one-fourth of a circumference, or 90°. A *minute* of the earth's circumference is called a *geographic mile.*

2. The divisions of the circumference are not of absolute length; they are merely *equal parts*, used to indicate the size of angles. Thus, a *quadrant*, whether the circle is large or small, measures a *right angle.*

WRITTEN EXERCISES.

Reduce

1. 45′ 20″ to seconds.
2. 12° 25′ to minutes.
3. 24° 37′ 48″ to seconds.
4. 3 S. 18° 15′ 22″ to seconds.
5. 2720″ to minutes.
6. 745′ to degrees.
7. 88668″ to degrees.
8. 389722″ to signs.

9. If Mars at a certain time is 35° 17′ 30″ east of Jupiter, how many seconds are they apart? *Ans.* 127050″.

TIME.

243. Time is a portion of duration. The *measures* of time are fixed by the revolution of the earth on its axis and around the sun.

1. The revolution of the earth on its axis fixes the *day*; its revolution around the sun fixes the *year.*

2. The day is divided into 24 hours; each hour, into 60 minutes, and each minute, into 60 seconds.

TABLE.

60 seconds (sec.)	= 1 minute	min.
60 minutes	= 1 hour	h.
24 hours	= 1 day.	da.
365 days	= 1 common year . .	yr.
366 days	= 1 leap year . . .	yr.
100 years	= 1 century	cen.

ALSO,

7 days	= 1 week . . .	wk.
4 weeks	= 1 lunar month,	mo.
13 lunar months, 1 da., 6 hr., or 12 calendar months	= 1 year . . .	yr.

The *unit of time* is the *day;* it is determined by the revolution of the earth on its axis. The *Sidereal Day* is the exact time of the revolution of the earth on its axis. The *Solar Day* is the time of the apparent revolution of the sun around the earth. The *Astronomical Day* is the solar day, beginning and ending at noon. The *Civil Day* is the average length of all the solar days of the year; it begins at 12 o'clock midnight, and consists of two periods of 12 hours each.

THE CALENDAR.

244. The **Calendar** is a division of time into periods adapted to the purposes of civil life.

245. The **Year** is divided into 12 calendar months, three of which constitute a period called a *Season.*

246. The seasons, months, and number of days in each are given in the following table:

No. of Mo.	Month.	Season.	No. of Days.
1	January	Winter	31
2	February	Winter	28 or 29
3	March	Spring	31
4	April	Spring	30
5	May	Spring	31
6	June	Summer	30
7	July	Summer	31
8	August	Summer	31
9	September	Autumn	30
10	October	Autumn	31
11	November	Autumn	30
12	December	Winter	31

The number of days in each month is easily remembered by the following stanza:

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February alone;
To which we twenty-eight assign,
Till leap year gives it twenty-nine.

ORAL EXERCISES.

1. How many seconds in 6 minutes? In 1 hour?

2. How many minutes in 420 seconds? In 5 hours?

3. How many hours in 840 minutes? In 6 days?

4. How many days in 48 hours? In 3 weeks?

5. How many days in 2 common years? In 2 leap years?

6. How many common years in 730 days?

7. Name the months which have 30 days each; those which have 31 days each.

8. How many days in the three spring months? In the three summer months?

WRITTEN EXERCISES.

How many

1. Seconds in 1 day?
2. Minutes in 1 week?
3. Min. in 3 da. 5 h. 45 min.?
4. Sec. in 21 h. 21 min. 30 sec.?
5. Days in 86400 sec.?
6. Weeks in 10080 min.?
7. Days in 4665 min.?
8. Hours in 76890 sec.?

9. In a common year how many hours? How many minutes? Seconds? *Ans.* 8760; 525,600; 31,536,000.

10. How many days from April 9th to September 11th? *Ans.* 155 days.

11. How many days from March 28th to December 16th? *Ans.* 263 days.

12. How many more days in the three summer months of 1776 than in the three winter months of that year?

13. How many days from Mar. 21st to July 16th? From April 14th to August 27th? From May 12th to October 27th?

ADJUSTMENT OF THE CALENDAR.

247. A **True** or **Solar Year** is the exact time in which the earth revolves around the sun. It consists of 365 da. 5 h. 48 min. 49.7 sec.

248. Since it is inconvenient to reckon the fractional part of a day each year, it is necessary to arrange a correct calendar in which each year may have a whole number of days. This is done by causing some years to consist of 365 days and others of 366 days. The former are called *Common years*, the latter *Bissextile* or *Leap years.*

249. The calendar is reckoned according to the following

Rule.—*Every year that is divisible by 4, except the centennial years, and every centennial year divisible by 400, is a leap year; all the others are common years.*

1. The centennial years are those whose expressions in figures end in two ciphers.

2. For a full explanation of the above rule, see Supplement.

ORAL EXERCISES.

1. How many centuries is it since the birth of Christ?

2. When did the 18th century end and the 19th century begin?

3. How many leap years and how many common years in every century?

4. Which of the following are leap years: 1700? 1760? 1776? 1800? 1876? 1880? 1890? 1894? 1900? 2000?

5. My watch ticks 4 times in a second; how many times will it tick in a day?

WRITTEN EXERCISES.

1. How many days in the three years 1894, 1895, and 1896? *Ans.* 1096 da.

2. How many days from the beginning of 1880 to the close of 1896? *Ans.* 6210 da.

3. If any year begins on Monday, on what day will the following year begin?

4. If the year 1876 began on Saturday, on what day did 1877 begin? *Ans.* Monday.

5. The 4th of July, 1776, fell on Thursday; on what day did the year begin? *Ans.* Monday.

6. If the year 1895 begins on Tuesday, on what day will the year 1900 begin? *Ans.* Monday.

7. How many days from January 20th, 1892, to September 24th of the same year? *Ans.* 248 days.

8. How many leap years from 1883 to 1893? From 1895 to 1905?

9. How many days from the beginning of 1896 to the end of 1905? *Ans.* 3652 days.

MEASURES OF VALUE.

250. Money is the measure of the value of things. It is of two kinds—coin and paper money.

251. Coin, or Specie, is metal prepared and authorized by government to be used as money.

252. Paper Money consists of printed promises to pay the bearer a certain amount duly authorized to be used as money.

253. Currency is whatever circulates as money. It is of two kinds—*specie currency* and *paper currency.*

254. Legal Tender is a term applied to money which is required by law to be accepted in payment of debts.

In coining gold and silver they are mixed with a baser metal called an *alloy*, for the purpose of rendering the coin harder and more durable. In coinage the alloy is considered as having no value.

UNITED STATES MONEY.

255. United States, or Federal Money, is the legal currency of the United States.

TABLE.

10 mills (m.)	= 1 cent, ct.	10 dimes	= 1 dollar, $.
10 cents	= 1 dime, d.	10 dollars	= 1 eagle, E.

The *unit* is the *dollar*. The currency is founded upon the decimal system, dimes, cents, and mills being written as decimals. This gives great simplicity to the operations.

ORAL EXERCISES.

1. How many cents in $\$\frac{1}{2}$? $\$\frac{1}{10}$? $\$\frac{3}{4}$? $\$\frac{3}{5}$? $\$\frac{4}{5}$? $\$\frac{3}{8}$? $\$\frac{5}{8}$?

2. What part of a dollar is 10 cts.? $12\frac{1}{2}$ cts.? 20 cts.? 25 cts.? $16\frac{2}{3}$ cts.? $33\frac{1}{3}$ cts.? $37\frac{1}{2}$ cts.? 50 cts.? $62\frac{1}{2}$ cts.? 75 cts.? $83\frac{1}{3}$ cts.?

3. What part of 4 eagles is 25 dimes? What part of 15 cents is $\frac{2}{3}$ of a dime?

4. How many eagles in 80 dollars? In 300 dimes? In 4560 cents? In 25,000 mills?

ENGLISH OR STERLING MONEY.

256. English, or Sterling Money, is the legal currency of England.

TABLE.

4 farthings (far. or qr.) .	=	1 penny	d.
12 pence	=	1 shilling	s.
20 shillings	=	1 pound or sovereign .	£.
21 shillings	=	1 guinea	G.

The *unit* is the *pound*, represented by the sovereign and £1 bank-note. Its value by late act of Congress is fixed at $4.8665.

ORAL EXERCISES.

How many

1. Farthings in 8 pence?

2. Pence in 7 shillings?

3. Shillings in 105 pence?

4. Shillings in 18 pounds?

5. Pounds in 320 shillings?

6. Pence in 1 guinea?

7. What part of 3 pence is 8 farthings? What part of 4 shillings is 7 pence?

8. What part of 18 pence is $\frac{3}{4}$ of a shilling? What part of a guinea is $\frac{3}{4}$ of a pound?

WRITTEN EXERCISES.

How many

1. Farthings in 7 s. 6 d. 2 far.?

2. Pence in £12 10 s. 10 d.?

3. Pence in £14 18 s. 11 d.?

4. Far. in £92 15 s. 8 d. 2 far.?

5. Shillings in 362 far.?

6. Pounds in 3010 d.?

7. Pounds in 3587 d.?

8. Pounds in 89074 far.?

9. Value in U. S. money of £25 6 s.? *Ans.* \$123.12 +.

10. In U. S. money of £16 18 s. 10 d.? *Ans.* \$82.45 —.

CANADA MONEY.

257. The **Currency of Canada** is nominally the same as that of the United States, the table and denominations being the same.

258. The decimal currency was adopted in 1858, the Act taking effect in 1859, previous to which the currency was the same as the English.

1. The coins are nominally equal to the corresponding coins of the United States money, but the intrinsic value is a little less. The *eagle* of the United States is the legal tender for sums of \$10 and upwards.

2. The coins consist of silver and copper. The *silver coins* are the *50-cent piece*, the *25-cent piece*, the *shilling* or *20-cent piece*, the *dime*, the *half-dime*. The *copper coin* is the *cent*. The gold coin used in Canada is the English sovereign, worth \$4.8665.

FRENCH AND GERMAN MONEY.

259. The **Unit** of French money is the *franc*, the value of which is 19.3 cents. The franc is divided into tenths and hundredths, called respectively *decimes* and *centimes*.

The *decime*, like our dime, is not used in business calculations, but is expressed by *centimes;* thus, instead of 5 decimes, we say 50 centimes.

260. The **Unit** of money in Germany is the *mark* (*Reichsmark*), worth 23.85 cents, and this is divided into 100 *pfennige*.

WRITTEN EXERCISES.

1. How many dollars in 125.25 francs? *Ans.* \$24.17+.

2. How many francs in \$145? *Ans.* 751.29+fr.

3. Dollars in 254 marks 25 pfennige? *Ans.* \$60.638+.

4. How many marks in \$754.50? *Ans.* 3163.52+m.

5. Bought an opera-glass for 20 francs; what did it cost in United States money? *Ans.* \$3.86.

6. Bought a lot of photographs in Munich for 200 marks; what are they worth in our money? *Ans.* \$47.70.

MISCELLANEOUS TABLES.

261. The following tables are frequently used, the first in counting certain kinds of articles, and the second in the paper trade:

COUNTING.	PAPER.
12 units = 1 dozen.	24 sheets = 1 quire.
12 dozen = 1 gross.	20 quires = 1 ream.
12 gross = 1 great-gross.	480 sheets = 1 ream.
20 units = 1 score.	

1. *Two* things of a kind are frequently called a *pair*, and *six* a *set*.
2. Paper is sold at retail by *sheets*, *quires*, and *reams*, and at wholesale by reams. Manufacturers and wholesale dealers usually sell paper by the pound.

BOOKS.

262. In printing books large sheets of paper are used, which are folded into leaves according to the size of the book.

263. The terms *folio*, *quarto*, *octavo*, etc., as applied to printed books, are based on sheets about 18 × 24 in., about half the sizes now generally used, and indicate the number of leaves into which such a sheet is folded.

A sheet folded in 2	leaves	is	called	a folio, makes 4 pages.
A sheet folded in 4	"	"	"	a quarto or 4to, makes 8 pages.
A sheet folded in 8	"	"	"	an octavo or 8vo, makes 16 pages.
A sheet folded in 12	"	"	"	a 12mo, makes 24 pages.
A sheet folded in 16	"	"	"	a 16mo, makes 32 "
A sheet folded in 18	"	"	"	an 18mo, makes 36 "
etc.				etc.

NOTE.—Printing-paper is made of many sizes, according to the requirements of the printer. In book-printing 24 × 38 inches, called *Double Medium*, is perhaps used most largely.

264. Clerks and copyists are often paid by the *folio* for making copies of legal papers, records, and documents.

72 words make 1 folio, or sheet of common law.
90 " " 1 " " " chancery.

ORAL EXERCISES.

1. How many pairs in 30? Scores in 80? Scores in 70? Sets in 42?

2. How many dozen in 2 gross? In $\frac{1}{2}$ a gross? In $\frac{1}{2}$ a great-gross?

3. How many sheets in 4 quires? In $2\frac{1}{2}$ quires? In $\frac{1}{2}$ a ream? In $\frac{1}{4}$ ream?

4. How many eggs in $4\frac{1}{2}$ dozen? In a quarter of a dozen? In a dozen and a half?

5. How many years in 4 score? In "3 score and 10"? In 4 score and a half?

6. How many sheets of paper will be required to make a 12mo book of 240 pages? Of 480 pages?

7. How many sheets will be required to make an octavo book of 160 pages? Of 480 pages?

8. How many octavo books will the paper for a quarto book make of the same number of pages?

WRITTEN EXERCISES.

1. How many fine black crayons are there in 54 boxes, each box containing 1 gross? *Ans.* 7776.

2. Sold 74 boxes of Stafford's writing fluid, each box containing 3 doz. bottles; how many gross? *Ans.* $18\frac{1}{2}$.

3. What would be the cost of 4800 sheets of foolscap at 25 cents a quire? *Ans.* \$50.

4. A lady copied in one month 594.5 chancery folios at 12¢ per folio; what did she receive? *Ans.* \$71.34.

5. Bought paper at \$7.50 per ream, and sold it at 25 cents a quire; what was the loss? *Ans.* \$2.50.

6. A printer used 5 reams, 9 quires, 16 sheets of paper for printing half-sheet posters; how many did he print, allowing 1 quire to a ream for waste? *Ans.* 5000.

7. How much paper would it require to print 3000 copies of a 12mo book of 504 pages, allowing 1 quire to each ream for waste?

REDUCTION OF COMPOUND NUMBERS.

265. Reduction is the process of changing a number from one denomination to another without changing its value.

266. There are **Two Cases**—*Reduction Descending* and *Reduction Ascending.*

These two cases have been considered in the examples under the tables, but we will present a few more problems under their proper heads.

REDUCTION DESCENDING.

267. Reduction Descending is the process of reducing a number to a lower denomination.

WRITTEN EXERCISES.

Reduce

1. 263 gal. 1 qt. to gills.	**7.** 2 T. 19 cwt. 99 lb. 3 oz. to oz.
2. 53 bu. 4 qt. 1 pt. to pints.	**8.** 4 mi. 89 rd. 4 yd. $2\frac{1}{2}$ ft. to in.
3. £11 8 s. 9 d. to pence.	**9.** 6 mi. 54 ch. 27 li. to li.
4. £68 13 s. 5 d. to farthings.	**10.** 2 cd. 118 cu. ft. 483 cu. in.
5. 15 lb. 14 pwt. 6 gr. to grains.	**11.** 36 sq. m. 242 A. 59 P. to P.
6. 13 ℔. 7℥ 6ʒ 2℈ 3 gr. to gr.	**12.** 3 mi. 244 rd. $1\frac{1}{2}$ yd. to feet.

13. If a hogshead of molasses contains 75 gallons, required the number of gills it contains. *Ans.* 2400 gi.

14. Reduce 47 cu. yd. 15 cu. ft. 1700 cu. in. to cubic inches. *Ans.* 2220452 cu. in.

15. Between the times of two observations the planet Venus was found to have passed over 5 S. 26° 50″; how many seconds in this distance? *Ans.* 633650″.

16. If my watch ticks 4 times a second, how many times does it tick in half a day? *Ans.* 172800.

17. If a person's pulse beats 75 times a minute, how often does it beat in a day? *Ans.* 108000 times.

18. What will it cost to fence a lot 36 rd. long by 24 rd. wide, at $12\frac{1}{2}$ cents a foot? *Ans.* $247.50.

19. If the distance from New York to Philadelphia is exactly 90 miles, required the number of inches. *Ans.* 5702400 in.

20. A ship driven out of her course by a storm changed her longitude 5° 24′ 18″; how many geographical miles was her longitude changed? *Ans.* $324\frac{3}{10}$ miles.

REDUCTION ASCENDING.

268. Reduction Ascending is the process of reducing a number to a higher denomination.

WRITTEN EXERCISES.

Reduce

1. 8424 gi. to gallons.

2. 3401 pt. to bushels.

3. 2745 d. to pounds.

4. 65924 far. to pounds.

5. 86742 gr. to lb. Troy.

6. 78643 gr. to ℔.

7. 95987 oz. to tons.

8. 271236 in. to miles.

9. 53427 li. to miles.

10. 646755 cu. in. to cords.

11. 3725179 P. to sq. mi.

12. 19870 ft. to miles.

13. How many common years are there in 67,434,486 seconds? *Ans.* 2 yr. 50 da. 11 h. 48 min. 6 sec.

14. A butter-dealer sold 5742 pounds of butter in a month; how many firkins (56 lb.) did he sell? *Ans.* 102 firk. 30 lb.

15. If the sun rises at 5 hr. 20 min. A. M., and sets 6 hr. 40 min. P. M., how long is the day? *Ans.* 13 hr. 20 min.

16. If a watch gains 2 min. 15 sec. a day, how much will it gain during July and August? *Ans.* 2 hr. $19\frac{1}{2}$ min.

17. A ship changed her longitude in a storm 864 geographical miles; what change had she made in her longitude? *Ans.* 14° 24′.

18. A miller ground a quantity of rye, and took 84 lb. of flour for toll, which was $\frac{1}{10}$ of the whole; how many bushels were there brought to mill if a bushel of grain produced 42 lb. of flour? *Ans.* 20 bushels.

ORAL EXERCISES.

1. What cost 15 oranges at 30 cents a dozen?

2. What cost 3 dozen pearl buttons at 25 cents a dozen?

3. What cost 6 tablespoons at the rate of \$11.52 a gross?

4. What is the price of shoe-tacks a dozen at \$2.88 a great-gross?

5. What cost 10 sheets of paper at 12 cents a quire?

6. What cost 15 quires of paper at \$4 a ream?

7. What cost ½ a gross of lead-pencils at 25 cents a dozen?

8. What will 3 reams of sand-paper cost at 18 cents a quire?

9. If a ream of emery-paper costs \$10, what is it worth a quire?

10. What will 6 gross of pens cost, if the wholesale rate is 2 pens for a cent?

11. A vessel was sunk in 12 fathoms of water; how many feet deep was it?

12. How many feet high is a horse which measures 15 hands high, one hand equaling 4 inches?

13. When apples sell at 20 cents a peck, how much are they worth a bushel?

14. How many vials, holding a gill each, can be filled with a gallon of eye-water?

15. How many half-pint bottles will be filled from 3 gallons of Stafford's writing fluid?

16. If I pay \$1.28 for a bushel of blackberries, how much do I pay per quart?

17. If it takes 6 buttons for 1 shirt, how many shirts will 2 gross of buttons trim?

18. The distance across a city park is 140 paces; how many rods is it, one pace being equal to 3 feet?

19. What will 25 pounds of flour cost, at the rate of \$2½ a hundredweight?

20. A boy bought a bushel of chestnuts for \$2.40, and sold them at 9 cents a quart; what did he gain?

21. I bought 50 lb. of flour for \$2.50; how much is that a hundredweight?

22. What part of 5 feet square is 5 square feet?

WRITTEN EXERCISES.

MISCELLANEOUS EXAMPLES.

1. What will be the cost of 3 cwt. 25 lb. of sugar at 5 cents a pound? *Ans.* $16.25.

2. How much rice, at 11¢ a pound, can be bought for $61,270? *Ans.* $278\frac{1}{2}$ tons.

3. How much must I pay for 7 tons of coal at $31\frac{1}{4}$¢ a hundred? *Ans.* $43.75.

4. How many rails 30 ft. long will be required to build 5 miles of railroad? *Ans.* 1760.

5. How many bushels of corn in a sack of corn weighing 2800 lb., reckoning 56 lb. to the bushel? *Ans.* 50 bu.

6. How many coffee-cups, holding $\frac{1}{2}$ pint each, can be filled from an urn holding 1 gal. 3 qt. 1 pt.? *Ans.* 30.

7. If there are 9 oz. of iron in the blood of a man, how many men would furnish iron enough to make an 18-pound ball? *Ans.* 32 men.

8. How many pills of 2 grains each can be made from 2 lb. 6℥ 3ʒ 2℈ of quinine? *Ans.* 7310.

9. How many times will a wagon-wheel 8 ft. 3 in. in circumference revolve in going 25 miles? *Ans.* 16,000.

10. Which is greater, and how much, 6 dozen dozen or a half a dozen dozen? *Ans.* 1st, by 792.

11. How many cannon-balls, each weighing 31 lb. 4 oz., can be made out of a ton of iron? *Ans.* 64.

12. At the rate of $3.50 a day of 10 hours, how much should be paid a man who works from quarter before 8 until 15 min. after 4? *Ans.* 2.97\frac{1}{2}$.

13. I heard the clap of thunder 8 seconds after seeing the flash; how far was the stroke away, sound moving 1092 feet a second? *Ans.* $1\frac{36}{55}$ miles.

14. If a lady takes an hour's nap every afternoon for 25 years, how much time will she lose if 6 of the years are leap years? *Ans.* 1 yr. $15\frac{11}{24}$ da.

15. How long will it take to hear a clap of thunder from a cloud 1 mi. 250 rds. 4 yd. distant, if sound travels 1092 ft. in a second? *Ans.* $8\frac{1}{2}$ + sec.

16. How long will it take to count $1,000,000 at the rate of $75 per minute, working 10 hr. a day? *Ans.* $22\frac{2}{9}$ da..

17. How many minutes longer was March, 1892, than February of the same year? *Ans.* 2880 min.

18. How many more seconds were there in 1892 than in a solar year? (See Art. 247.) *Ans.* 65470.3 sec.

19. How many half-pint bottles will be required to hold 5 gallons of writing fluid? *Ans.* 80.

20. If a druggist puts up in prescriptions an average of 1 ℔. 3℥ 1ʒ 2℈ 15 gr. of drugs daily, how much did he use during February, 1892? *Ans.* 36 lb. 9℥ 7ʒ 1℈ 15 gr.

21. How many ounces of calomel will it take to make 750 pills of 5 grains each? *Ans.* 7℥ 6ʒ 1℈ 10 gr.

22. How many minutes more in the summer of every common year than in the winter? *Ans.* 2880 min.

23. How many pages are there in a 12mo book which requires 18 fully-printed sheets? *Ans.* 432 pages.

24. Mr. Jones's income averages 5 cents a minute; what will it be during the three autumn months? *Ans.* $6552.

25. If a factory makes 43200 steel pens in a day, how many gross will be made in the month of February in a common year, omitting Sundays? *Ans.* 7200 gross.

26. If a comet passes through an arc of 9° 30′ a day, how long will it take to pass through an arc of 300°? *Ans.* $31\frac{11}{19}$ da.

27. If the fore wheel of a carriage is 4 ft. 9 in. in circumference, and the hind wheel 5 ft. 5 in., how many more turns

will the fore wheel make than the hind wheel in going 1 mile? *Ans.* $136\frac{200}{247}$.

28. If a grocer's pound-weight weighs only $15\frac{3}{4}$ oz., how much is his fraudulent gain on 3 bags of Java coffee of 112 lb. each, true weight, @ 27¢ a pound? *Ans.* $1.44.

29. If a daily newspaper of two sheets prints every day 7000 copies, how many reams of paper will be used every day, making no allowance for waste? *Ans.* 29 reams, 3 quires, 8 sheets.

30. A farmer sold 17 bu. 3 pk. 2 qt. of raspberries at 8¢ a quart, and agreed to take flour in payment at $4.50 a barrel as far as it would make an exact number of barrels, the remainder to be paid in cash; how many barrels and how much cash did he receive? *Ans.* 10 bbl.; 60¢.

ADDITION OF COMPOUND NUMBERS.

269. Addition of Compound Numbers is the process of finding the sum of two or more similar compound numbers.

1. Find the sum of 12 lb. 5 oz. 15 pwt. 22 gr., 10 lb. 6 oz. 17 pwt. 18 gr., 3 lb. 9 oz. 20 gr., and 18 lb. 1 oz. 12 pwt. 16 gr.

OPERATION.

lb.	oz.	pwt.	gr.
12	5	15	22
10	6	17	18
3	9	0	20
18	1	12	16
44	11	7	4

SOLUTION.—We write the numbers so that similar units shall stand in the same column, and begin at the right to add. 16 gr. plus 20 gr. plus 18 gr. plus 22 gr. are 76 gr., which, by reduction, we find equals 3 pwt. and 4 gr.; we write the 4 gr. in the grains column, and reserve the 3 pwt. to add to the column of pennyweights: 3 pwt. plus 12 pwt. plus 17 pwt. plus 15 pwt. are 47 pwt., which, by reduction, we find equals 2 oz. and 7 pwt.; we write the 7 pwt. in the column of pennyweights, and add the 2 oz. to the column of ounces: 2 oz. plus 1 oz. plus 9 oz. plus 6 oz. plus 5 oz. are 23 oz., which we find, by reduction, to be equal to 1 lb. 11 oz.; we write the 11 oz. in the column of ounces and reserve the 1 lb. to add to the column of pounds: 1 lb. plus 18 lb. plus 3 lb. plus 10 lb. plus 12 lb. equal 44 lb., which we write under the pounds.

1. Let the student derive a rule from the example solved.

2. In writing the numbers, if any places are wanting, supply them with a cipher.

WRITTEN EXERCISES.

(2)

lb.	oz.	pwt.	gr.
18	7	16	15
19	9	17	17
14	10	18	23
11	11	13	21

(3)

T.	cwt.	lb.	oz.
25	15	75	12
14	19	38	11
15	17	54	15
54	18	65	13

(4)

℔.	℥	ʒ	℈	gr.
17	10	7	1	15
18	9	6	2	12
22	7	4	0	16
45	3	0	0	19

(5)

gal.	qt.	pt.	gi.
56	3	1	2
77	2	0	1
87	1	1	3
95	2	1	2
120	0	0	0

(6)

bu.	pk.	qt.	pt.
57	3	7	1
98	1	5	0
24	2	2	0
37	0	1	1
61	1	4	1

(7)

cd.	cu. ft.	cu. in.
25	115	1246
18	104	1564
17	96	1372
65	75	396
49	120	1142

(8)

m.	rd.	yd.	ft.	in.
17	160	5	1	11
18	310	4	2	10
28	240	2	1	7
45	125	5	1	8
110	198	$1\frac{1}{2}$	2	0
		$\frac{1}{2}$=	1	6
110	198	2	0	6

(9)

A.	P.	sq. yd.	sq. ft.	sq. in.
52	125	30	4	112
18	104	28	6	72
34	45	16	8	68
47	117	18	5	108
153	74	$3\frac{1}{4}$	7	72
		$\frac{1}{4}$=	2	36
153	74	4	0	108

10. Find the sum of 15 mi. 300 rd. 4 yd. 2 ft. 3 in., 21 mi. 120 rd. 3 yd. 1 ft. 8 in., 18 mi. 220 rd. 1 yd. 2 ft. 6 in., 14 mi. 180 rd. 5 yd. 6 in. *Ans.* 70 mi. 182 rd. 4 yd. 11 in.

11. Find the sum of 9 sq. mi. 500 A. 50 P. 4 sq. yd. 5 sq. ft. 120 sq. in., 7 sq. mi. 120 A. 30 sq. yd. 7 sq. ft. 9 sq. in., 17 sq. mi. 240 A. 150 P. 20 sq. yd. 8 sq. ft. 100 sq. in., 24 sq. mi. 100 A. 64 P. 7 sq. ft. 112 sq. in.

Ans. 58 sq. m. 321 A. 105 P. 27 sq. yd. 17 sq. in.

SUBTRACTION OF COMPOUND NUMBERS.

270. **Subtraction of Compound Numbers** is the process of finding the difference between two similar compound numbers.

1. From 16 oz. 11 pwt. 15 gr. take 8 oz. 12 pwt. 14 gr.

OPERATION.

oz.	pwt.	gr.
16	11	15
8	12	14
7	19	1

SOLUTION.—We write the subtrahend under the minuend, placing similar units in the same column, and begin at the lowest denomination to subtract; 14 gr. subtracted from 15 gr. leaves 1 gr., which we write under the grains: 12 pwt. from 11 pwt. we cannot take; we will therefore take 1 oz. from the 16 oz., leaving 15 oz.; 1 oz. equals 20 pwt., which, added to 11 pwt., equals 31 pwt.; 12 pwt. subtracted from 31 pwt. equals 19 pwt., which we write under the pwt.; 8 oz. from 15 oz. (or, since it will give the same result, we may add 1 oz. to 8 oz., and say 9 oz. from 16 oz.) leaves 7 oz.

1. Let the student derive a rule from the example solved.
2. The pupil will notice that the general principle of addition and subtraction is the same as in simple numbers, the difference being in the irregularity of the scale, the units themselves being expressed in the decimal scale.

WRITTEN EXERCISES.

(2)

lb.	oz.	pwt.	gr.
122	10	14	20
84	9	15	23

(3)

T.	cwt.	lb.	oz.
326	15	74	12
145	17	36	15

(4)

lb.	℥	ʒ	℈	gr.
115	7	4	1	18
99	5	7	2	15

(5)

gal.	qt.	pt.	gi.
75	3	1	2
44	2	0	3

(6)

yr.	mo.	wk.	da.	h.
214	11	3	6	20
142	9	1	5	23

(7)

sq. yd.	sq. ft.	sq. in.
417	7	125
216	8	130

(8)

mi.	rd.	yd.	ft.	in.
95	280	0	3	6
46	175	2	1	9
49	104	$3\frac{1}{2}$	1	9
		$\frac{1}{2}=$	1	6
49	104	4	0	3

(9)

A.	P.	sq. yd.	sq. ft.	sq. in.
149	125	14	7	124
135	116	18	8	129
14	8	$25\frac{1}{4}$	7	139
		$\frac{1}{4}=$	2	36
14	8	26	1	31

10. Subtract 18 mi. 234 rd. 5 yd. 1 ft. 9 in. from 64 mi. 145 rd. 3 yd. 3 ft. 7 in. *Ans.* 45 mi. 230 rd. 4 yd. 4 in.

11. Mr. Brown had 500 bu. of corn, and sold 56 bu. 3 pk. 6 qt. 1 pt. to Mr. Black, and 59 bu. 2 pk. 7 qt. 1 pt. to Mr. White; how much had he remaining? *Ans.* 383 bu. 1 pk. 2 qt.

12. A Colorado miner, having 150 lb. of gold, sent his mother 18 lb. 9 oz. 15 pwt. 20 gr., and 4 lb. 18 pwt. less to his sister; how much did he retain? *Ans.* 116 lb. 5 oz. 6 pwt. 8 gr.

271. To find the time between different hours of the day.

WRITTEN EXERCISES.

1. How many hours from 10 o'clock Sunday morning to 9 o'clock Monday evening? *Ans.* 35 hours.

2. How many hours and minutes from 6.15 A. M. Monday to 5.30 P. M. Tuesday? *Ans.* 35 hr. 15 min.

3. How many hours and minutes from 3.30 A. M. Wednesday to 8.45 P. M. Friday? *Ans.* 65 hr. 15 min.

4. How long is it from 3.45 P. M. Sunday to 9.15 A. M. the following Monday? *Ans.* 17 hr. 30 min.

5. A man left home at 8.15 A. M. Monday, and was gone 34 hr. 25 min.; when did he return? *Ans.* 6.40 P. M. Tuesday.

6. What time is it from 9.45 P. M. Thursday to 10.20 A. M. the following Saturday? *Ans.* 36 hr. 35 min.

7. A man goes to work at 7.30 A. M. and leaves off at 6.30 P. M.; how many hours does he work, allowing an hour at noon? *Ans.* 10 hours.

272. To find the difference of time between two dates.

1. Find the time from April 20th to July 16th.

SOLUTION.—There remain 10 days in April after the 20th; in May there are 31 days, in June 30, and, adding in the 16 days in July, we find the time to be 87 days.

OPERATION.

April	10 days.
May	31 "
June	30 "
July	16 "
Ans.	87 days.

How many days from

2. Jan. 15th to July 10th?
3. Aug. 31st to Dec. 1st?
4. Sept. 10th to May 16th?
5. May 5th to Nov. 17th?
6. Oct. 21st to Mar. 15th?
7. Feb. 12th to June 16th?
8. July 9th to Dec. 15th?
9. Nov. 15th to Apr. 7th?

10. Lincoln was born Feb. 12th, 1809, and died April 14th, 1865; what was his age?

SOLUTION.—Dates are expressed in the *number* of the year, the month, and the day; hence the date of his birth is 1809 yr. 2 mo. 12 da., and the date of his death is 1865 yr. 4 mo. 14 da.; and the difference of these two dates will equal his age, which we find to be 56 yr. 2 mo. 2 da.

OPERATION.

yr.	mo.	da.
1865	4	14
1809	2	12
56	2	2

NOTE.—In this method we reckon 30 days to the month; when greater accuracy is required, we reckon the actual number of days in each month.

WRITTEN EXERCISES.

11. Washington was born Feb. 22d, 1732, and died Dec. 14th, 1799; what was his age? *Ans.* 67 yr. 9 mo. 22 da.

12. What is the exact difference in time from Dec. 25th, 1880, to Jan. 1st, 1890? *Ans.* 9 yr. 7 da.

13. What is the exact time a note has to run dated Nov. 30th, 1894, and payable Jan. 6th, 1896? *Ans.* 1 yr. 37 da.

14. Wm. Henry Harrison was born Feb. 9th, 1773, and died April 4th, 1841; what was his age? *Ans.* 68 yr. 1 mo. 25 da.

15. Thomas Jefferson was born April 2d, 1743, and died July 4th, 1826; what was his age? *Ans.* 83 yr. 3 mo. 2 da.

16. How old was Thomas Jefferson at the birth of Abraham Lincoln, Feb. 12th, 1809? *Ans.* 65 yr. 10 mo. 10 da.

17. The Revolution began April 19th, 1775, and ended Jan. 20th, 1783; how long did it last? *Ans.* 7 yr. 9 mo. 1 da.

18. A man works at a job from the morning of the 8th till the evening of the 13th; how much will he earn at $2.50 a day? *Ans.* $15.

19. The Civil War began April 11th, 1861, and closed April 9th, 1865; how long did it last? *Ans.* 3 yr. 11 mo. 28 da.

20. How many years, months, and days from your birthday to the present date?

MULTIPLICATION OF COMPOUND NUMBERS.

273. Multiplication of Compound Numbers is the process of finding the product when the multiplicand is a compound number.

1. Multiply 25 lb. 11 oz. 7 pwt. 16 gr. by 6.

OPERATION.

lb.	oz.	pwt.	gr.
25	11	7	16
			6
155	8	6	0

SOLUTION.—We write the multiplier under the lowest denomination of the multiplicand, and begin at the right to multiply. 6 times 16 gr. are 96 gr., which, by reduction, we find equals 4 pwt. and no gr.; we write 0 under the grains and reserve the 4 pwt. to add to the next product: 6 times 7 pwt. are 42 pwt., which, added to the 4 pwt., equals 46 pwt., which we find, by reduction, equals 2 oz. and 6 pwt.; we write the 6 pwt. under the pennyweights and reserve the 2 oz. to add to the next product; 6 times 11 oz. are 66 oz., which, added to the 2 oz., equals 68 oz., which we find, by reduction, equals 5 lb. and 8 oz.; we write the 8 oz. under ounces, and reserve the 5 lb. to add to the next product; 6 times 25 lb. equals 150 lb., plus the 5 lb. equals 155 lb., which we write under the pounds.

1. Let the student derive a rule from the example solved.
2. If the multiplier is a large composite number, it will be more convenient to multiply by its factors.

WRITTEN EXERCISES.

(2)

cwt.	lb.	oz.
16	98	12
		5

(3)

lb.	oz.	pwt.	gr.
26	9	17	18
			3

(4)

mo.	da.	h.	min.	sec.
250	20	21	30	50
				7

(5)

£	s.	d.	far.
15	14	10	3
			8

(6)

gal.	qt.	pt.	gi.
54	2	1	3
			9

(7)

℔.	℥	ʒ	℈	gr.
28	9	6	2	10
				11

Multiply

8. 15 L. 1 mi. 135 rd. by 12. *Ans.* 185 L. 2 mi. 20 rd.

9. 124 bu. 3 pk. 6 qt. by 13. *Ans.* 1624 bu. 6 qt.

10. 8 mi. 120 rd. 4 yd. by 16. *Ans.* 134 mi. 11 rd. $3\frac{1}{2}$ yd.

11. A farmer sold 10 loads of hay, each containing 1 T. 14 cwt. 50 lb.; how much did he sell? *Ans.* 17 T. 5 cwt.

12. If a boy walks 15 mi. 200 rd. in each of 21 days, how far will he walk in all? *Ans.* 328 mi. 40 rd.

13. If a farmer raises 80 bu. 2 pk. 7 qt. 1 pt. of grain on 1 acre, how much can he raise at the same rate on 48 acres? *Ans.* 3875 bu. 1 pk.

DIVISION OF COMPOUND NUMBERS.

274. Division of Compound Numbers is the process of finding the quotient when the dividend is a compound number.

275. There are two cases:

1st. To divide a compound number into equal parts.

2d. To divide one compound number by a similar one.

CASE I.

276. To divide a compound number into a number of equal parts.

1. Divide 97 lb. 16 pwt. 21 gr. by 5; that is, take $\frac{1}{5}$ of it.

OPERATION.

	lb.	oz.	pwt.	gr.
5)	97	0	16	21
	19	4	19	9

SOLUTION.—We write the divisor at the left of the dividend, and begin at the highest denomination to divide. $\frac{1}{5}$ of 97 lb. is 19 lb. and 2 lb. remaining; 2 lb. equal 24 oz.; $\frac{1}{5}$ of 24 oz. is 4 oz. and 4 oz. remaining; 4 oz. equal 80 pwt., which, added to 16 pwt., equals 96 pwt.; $\frac{1}{5}$ of 96 pwt. is 19 pwt. and 1 pwt. remaining; 1 pwt. equals 24 gr., which, added to 21 gr., equals 45 gr.; $\frac{1}{5}$ of 45 gr. is 9 gr.

1. Let the student derive a rule from the example solved.

2. When the divisor is a *large number* and *composite*, the factors being not greater than 12, it is perhaps more convenient to divide by the factors

WRITTEN EXERCISES.

(2)

	lb.	oz.	pwt.	gr.
4)	76	10	14	12
	19	2	13	15

(3)

	T.	cwt.	lb.
6)	112	16	66
	18	16	11

(4)

	℔	℥	ʒ	℈
7)	29	5	3	1

(5)

	S.	°	′
8)	125	24	12

(6)

	gal.	qt.	pt.	gi.
9)	342	2	1	2

(7)

	mi.	rd.	yd.	ft.
11)	120	313	3	2

8. Three sons share 115 A. 134 P. 15 sq. yd. of land; how much does each receive? *Ans.* 38 A. 98 P. 5 sq. yd.

9. If 10 farmers raise 2137 bu. 3 pk. 6 qt. 1 pt. of grain, what is the average? *Ans.* 213 bu. 3 pk. 1 qt. $\frac{9}{10}$ pt.

10. A miner sends 97 lb. 11 oz. 17 pwt. 19 gr. of gold to his 5 sisters; how much does each receive?

Ans. 19 lb. 7 oz. 3 pwt. 13$\frac{2}{5}$ gr.

CASE II.

277. To divide one compound number by a similar one.

1. Divide 49 lb. 5 oz. 15 pwt. by 9 lb. 10 oz. 15 pwt.

OPERATION.

$$\frac{49 \text{ lb. } 5 \text{ oz. } 15 \text{ pwt.} = 11875 \text{ pwt.}}{9 \text{ lb. } 10 \text{ oz. } 15 \text{ pwt.} = 2375 \text{ pwt.}}$$

$$\frac{11875 \text{ pwt.}}{2375 \text{ pwt.}} = 5. \quad \textit{Ans.}$$

SOLUTION.—49 lb. 5 oz. 15 pwt. equals 11875 pwt.; 9 lb. 10 oz. 15 pwt. equals 2375 pwt.; 11875 pwt. divided by 2375 pwt. equals 5.

1. Let the student derive the rule from the example solved.
2. The division may also be made without reducing to the lowest denomination, and this will be shorter when the quotient is integral.

WRITTEN EXERCISES.

2. Divide 48 T. 9 cwt. 23 lb. 8 oz. by 6 T. 1 cwt. 15 lb. 7 oz. *Ans.* 8.

3. Divide 105 cd. 111 cu. ft. 5$\frac{1}{3}$ cu. in. by 15 cd. 112 cu. ft. 1124 cu. in. *Ans.* 6$\frac{2}{3}$.

4. If a horse eats 1 pk. 5 qt. 1 pt. of oats a day, how long will 4 bu. 7 qt. last him? *Ans.* 10 days.

5. If a tun of wine contain 129 gal. 1 qt. 1 pt., how many demijohns of 2 gal. 3 qt. 1 pt. each can be filled from it? *Ans.* 45.

LATITUDE AND LONGITUDE.

278. The **Latitude** of a place is its distance from the equator, north or south. It is reckoned in degrees, minutes, and seconds, and cannot exceed 90°, or a quadrant.

279. The **Longitude** of a place is its distance, east or west, from a given meridian. It is reckoned in degrees, minutes, and seconds, and cannot exceed 180°, or a semicircumference.

Longitude is usually reckoned from the meridian passing through Greenwich, which is a little east of London. In this country longitude is sometimes reckoned from the meridian of Washington, which is 77° 2′ west from Greenwich.

NOTE.—In adding two longitudes, if their sum exceeds 180 degrees, it must be subtracted from 360 degrees for the correct difference of longitude.

ORAL EXERCISES.

1. From what is latitude reckoned? From what is longitude reckoned?

2. What is the greatest latitude a place may have? What is the greatest longitude a place may have?

3. What places have no latitude? What places have no longitude? What place has neither latitude nor longitude?

4. What is the use of latitude and longitude? Has every place a meridian of longitude?

5. What is the latitude of the equator? the latitude of the poles? the longitude of the poles?

280. From the above principles, to find the difference of latitude or longitude, we have the following

Rule.—*When the latitudes or longitudes are both of the same name, subtract the less from the greater; when they are of different names, take their sum.*

WRITTEN EXERCISES.

1. The latitude of Baltimore is 39° 17′ 23″ N.; that of St. Louis is 38° 37′ 28″ N.; what is the difference of latitude? *Ans.* 39′ 55″.

2. The longitude of St. Paul is 93° 5′ W., and of Mobile 87° 59′ W.; what is the difference of longitude? *Ans.* 5° 6′.

3. The latitude of the Cape of Good Hope is 34° 22′ S.; that of Cape Horn, 55° 58′ 40″ S.; what is the difference of latitude? *Ans.* 21° 36′ 40″.

4. The longitude of New Orleans is 90° 5′ W., and the longitude of Paris is 2° 20′ 22″ E.; what is the difference of longitude? *Ans.* 92° 25′ 22″.

5. The longitude of Portland, Me., is 70° 15′ 40″ W., and of Portland, Ore., 122° 27′ 30″ W.; what is the difference of longitude? *Ans.* 52° 11′ 50″.

6. What is the difference of longitude of Calcutta, 88° 23′ 30″ E., and of Portland, Oregon? *Ans.* 149° 9′.

7. The longitude of Vienna is 16° 23′ E., and of New York 74° 3″ W.; what is the difference of longitude?

Ans. 90° 23′ 3″.

LONGITUDE AND TIME.

281. The earth revolves on its axis from west to east once in 24 hours, and this causes the sun to appear to revolve around the earth from east to west in the same time.

Places on the east of a certain point have later time, those on the west earlier time, since the sun appears to those on the east first.

282. The circumference of a circle contains 360°; hence the sun appears to travel through 360° in 24 hours, and in 1 hour it travels $\frac{1}{24}$ of 360° = 15°; in 1 minute it travels $\frac{1}{60}$ of 15° = 15′; and in 1 second it travels $\frac{1}{60}$ of 15′ = 15″. Hence the following table:

TABLE OF LONGITUDE AND TIME.

15° of longitude	=	1 hour of time.
15′ of "	=	1 minute of time.
15″ of "	=	1 second of time.

ORAL EXERCISES.

1. In what time does the earth revolve on its axis? What part of a revolution does it make in 12 hours? in 8 hours?

2. In what direction does the earth turn on its axis? In what direction does the sun appear to move?

3. Does the sun appear first to places east or west of a given point?

4. When it is noon with us, is it earlier or later east of us? west of us?

5. How many degrees of the earth's surface pass directly under the sun's rays in 24 h.? in 12 h.? in 4 h.?

6. How many degrees of longitude make a difference of 1 hour in time? 2 hours? 3 hours? 4 hours?

7. When it is noon at Boston, what is the time 15° east of Boston? 15° west? 30° east? 30° west?

8. What difference in longitude makes a difference of 1 hour of time? of 1 minute? of 1 second?

9. What is the difference of longitude between two cities if the difference of time is 1 hour? 1 h. 30 min.? 2 h.? 2 h. 45 min.?

10. If I start at New York and travel until my watch is 1 h. 30 min. too fast, in what direction and how far do I go?

11. If I start at Chicago and travel until my watch is 2 h. 15 min. too slow, how far and in what direction do I travel?

CASE I.

283. To find the difference of time of two places when their difference of longitude is given.

1. The difference of longitude between two places is 42° 18′; what is their difference of time?

OPERATION.

$$15\overline{)42^\circ\ 18'\ 00''}$$
$$\quad 2 \quad 49 \quad 12$$

SOLUTION.—Since 15° of longitude correspond to 1 h. of time, 15′ of longitude to 1 min. of time, and 15″ of longitude to one second of time, $\frac{1}{15}$ of the number of *degrees*, *minutes*, and *seconds* will equal the number of *hours*, *minutes*, and *seconds* difference in time. Dividing by 15, we have 2 h. 49 min. 12 sec. Hence the following

Rule.—*Divide the difference of longitude expressed in ° ′ ″ by* 15; *the result will be the difference of time in* H., MIN., SEC.

WRITTEN EXERCISES.

2. The difference of longitude of two places is 65°; what is their difference of time? *Ans.* 4 h. 20 min.

3. The longitude of Washington is 77° 2′ 48″ W., and of San Francisco 122° 24′ 15″ W.; what is their difference of time? *Ans.* 3 h. 1 min. $25\frac{4}{5}$ sec.

4. The longitude of New York is 74° 0′ 3″ W., and of Rome 12° 27′ 14″ E.; what is the difference in time?

Ans. 5 h. 45 min. $49\frac{2}{15}$ sec.

5. The longitude of Chicago is 87° 37′ 30″ W., and of Philadelphia 75° 10′ W.; what is the time at Philadelphia when it is noon at Chicago? *Ans.* 12 h. 49 min. 50 sec.

6. Calcutta is 88° 23′ 30″ east of London; if the people in London hold a celebration at noon, at what hour must it be held in Calcutta to occur at the same time?

Ans. 5 h. 53 min. 34 sec. P. M.

CASE II.

284. To find the difference of longitude of two places when their difference of time is given.

1. The difference of time between two places is 35 min. 42 sec.; what is their difference of longitude?

SOLUTION.—Since 1 h. of time corresponds to 15° of longitude, and 1 min. of time to 15′ of longitude, and 1 sec. of time to 1″ of longitude, 15 times the number of *hours*, *minutes*, and *seconds* difference in time will equal the number of *degrees*, *minutes*, and *seconds* difference in longitude. Multiplying by 15, we have 8° 55′ 30″. Hence the following

OPERATION.

h.	min.	sec.
0	35	42
		15
8°	55′	30″

Rule.—*Multiply the difference of time expressed in* H., MIN., SEC. *by* 15; *the result will be the difference of longitude in* ° ′ ″.

WRITTEN EXERCISES.

2. The difference of time between Springfield, Mass., and Springfield, Ill., is 1 h. 7 min. 9 sec.; what is the difference in longitude? *Ans.* 16° 47′ 15″.

3. The time at Galveston is about 1 h. 11 min. $8\frac{4}{5}$ sec. earlier than the time at Washington; what is the difference in longitude? *Ans.* 17° 47′ 12″.

4. A vote taken in the British House of Commons at 1 o'clock A. M., May 5th, and telegraphed immediately to New York, is received at 8 h. 4 min. P. M., May 4th; what is the difference of longitude? *Ans.* About 74°.

5. In traveling from Boston to Chicago, I find on arriving that my watch is 1 h. 6 min. 16 sec. ahead of exact time at Chicago; what is the difference of longitude? *Ans.* 16° 34′.

STANDARD TIME.

285. Standard Time is time reckoned from certain fixed meridians instead of from the meridian of the place.

For the convenience of the business world the country has been divided into four great time-belts, each 15° wide, and the "local time" of the central meridian of each belt is made the "standard time" for the entire belt. Places within 7° 30′ on each side of the central meridian have the time of the meridian.

286. Standard time embraces four divisions—*Eastern Time, Central Time, Mountain Time,* and *Pacific Time.*

1. EASTERN TIME is the time of the 75th meridian.
2. CENTRAL TIME is the time of the 90th meridian.
3. MOUNTAIN TIME is that of the 105th meridian.
4. PACIFIC TIME is that of the 120th meridian.

To the east of the United States is also the time-belt of the 60th meridian, known as the *Intercolonial.*

287. Standard time, as is evident, coincides with local time in very few places, and therefore it is frequently necessary, in accurate calculations, to obtain the local time.

By finding the difference of longitude between any place and its standard meridian, local time can be reduced to standard time or standard time to local time.

NOTES.—1. *Eastern Time* coincides very nearly with the local time of Philadelphia, Pa., and Cape May, N. J.

2. *Central Time* coincides very nearly with the local time of St. Louis, Mo., Memphis, Tenn., and New Orleans, La.

3. *Mountain Time* is very nearly the same as the local time of Denver, Col., and Cheyenne, Wyoming.

4. *Pacific Time* is very nearly the same as the local time of Reno and Carson City, Nev., and Santa Barbara, Cal.

5. The limit of any belt is not a straight line, as it is convenient to select prominent cities, not on the same meridian, as the places for making a change of time.

WRITTEN EXERCISES.

1. The longitude of Cincinnati is about 84° 26′ W.; what is the difference between standard time and local time?

SOLUTION.—Cincinnati, whose longitude is within 7° 30′ of 90°, is in the Central time-belt; hence, its standard time is that of the meridian of 90°. The difference of longitude between Cincinnati and the 90th meridian is 5° 34′; reducing this, we find the difference of time to be 22 min. 16 sec.

OPERATION.

$$\begin{array}{r} 90^\circ\ 00' \\ \underline{84^\circ\ 26'} \\ 15)\underline{5^\circ\ 34'} \\ 22 \text{ min. } 16 \text{ sec.} \end{array}$$

Rule.—*Find the difference between the longitude of a place and that of its standard meridian, and find the time corresponding to this difference.*

To determine the standard meridian of a place, see Art. 286, noticing that each belt extends 7° 30′ on each side of the established meridian.

2. The longitude of St. Paul is 93° 5′ W.; what is the difference between the local and the standard time of St. Paul? *Ans.* 12 min. 20 sec.

3. The longitude of Boston is 71° 3′ 30″; when it is noon by standard time, what is the local time? *Ans.* 12 h. 15 min. 46 sec.

4. The longitude of Galveston is 94° 50′; when it is noon there by local time, what hour is it by standard time?

Ans. 12 h. 19 min. 20 sec.

5. The local time at Mobile is 6 h. 8 min. 4 sec. when it is 6 o'clock A. M. by standard time; what is the longitude of Mobile? *Ans.* 87° 59′.

6. The longitude of San Francisco is 122° 24′ 15″, and of Portsmouth, N. H., 70° 45′ 50″; when it is 4 h. 30 min. P. M. local time at San Francisco, what will be the local time at Portsmouth? *Ans.* 7 h. 56 min. $33\frac{2}{3}$ sec.

REDUCTION OF DENOMINATE FRACTIONS.

288. A **Denominate Fraction** is one in which the unit of the fraction is denominate; as, $\frac{3}{4}$ of a pound.

289. **Denominate Fractions** may be expressed either as *common* fractions or as *decimals.*

290. The following cases are presented to show pupils the most convenient methods of operation.

NOTE.—For answers not given, see numbers in problems of corresponding cases.

291. To reduce a common denominate fraction to a fraction of a lower denomination.

1. Reduce $\frac{1}{1440}$ of an oz. to the fraction of a grain.

OPERATION.

$$\frac{1}{1440} \times 20 \times 24 = \frac{1}{3} \text{ gr.}$$

SOLUTION.—Since there are 20 pennyweights in 1 ounce, 20 times the number of ounces equals the number of pennyweights; and since there are 24 grains in 1 pennyweight, 24 times the number of pennyweights equals the number of grains; hence, $\frac{1}{1440}$ of an ounce equals $\frac{1}{1440} \times 20 \times 24$ grains, which, by cancelling and multiplying, becomes $\frac{1}{3}$ of a grain.

WRITTEN EXERCISES.

Reduce	*Ans.*		*Ans.*
2. $\frac{5}{288}$ gal. to gills.	$\frac{5}{9}$ gi.	6. $\frac{1}{12000}$ ton to oz.	$2\frac{2}{3}$ oz.
3. $\frac{1}{640}$ da. to min.	$2\frac{1}{4}$ min.	7. $\frac{7}{18720}$ mi. to in.	$23\frac{9}{13}$ in.
4. $\frac{3}{160}$ bu. to pints.	$1\frac{1}{5}$ pt.	8. $\frac{5}{64152}$ sq. rd.	$3\frac{1}{18}$ sq. in.
5. £$\frac{1}{1280}$ to pence.	$\frac{3}{16}$ d.	9. $\frac{7}{216000}$ S. to sec.	$3\frac{1}{2}$ sec.

292. To reduce a common denominate fraction to a common fraction of a higher denomination.

1. Reduce $\frac{1}{2}$ of a grain to the fraction of an ounce.

OPERATION.

$$\frac{1}{2} \times \frac{1}{24} \times \frac{1}{20} = \frac{1}{960} \text{ oz.}$$

SOLUTION.—There are 24 grains in 1 pennyweight, therefore $\frac{1}{24}$ of the number of grains equals the number of pennyweights; there are 20 pennyweights in 1 ounce, therefore $\frac{1}{20}$ of the number of pennyweights equals the number of ounces; hence, 1 grain equals $\frac{1}{20} \times \frac{1}{24} \times \frac{1}{2} = \frac{1}{960}$ of an ounce.

WRITTEN EXERCISES.

Reduce	*Ans.*		*Ans.*
2. $\frac{8}{9}$ gi. to gal.	$\frac{1}{36}$ gal.	6. $\frac{15}{16}$ min. to da.	$\frac{1}{1536}$ da.
3. $\frac{11}{15}$ ft. to mile.	$\frac{1}{7200}$ mi.	7. $4\frac{2}{7}$ oz. to ton.	$\frac{3}{22400}$ T.
4. $\frac{6}{7}$ in. to rod.	$\frac{1}{231}$ rd.	8. $1\frac{1}{5}$ pt. to bu.	$\frac{3}{160}$ bu.
5. $\frac{5}{11}$ sec. to deg.	$\frac{1}{7920}$ deg.	9. $2\frac{1}{4}$ min. to da.	$\frac{1}{640}$ da.

10. What part of a cord is a pile of wood containing 72 cubic feet? *Ans.* $\frac{9}{16}$.

293. To reduce a common denominate fraction to integers of lower denomination.

1. What is the value of $\frac{5}{7}$ of a pound Troy?

OPERATION.

lb.	oz.	pwt.	gr.
7)5	0	0	0
	8	11	$10\frac{2}{7}$

SOLUTION.—$\frac{5}{7}$ of a pound equals $\frac{1}{7}$ of 5 lb.; and $\frac{1}{7}$ of 5 lb. we find, by dividing, is 8 oz. 11 pwt. $10\frac{2}{7}$ gr.

WRITTEN EXERCISES.

Find the value in integers

2. Of $\frac{5}{9}$ lb. Troy.
3. Of $\frac{3}{7}$ bushel.
4. Of $\frac{5}{9}$ day.
5. Of $\frac{27}{40}$ bushel.
6. Of $\frac{2}{3}$ mile.
7. Of $\frac{5}{9}$ acre.
8. Of $\frac{2}{3}$ cord.
9. Of $\frac{33}{40}$ rod.

10. What part of 5 A. is a field containing 25 P.? *Ans.* $\frac{1}{32}$.

294. To reduce a compound number to a common fraction of a higher denomination.

1. Reduce 3 cwt. 25 lb. to the fraction of a ton.

SOLUTION.—By reduction, we find 3 cwt. 25 lb. equal to 325 lb., and also 1 T. = 2000 lb.; 1 lb. is $\frac{1}{2000}$ of a ton, and 325 lb. equal 325 times $\frac{1}{2000} = \frac{325}{2000}$, which, reduced to its lowest terms, equals $\frac{13}{80}$.

OPERATION.

3 cwt. 25 lb. = 325 lb.
1 T. = 2000 lb.
$\frac{325}{2000} = \frac{13}{80}$ T. *Ans.*

WRITTEN EXERCISES.

Reduce the following to common fractions:

2. 6 oz. 13 pwt. 8 gr. to lb.

3. 1 pk. 5 qt. $1\frac{3}{7}$ pt. to bu.

4. 13 h. 20 min. to day.

5. 2 pk. 5 qt. $1\frac{1}{5}$ pt. to bu.

6. 213 rd. 1 yd. 2 ft. 6 in. to mi.

7. 88 P. 26 sq. yd. 8 sq. ft.

8. 85 cu. ft. 576 cu. in. to cd.

9. 4 yd. 1 ft. $7\frac{7}{20}$ in. to rod.

10. What part of 5 inches square are 5 square inches? What part of a $4\frac{1}{2}$-inch cube are $4\frac{1}{2}$ cubic inches? *Ans.* $\frac{1}{5}$; $\frac{4}{81}$.

295. To reduce a denominate decimal to integers of lower denomination.

1. Reduce .375 lb. Troy to integers of lower denominations.

SOLUTION.—There are 12 ounces in 1 pound, therefore 12 times the number of pounds equals the number of ounces; 12 times .375 equals 4 oz. and .5 oz.; there are 20 pwt. in 1 ounce, therefore 20 times the number of ounces equals the number of pwt.; 20 times .5 equals 10. Therefore, .375 lb. equals 4 oz. 10 pwt.

OPERATION.

$$\begin{array}{r} .375 \\ \underline{12} \\ 4.500 \\ \underline{20} \\ 10.000 \end{array}$$

WRITTEN EXERCISES.

Find the value in integers

2. Of .675 lb. Troy.

3. Of .825 rd.

4. Of .425 gal.

5. Of $.7343\frac{3}{4}$ bu.

6. Of .6725 bu.

7. Of $.28541\frac{2}{3}$ lb.

8. Of .3218 ton.

9. Of .92 da.

296. To reduce a compound number to a decimal of a higher denomination.

1. Reduce 3 oz. 8 pwt. 12 gr. to the decimal of an ounce.

SOLUTION.—There are 24 grains in 1 pennyweight, hence $\frac{1}{24}$ of the number of grains equals the number of pennyweights; $\frac{1}{24}$ of 12 equals .5, which, with 8 pennyweights, equals 8.5 pennyweights; there are 20 pwt. in an ounce, hence $\frac{1}{20}$ the number of pwt. equals the number of ounces; $\frac{1}{20}$ of 8.5 equals .425, which with 3 oz. equals 3.425 oz.

OPERATION.

$$\begin{array}{r|l} 24 & 12 \\ \hline 20 & 8.5 \\ \hline & 3.425 \end{array}$$

WRITTEN EXERCISES.

Reduce the following to decimals:

2. 8 oz. 2 pwt. to lb.

3. 4 yd. 1 ft. $7\frac{7}{20}$ in. to rod.

4. 1 qt. 1 pt. $1\frac{3}{5}$ gi. to gal.

5. 2 pk. 7 qt. 1 pt. to bu.

6. 2 pk. 5 qt. $1\frac{1}{25}$ pt. to bu.

7. 3 oz. 8 pwt. 12 gr. to lb.

8. 6 cwt. 43 lb. $9\frac{3}{5}$ oz. to ton.

9. 22 h. 4 min. 48 sec. to day.

10. What decimal part of an acre is 45 P. 19 sq. yd. 2 sq. ft. 36 sq. in.? *Ans.* $.2852\frac{8}{11}$.

ORAL EXERCISES.

MISCELLANEOUS EXAMPLES.

1. Eliza picked a peck of huckleberries, and sold them at 5 cents a pint; how much did she receive for them?

2. A sailor took the sounding, and found the water 13 fathoms deep; how many feet was it?

3. My grandfather's age is "three-score years and ten;" how many years old is he?

4. How many pages of a quarto volume can be printed on 5 quires of paper?

5. Mrs. Bell put up 70 quart-cans of blackberries; how many pecks did she put up?

6. A hostler at a hotel stable fed out 36 half-peck measures of oats; how many bushels did he feed?

7. When the sun has seemed to pass over $2\frac{1}{2}$ signs, how many seconds has it appeared to move?

8. How many seconds are there in the circumference of a bicycle wheel?

9. A merchant buys $\frac{1}{2}$ dozen hand-saws, at \$16 a dozen; how shall he sell them to gain 50 cents apiece?

10. How many leap years in every century? At what time did the 19th century begin?

11. How many cubic inches in a 3-inch cube? in a 5-inch cube? in a 6-inch cube?

12. A huckster-woman sold one day a bushel of pea-nuts at 5 cents a half-pint; how much did she receive?

13. A lady bought 3 pairs of vases, 2 sets of chairs, and a dozen knives and forks; how many individual articles did she buy?

14. What must I pay a gross for hair-pins, that I may sell them for 3 cents a dozen and gain 1 cent a dozen?

15. If Daniel Webster spoke 80 words in a minute, how many words would he speak in an address of one hour?

16. A merchant bought $\frac{1}{2}$ dozen rakes, at \$6.60 a dozen; how shall he retail them to gain 15 cents apiece?

17. A grocer paid \$6.40 for a bushel of cranberries; how shall he sell them a quart to gain 3 cents a pint?

18. Bought paper-collars at 30 cents a box, each box containing a dozen; how much will I gain on each collar by selling them for 4 cents apiece?

WRITTEN EXERCISES.

MISCELLANEOUS EXAMPLES.

1. What is the sum of $\frac{1}{5}$ yd., $\frac{1}{5}$ ft., and $\frac{1}{5}$ in.? *Ans.* $9\frac{4}{5}$ in.

2. Subtract 1 pwt. 15 gr. from $\frac{3}{4}$ lb. Troy.
Ans. 8 oz. 18 pwt. 9 gr.

3. Find the value of $6\frac{1}{2}$ d. $\times$ $8.33\frac{1}{3}$ minus 3 s. 6 d.
Ans. 1s. $\frac{1}{6}$ d.

4. Add $.00\frac{7}{8}$ sq. yd., $.04\frac{3}{4}$ sq. ft., and .0008 sq. in.
Ans. 18.1808 sq. in.

5. How many cubic feet in 75.125 cords? *Ans.* 9616 cu. ft.

6. A man bought 25 gal. 3 qt. of sherry at \$4.25 per gallon; what did it cost? *Ans.* $\$109.43\frac{3}{4}$.

7. If 9 bales of goods weigh 20 cwt. 75 lb., what will 144 bales of the same size weigh? *Ans.* 332 cwt.

8. A grocer shipped to Boston 8 barrels containing 8160 eggs; how many did he pack in a barrel? *Ans.* 85 doz.

9. How many steps of 30 inches each will a man take in walking 8 miles? *Ans.* 16896.

10. I bought 27 cwt. 96 lb. of sugar at $5.50 per cwt.; what did I pay for the sugar? *Ans.* $153.78.

11. A druggist bought 15 lb. $7\frac{1}{2}$ oz. of quinine at $1.50 an ounce; what did it cost? *Ans.* $371.25.

12. How many cubic feet of ice does it take to make a ton, if 1 cubic foot weighs 56 lb. 14 oz.? *Ans.* $35\frac{15}{91}$ cu. ft.

13. An occultation of Jupiter by the moon is seen by one observer at 10 P. M., and by another at 11.15 P. M.; required their difference of longitude. *Ans.* 18° 45′.

14. A silversmith has a bar of silver weighing 11 lb. 8 oz. 8 pwt.; how many dozen spoons, each weighing 2 oz. 12 pwt., can he make from it? *Ans.* $4\frac{1}{2}$ doz.

15. If 3.84 cwt. of sugar cost $15, what will 11 cwt. 30 lb. cost at the same rate? *Ans.* 44.14\frac{1}{16}$.

16. If a man walks 12 mi. 16 rd. in 3 h. 20 min., in what time will he walk 120 mi. 160 rd.? *Ans.* 33 h. 20 min.

17. How much gold can be obtained from a ton of quartz rock if it yields .0032 of its weight in gold? *Ans.* 6.4 lb. Av.

18. Since noon the sun has seemed to pass over 9° 32′ 24″; what time is it? *Ans.* 12 h. 38 min. $9\frac{3}{5}$ sec.

19. If a river-current carries a raft of lumber at the rate of 4 mi. 265 rd. per hour, how long will it be in carrying it a distance of 309 miles? *Ans.* 2 da. 16 h.

20. If I start at Philadelphia, latitude 39° 56′ 39″, and travel due north 1200 miles, what latitude do I reach?
Ans. 57° 17′ 42.85″.

21. In what time can a man travel 150 mi. 130 rd. if he goes 3 mi. 40 rd. in 1 hour? *Ans.* 48 h. 7 min. 48 sec.

22. If 6 lb. 6 oz. of opium cost \$62.80, what will 31 lb. 14 oz. cost at the same rate? *Ans.* \$314.

23. What is the weight of \$1,000,000 in gold dollars of 25.8 gr. each? *Ans.* 4479 lb. 2 oz.

24. What is the weight of \$1,000,000 in silver dollars of $412\frac{1}{2}$ gr. each? *Ans.* 71614 lb. 7 oz.

25. How many tons weight in \$1,000,000 in gold dollars, and also in silver dollars, if a pound avoirdupois equals 7000 grains Troy?

Ans. 1 T. 16 cwt. 85 lb. $11\frac{3}{7}$ oz.; 29 T. 9 cwt. 28 lb. $9\frac{1}{7}$ oz.

MISCELLANEOUS PROBLEMS.

1. How many gallons of ice-cream will it take to serve 84 persons, allowing a half-pint for each person? *Ans.* $5\frac{1}{4}$ gal.

2. If my watch gains 1 min. 10 sec. a day, how much will it gain during June and July? *Ans.* 1 h. 11 min. 10 sec.

3. If the rent of a house is \$65 a month, what will it amount to in 2 yr. 3 mo. 12 da.? *Ans.* \$1781.

4. What will a family, using $2\frac{1}{2}$ quarts a day, pay for milk during April at 20¢ a gallon? *Ans.* \$3.75.

5. How many spoons can be made from 8 lb. of silver if each spoon weighs 60 pwt.? *Ans.* 32 spoons.

6. If a boy bought $\frac{5}{8}$ of a bushel of walnuts for \$2, and sold them at 16¢ a quart, what was his gain? *Ans.* \$1.20.

7. If a man requires 2 hours 15 minutes to hoe a row of corn, how many rows would he hoe in 5 days of 10 hours each? *Ans.* $22\frac{2}{9}$ rows.

8. What will a dozen silver spoons cost at \$1.50 an ounce, if each spoon weighs 1 oz. 8 pwt. 20 gr.? *Ans.* \$25.95.

9. A 5-gallon can of oil lacks 3 pints of being full; what is the oil worth at 9¢ a gallon? *Ans.* $41\frac{5}{8}$¢.

10. A grocer buys 64 dozen eggs @ 15¢ a dozen, and sells them at the rate of 16 for 25¢; how much does he make on the sale? *Ans.* \$2.40.

11. A stationer bottled 120 gal. of ink in bottles holding $\frac{2}{3}$ of a pint, and sold it for 15¢ a bottle; how much did he receive? *Ans.* \$216.

12. If a man's wages are at the rate of \$2.50 for a day of 10 hours, what should he receive if he works from 7.45 A. M. to 10.15 P. M.? *Ans.* $\$3.62\frac{1}{2}$.

13. The sun sets at Philadelphia, Dec. 21, at 4 h. 37 min. P. M., and rises next morning at 7 h. 19 min. A. M.; how long is the night? *Ans.* 14 h. 42 min.

14. James has $9\frac{5}{8}$ bu. of chestnuts which he wishes to put into paper bags holding 1 pint each; how many bags can he fill? *Ans.* 616.

15. A grocer bought 8 bu. of potatoes at 75¢ a bushel, and sold them at 15¢ a half-peck; what was his gain? *Ans.* \$3.60.

16. Required the total cost of 5 dozen rockets at \$7.25 a gross, 6 dozen Roman candles at \$9.20 a gross, and 8 dozen pin-wheels at \$1.25 a gross. *Ans.* $\$8.45\frac{5}{12}$.

17. From a piece of cloth measuring $31\frac{1}{4}$ yards there have been sold $3\frac{1}{2}$ yd., $7\frac{3}{8}$ yd., $12\frac{7}{8}$ yd.; what is the value of the whole piece, if the remainder is worth \$18.75? *Ans.* $\$78.12\frac{1}{2}$.

18. If a button manufactory makes 75 dozen buttons a day, how many great-gross of these buttons will it make in 26 weeks? *Ans.* $81\frac{1}{4}$ gr.-gross.

19. A druggist buys potassium chlorate at 50¢ a lb. Av., and retails it in powders of 25 gr. at 5¢ each; what is his profit on 4 lb.? *Ans.* \$54.

20. What is the difference in weight between 1 dozen silver tablespoons, weighing 1 lb. 11 oz. 2 pwt. 16 gr., and 3 dozen silver teaspoons, weighing 7 oz. 5 pwt. 16 gr. a dozen?

Ans. 1 oz. 5 pwt. 16 gr.

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21. A cistern holding 30 hhd. (63 gal.) is full of water; if two pipes are opened, by one of which 4 gal. run in per minute, and by the other $8\frac{1}{2}$ gal. run out per minute, how long will it take to empty the cistern? *Ans.* 7 hours.

22. I have a clock which gains $1\frac{1}{2}$ sec. in 24 hours, and another which loses 2 sec. in the same time; if both are set right Saturday evening at 6 o'clock, how far apart will they be by the next Wednesday noon? *Ans.* $13\frac{1}{8}$ sec.

23. A milkman paid $4.80 for 15 2-gal. cans of milk, but had the misfortune to spill 5 gallons; for what must he sell the remainder per quart to gain 4 cents a gallon on what he bought? *Ans.* 6¢.

24. A boarding-school uses $3\frac{1}{2}$ pecks of potatoes daily; how much was expended for potatoes during January, February, and March, 1892, if they cost 75¢ a bushel? *Ans.* $$59.71\frac{7}{8}$.

25. A merchant received from California a lot of pears amounting to 36 bu. 2 pk. 5 qt., but only $\frac{7}{9}$ of the fruit was in good condition; how many bushels had to be thrown away? *Ans.* 8 bu. $4\frac{2}{9}$ qt.

26. A man breathes on an average 17 times a minute, and takes in about $\frac{6}{7}$ of a quart of air at each breath; how many gallons of air will he breathe in 12 hours? *Ans.* $2185\frac{5}{7}$.

27. If it costs $1.97 to make a barrel of flour into bread, and flour is worth $4.75 a barrel (196 lb.), what should be the price of a loaf of bread containing $1\frac{3}{4}$ lb. of flour? *Ans.* 6¢.

28. The weight of a cubic foot of water is 1000 ounces, what is the weight of a cubic foot of gold, if the weight of gold is 19.4 times that of water? *Ans.* 1212 lb. 8 oz. Av.

29. A man started from New York, long. 74° 3″, at 10 h. 30 min. P.M., Feb. 27, 1892, local time, and arrived at San Francisco, long. 122° 24′ 15″, 10 h. 40 min. A. M., Mar. 5, local time; how long was he on his journey? *Ans.* 6 da. 15 h. 23 min. $36\frac{4}{5}$ sec.

SECTION VII.

PRACTICAL MEASUREMENTS.

297. The **Application of Measures** to the farm, the household, the mechanic arts., etc. is so extensive that we now present a section called Practical Measurements.

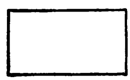
MEASURES OF SURFACE.

298. A **Surface** is that which has length and breadth without thickness.

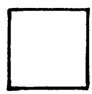
299. A **Plane Figure** is a portion of a plane surface bounded by straight or curved lines.

THE RECTANGLE.

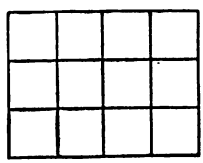
300. A **Rectangle** is a plane figure having four straight sides and four right angles. A slate, a door, the sides of a room, etc. are examples of rectangles.



301. A **Rectangle** has two *dimensions*, length and breadth. A *Square* is a rectangle in which the sides are are all equal.



302. The **Area** of a rectangle is the surface included within its sides. It is expressed by the number of times it contains a small square as a *unit of measure*.



Principle.—*The area of a square or rectangle is equal to the product of its length by its breadth.*

For in the rectangle above the whole number of little squares is equal to the number in each row multiplied by the number of rows, which is equal to the number of linear units in the length multiplied by the number in the breadth.

Hence, to find either side of a square or rectangle, divide the area by the other side.

1. The sides multiplied must be of the *same denomination*, and the product will be *square units* of that denomination, which may be reduced, if necessary, to higher denominations.

2. In dividing, the *linear unit* of the side must be of the same name as the *square unit* of the area, and the quotient will be linear units of the same denomination.

WRITTEN EXERCISES.

1. How many square yards in the surface of a blackboard 36 ft. long by 5 ft. wide?

SOLUTION.—To find the area, we multiply the length by the breadth; $36 \times 5 = 180$; hence, the area is 180 sq. ft.; reducing this to square yards, we have 20 sq. yd.

2. How many square yards in the floor of a room 32 ft. long by 24 ft. wide? *Ans.* $85\frac{1}{3}$ sq. yd.

3. A man has a garden 216 ft. long by 108 ft. wide; how many square yards does it contain? *Ans.* 2592 sq. yd.

4. Required the width of a room 28 ft. long, whose floor contains 700 square feet. *Ans.* 25 ft.

5. How many square feet in the walls and ceiling of a room 24 ft. long, 14 ft. wide, and 10 ft. high? *Ans.* 1096 sq. ft.

6. How many sq. ft. in the surface of a box 3 ft. 9 in. long, 2 ft. 6 in. wide, and 1 ft. 9 in. high? *Ans.* $40\frac{5}{8}$ sq. ft.

7. What is the surface of a cubical box each of whose dimensions is 2 ft. 3 in.? *Ans.* $30\frac{3}{8}$ sq. ft.

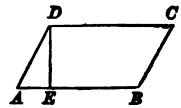
8. Measure a crayon box, and calculate the number of square inches on the bottom and four sides.

9. The floor of a parlor contains $60\frac{1}{2}$ sq. yd.; one side is 13 ft. 6 in. long; what is the other side? *Ans.* 40 ft. 4 in.

SUGGESTION.—In Practical Measurements, whenever possible, *express the operations* and *cancel common factors.* Thus, in Ex. 9 express $60\frac{1}{2} \times 9 \div 13\frac{1}{2}$ in a complex fraction and cancel.

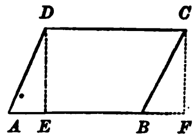
THE PARALLELOGRAM.

303. A **Parallelogram** is a plane figure having four sides, its opposite sides being parallel. The *altitude* is the perpendicular distance between its two opposite sides.



Thus, *ABCD* is a parallelogram; *AB* is the base and *DE* the altitude.

304. If we extend the side *AB* to *F*, and draw *DE* and *CF* perpendicular to *AF*, we have the rectangle *EFCD*.



Principle.—*The area of a parallelogram is equal to its base multiplied by its altitude.*

For it is readily shown that the parallelogram *ABCD* is equal to the rectangle *EFCD*, and the base *AB* equals the base *EF*, the altitude of each being *DE*.

WRITTEN EXERCISES.

1. What is the area of a parallelogram whose base is 25 inches and altitude 18 inches?

Solution.—To find the area, we multiply the base by the altitude; $25 \times 18 = 450$, hence the area is 450 square inches.

2. How many square yards in a parallelogram whose base is 60 yards and altitude 35 yards? *Ans.* 2100 sq. yd.

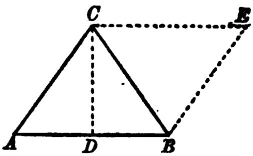
3. A lot is in the form of a parallelogram, the base being 125 ft. and the altitude 25 ft.; what is the area? *Ans.* 3125 sq. ft.

4. If the area of a parallelogram is 675 sq. ft. and its altitude 25 ft., what is its base? *Ans.* 27 ft.

5. A lady has a flower-bed in the form of a parallelogram; if the area is 25 sq. ft. and the base 10 ft., what is the altitude? *Ans.* $2\frac{1}{2}$ ft.

6. The area of a field in the form of a parallelogram is 4840 sq. yd., and each of the two longer sides is 264 feet; what is the altitude? *Ans.* 165 ft.

THE TRIANGLE.



305. A **Triangle** is a plane figure having three sides and three angles; as, *ABC*.

306. The **Base** is the side upon which it seems to stand, as *AB*. The **Altitude** is a line perpendicular to the base drawn from the angle opposite; as, *CD*.

Principle.—*The area of a triangle is equal to the product of the base by one-half of the altitude.*

For it may readily be shown that the triangle *ABC* is $\frac{1}{2}$ the parallelogram *ABEC*. But the area of the parallelogram is equal to the base multiplied by the altitude; hence, the area of the triangle is equal to $\frac{1}{2}$ the product of the base by the altitude.

To find the base or altitude of a triangle, divide the area by one-half the other dimension.

WRITTEN EXERCISES.

1. What is the area of a triangle whose base is 35 inches and altitude 20 inches?

Solution.—To find the area, we multiply the base by one-half the altitude; $35 \times 10 = 350$; hence the area is 350 sq. in.

2. How many square feet in a triangle whose base is 21 ft. 3 in. and altitude 11 ft. 6 in.? *Ans.* 122 sq. ft. 27 sq. in.

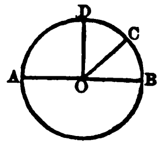
3. The area of a triangular bed of flowers is 29 sq. ft., and its base is 12 ft.; what is the altitude? *Ans.* 4 ft. 10 in.

4. The area of a triangular lot is 425 square yards, and its base is 153 ft.; what is its altitude? *Ans.* 50 ft.

5. What is the area of the gable end of a house 42 ft. wide, the ridge being 16 ft. higher than the top of the wall? *Ans.* 336 sq. ft.

6. The area of the gable of a house is 252 sq. ft., the base being 12 yards; what is the height of the ridge? *Ans.* 14 ft.

THE CIRCLE.



307. A **Circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within, called the *centre*.

308. The **Circumference** of a circle is the bounding line; any part of the circumference, as BC, is an *Arc*. An arc of one-fourth of the circumference is called a *Quadrant*.

309. The **Diameter** is a line passing through the centre and terminating in the circumference; as, AB. The *Radius* is a line drawn from the centre to the circumference; as, OB.

Principles.—I. *The circumference of a circle is equal to the product of the diameter by* 3.1416.

II. *The area of a circle is equal to the product of the circumference by one-fourth of the diameter.*

1. To find the diameter of a circle, divide the circumference by 3.1416, or multiply it by .3183.

2. The area of a circle is also equal to the *square of the radius multiplied by* 3.1416.

WRITTEN EXERCISES.

1. The diameter of a circle is 18 feet; what is its circumference?

Solution.—To find the circumference, we multiply the diameter by 3.1416; 3.1416×18 equals 56.5488; hence the circumference equals 56.5488 ft.

2. The distance across a circular reservoir is 50 feet; what is the distance around it? *Ans.* 157.08 ft.

3. I have a circular grass-plat 40 feet in diameter; what is the area of the plat? *Ans.* 1256.64 sq. ft.

4. The distance round a circular reservoir is 625 feet; what is the distance across? *Ans.* 198.94— ft.

5. A horse is fastened to a stake by a rope 18 feet long; what space can he graze over? *Ans.* 113.0976 sq. yd.

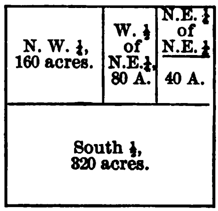
6. It takes 300 steps, $2\frac{1}{2}$ ft. each, to walk around a circular pond; what is the diameter of the pond? *Ans.* 238.725 ft.

7. If the equatorial diameter of the earth is 7925.75 mi., what is its circumference at the equator? *Ans.* 24899.536 + mi.

MEASUREMENT OF LAND.

310. The **Unit of Measure** of land is the *Acre*, which is sometimes divided into *square rods* and sometimes into *square chains*. Hundredths of an acre are also frequently used.

Government lands are divided by parallels and meridians into *townships*, which contain 36 square miles or *sections*, and each section is subdivided into *quarter-sections*. Hence, 640 acres make a *section*, and 160 acres a *quarter-section*. The quarter-sections are still further subdivided into *half-quarter-sections*, *quarter-quarter-sections*, and *lots*. Lots are often of irregular form on account of natural boundaries, but contain, as near as may be, a quarter-quarter-section.



1 section = 640 acres.

1. The pupil will remember that *rods* multiplied by *rods* give *square rods*, *chains* by *chains* give *square chains;* also, that 1 acre = 10 square chains or 160 square rods.

2. Each *section* is a square one mile on a side; a *quarter-section* is half a mile on the side; and a *quarter-quarter-section* is $\frac{1}{4}$ of a mile on a side.

WRITTEN EXERCISES.

1. How many acres in a farm, rectangular in shape, 625 rods long and 375 rods wide?

Solution.—The area equals 625 × 375, or 234375 sq. rd.; reducing to acres, we have 1464 A. 135 P.

2. I have a rectangular field 18.5 ch. long and 8.75 ch. wide; how much land does it contain? *Ans.* 16 A. 30 P.

3. What is the area of a rectangular reservoir 153 rd. 2 yd. long and 114 rd. $2\frac{1}{2}$ yd. wide? *Ans.* 109 A. 113 P. 5 sq. yd.

4. Mr. James bought a farm 248.7 rods long and 132.5 rods wide, at $\$75\frac{1}{2}$ an acre; what did he pay for the farm? *Ans.* \$15549.58 —.

5. Mr. Ashton bought 85 A. 110 P. of Minnesota prairie-land for $4.50 per acre, and sold it for $6.75 per acre; what was his gain? *Ans.* $192.80 —.

6. If a Western township is equally divided among 320 families, what part of a section does each receive, and how many acres? *Ans.* $\frac{9}{80}$; 72 acres.

7. Mr. Pyle owns a farm in the form of a rectangle, whose area is 193 A. 2 ch., and the length is 84 ch.; how many rods of fence will enclose it? *Ans.* 856 rods.

8. How many rails are required to fence a quarter-quarter-section, the fence being 5 rails high, and each rail 8 ft. long, and what will be the cost at $35 per thousand rails?

Ans. 3300 rails; $115.50.

PLASTERING, PAINTING, AND KALSOMINING.

311. Plastering, Painting, and Kalsomining are usually computed by the square yard.

Allowance is sometimes made for openings in the walls, as doors and windows, but custom varies so much in different localities that no general rule can be given. Half the area of the doors and windows is often deducted.

WRITTEN EXERCISES.

1. What will it cost to plaster the walls and ceiling of a room 24 ft. long, 15 ft. wide, and 10 ft. high, there being two doors, $6\frac{1}{2} \times 3$ ft., and 4 windows, 6×3 ft., at $.37 a sq. yd., deducting half the area of doors and windows?

SOLUTION.

Area of side walls $= 2 \times 24 \times 10 = 480$
Area of end walls $= 2 \times 15 \times 10 = 300$
Area of ceiling $= 24 \times 15 = \underline{360}$
Whole area $= 1140$
Area of doors $= 2 \times 6\frac{1}{2} \times 3 = 39$
Area of windows $= 4 \times 6 \times 3 = \underline{72}$
Area to be deducted $= \frac{1}{2} \times 111 = \underline{55.5}$
Area to be plastered $= 1084.5$ sq. ft.
$1084.5 \div 9 = 120.5$ sq. yd.
Cost $= 120.5 \times \$.37 = \$44.58\frac{1}{2}$.

2. What will it cost to kalsomine a room 30 ft. long, 16 ft. wide, and 12 ft. high, at 5¢ a sq. yd.? *Ans.* \$8.80.

3. What will be the cost of kalsomining a hall 32 ft. long, 7 ft. wide, and 10 ft. high with 2 coats at 8¢ a sq. yd. each? *Ans.* \$17.84$\frac{8}{9}$.

4. What will be the cost of painting the outside of a frame house 50 ft. long, 32 ft. wide, and 25 ft. high, with 2 coats, at 8$\frac{1}{2}$¢ a square yard each? *Ans.* \$77.44$\frac{4}{9}$.

5. What will it cost to plaster a school-room 40 ft. long, 20 ft. wide, and 11 ft. high, having 2 doors, 7 ft. by 3 ft., and 8 windows, 5 ft. by 3 ft., at \$.36 a square yard, allowing half for openings? *Ans.* \$81.56.

6. What will be the cost of painting the walls and ceiling of the room in Example 5 with 3 coats at 7¢ a sq. yd. each, making no allowance for openings? *Ans.* \$49.46$\frac{2}{3}$

ROOFING, FLOORING, PAVING, ETC.

312. Paving and **ceiling** are estimated by the square foot or square yard.

313. Roofing, flooring, partitioning, slating, etc. are generally reckoned by the *square,* which consists of 100 *square feet,* but sometimes by the square foot or yard.

314. Shingles, which commonly measure 18 in. by 4 in., are estimated by the *thousand* or *bundle.* 1000 shingles are generally allowed to a *square* of 100 sq. ft.

WRITTEN EXERCISES.

1. What will be the expense of paving a sidewalk 303 ft. long and 7$\frac{1}{2}$ ft. wide, at \$2.25 per square yard?

SOLUTION.—The area equals $303 \times 7\frac{1}{2}$, or 2272$\frac{1}{2}$ sq. ft., which equals 252$\frac{1}{2}$ sq. yd.; hence, the cost is \$$2.25 \times 252\frac{1}{2}$, or \$568.12$\frac{1}{2}$.

2. What is the cost of slating a roof 52 ft. 10 in. long, each side being 20 ft. from eaves to ridge, at \$15.25 per square? *Ans.* \$322.28$\frac{1}{3}$.

3. Required the cost of the shingles to shingle a roof 60 ft. long and 30 ft. from eaves to ridge on both sides, at \$6½ a thousand. *Ans.* \$234.

4. What is the cost of wainscoting a room 28 ft. long by 15 ft. 4 in. wide, to a height of 4 ft. 3 in., at \$0.45 per square yard? *Ans.* \$18.41⅔.

5. What will it cost to shingle a roof 64 ft. long and 32 ft. from eaves to ridge, the shingles costing \$5¾ a thousand, and the shingling costing \$2½ a square? *Ans.* \$337.92.

6. Which would cost most, to lay a brick sidewalk 6½ ft. wide and 540 ft. long, at \$1.17 a sq. yd., or to lay a stone sidewalk of the same dimensions, at 20¢ a sq. ft.? *Ans.* The stone, \$245.70.

7. A cistern 7 ft. long, 6 ft. wide, and 4 ft. deep is to be lined with zinc costing 12¢ a pound, allowing 5 lb. to the square foot; what will be the expense? *Ans.* \$87.60.

CARPETING.

315. In **Carpeting** we take into consideration the width of the carpet, the allowance for matching the figures, and whether the strips run lengthwise or crosswise.

1. Carpets are usually either 1 yd. or ¾ yd. wide; but matting, oil-cloth, and other materials used for covering floors are of various widths.

2. To match the figures we must often turn under or cut off one of the ends. When an exact number of strips is a little too wide for the room, a part of one breadth is turned under.

Rule.—*Find the number of strips required, and multiply the number of yards in each strip by the number of strips.*

WRITTEN EXERCISES.

1. How many yards of ingrain carpet, 1 yd. wide, running lengthwise, will be required to carpet a floor 16 ft. 6 in. by 14 ft. 6 in.?

SOLUTION.—The room is nearly 5 yd. wide, hence it will require 5 strips, or breadths; and since each strip is 16½ ft. long, it will take 16½ ft. × 5 = 82½ ft., or 27½ yds.

2. What will it cost to carpet a room 24 ft. by 18 ft. with carpet 1 yd. wide, at $1.50 a yard? *Ans.* $72.

3. How many yards of carpet, $\frac{3}{4}$ yd. wide, will it require to carpet, lengthwise, a room 21 ft. by 15 ft.? *Ans.* 49 yd.

4. How many yards of carpet, $\frac{7}{8}$ yd. wide, will it take to carpet, crosswise, a hall 42 ft. by 30 ft.? *Ans.* 160 yd.

5. How many yards of matting $3\frac{1}{2}$ ft. wide will cover a floor 31 ft. by 21 ft., lengthwise? crosswise? *Ans.* 62 yd.; 63 yd.

6. How many yards of carpet, $\frac{3}{4}$ yd. wide, will carpet lengthwise a parlor 35 ft. by 18 ft., the matching of figures requiring 6 in. waste in each strip? *Ans.* $94\frac{2}{3}$ yd.

7. A lady wishes to carpet (lengthwise) a room 14 ft. 6 in. long by 11 ft. wide, with Brussels carpet $\frac{3}{4}$ of a yard wide, at $1.25 a yard; what will it cost, allowing 15 in. waste in each strip for matching? *Ans.* $32.81\frac{1}{4}$.

PAPERING.

316. Wall-paper is sold only by the roll, and in estimates a part of a roll is reckoned as a whole roll.

317. A roll of American paper is commonly 8 yd. long and $\frac{1}{2}$ yd. wide.

1. Paper is now usually put up in double rolls 16 yd. long.
2. Borders and friezes are sold by the yard, and vary in width from 3 in. to 18 in.
3. On account of waste, the cost of papering a room can only be approximately estimated.

Rule.—I. *Find the entire distance around the room in yards, and multiply this by 2, to find the number of half-yards or strips.*

II. *Divide the number of strips required for the room by the number of strips that can be cut from a roll; the quotient will be the number of rolls required.*

Since there are 24 feet in a roll, if the length of the strips is 8 feet or less, 3 strips can be cut from a roll; if between 8 ft. and 12 ft., 2 strips, etc.

WRITTEN EXERCISES.

1. How many rolls of paper will cover the walls of a room 30 ft. long, $22\frac{1}{2}$ ft. wide, and 10 ft. 8 in. high?

SOLUTION.—The distance around the room is $2\times(30+22\frac{1}{2})=105$ ft. $=35$ yd. $=70$ half-yards. Since the height is $10\frac{2}{3}$ ft., we can cut only 2 full strips from a roll of 24 ft. Hence, the number of rolls will equal $70\div2=35$ rolls.

OPERATION.

$$\begin{aligned} 2\times(30+22\tfrac{1}{2}) &= 105 \text{ ft.} \\ &= 35 \text{ yd.} \\ &= 70 \text{ half-yd.} \\ 70\div2 &= 35 \text{ rolls.} \end{aligned}$$

2. What will be the cost of the paper for papering the above room at 50¢ a roll, including also a gilt moulding around the top of the walls at 6 cents a foot? *Ans.* \$23.80.

3. What will the paper cost, at 35¢ a roll, to paper the walls of a room 24 ft. long, 15 ft. wide, and 10 ft. high, deducting 3 rolls for doors and windows? *Ans.* \$8.05.

NOTE.—The distance around the room $=2\times(24+15)\div3=26$ yd. $=52$ half-yd. = No. of strips. The No. of rolls $=52\div2=26$, etc.

4. What will be the cost of the paper required to paper the walls of a room 25 ft. long, 20 ft. wide, and 8 ft. high, at 70¢ a double roll, making a deduction of 2 double rolls for doors and windows? *Ans.* \$5.60.

NOTE.—A double roll will make 6 full strips. Distance around room = 60 half-yd.; No. of double rolls $=60\div6-2=8$.

5. Required the cost of papering the walls of a parlor 36 ft. long, 18 ft. wide, 10 ft. high, with base-board 1 ft. wide, at \$2.50 a double roll, having also a border 18 in. wide, at 45¢ per yard, the price including the cost of putting on the paper and border. *Ans.* \$46.20.

NOTE.—No. of strips = 72. Deducting base-board and border, length of strip = $7\frac{1}{2}$ ft.; hence, 6 full strips can be cut from a double roll.

6. Required the cost of papering 3 rooms 28 ft. by 14 ft., 10 ft. high, having a dado 3 ft. wide, paper costing \$3 a double roll, border 25 cts. a yard, labor $\$5\frac{1}{4}$, allowing 3 rolls for doors and windows. *Ans.* \$101.25.

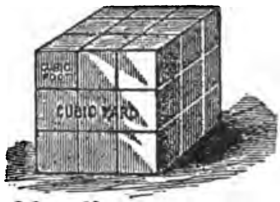
SUGGESTION TO TEACHER.—Have the pupils apply these problems to the school-room and the rooms in their homes also.

MEASURES OF VOLUME.

318. A **Volume** is that which has length, breadth, and thickness or height. These three elements are called *dimensions.* A volume is also called a *solid.*

319. A **Rectangular Volume** or **Solid** is a volume bounded by six rectangles. The bounding rectangles are called *faces.* Cellars, boxes, rooms, etc. are examples of rectangular volumes.

8 feet high.



3 feet wide.

320. A **Cube** is a volume bounded by six equal squares. Or, a cube is a rectangular volume whose faces are all equal.

321. By the **Contents** of a volume we mean the amount of space it contains. The contents of a volume are expressed by the number of times it contains a *cube* as a *unit of measure.*

Principle.—*The contents of a cube or rectangular volume are equal to the product of its length, breadth, and height.*

For in the volume above the number of cubic units on the base equals the length multiplied by the breadth, or $3 \times 3 = 9$, and the whole number of cubic units equals the number on the base multiplied by the number of layers of these cubes, or $9 \times 3 = 27$; hence, the whole number of cubes, or the contents, equals the product of the length, breadth, and height.

NOTE.—To find either dimension, divide the contents by the product of the other two dimensions.

WRITTEN EXERCISES.

1. Required the contents of a room 15 ft. long, 11 ft. wide, and 8 ft. high.

SOLUTION.—To find the contents, we multiply the length, breadth, and height together, and we have $15 \times 11 \times 8 = 1320$ cu. ft.; reducing this to cubic yards, we have 48 cu. yd. 24 cu. ft.

2. Required the contents of a cube whose edge measures 2 yd. 2 ft. *Ans.* 18 cu. yd. 26 cu. ft.

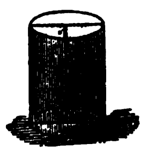
3. Required the depth of a cistern 10 ft. square which contains 600 cubic feet. *Ans.* 6 ft.

4. How much earth will be dug out of a cellar 54 ft. long, 32 ft. wide, and 8 ft. 3 in. deep? *Ans.* 528 cu. yd.

5. How many cubic yards of air in a room 36 ft. long, 18 ft. 9 in. wide, and 10½ ft. high? *Ans.* 262½ cu. yd.

6. How many cubic inches in a rectangular block of marble 8 ft. long, 6 ft. wide, and 3½ ft. thick? *Ans.* 290304.

THE CYLINDER.



322. A **Cylinder** is a round body of uniform size, with equal and parallel circles for its ends. The two circular ends are called *bases.*

323. The **Altitude** of a cylinder is the distance from the centre of one base to the centre of the other.

324. The **Convex Surface** of a cylinder is the surface of the curved part.

Principles.—I. *The convex surface of a cylinder is equal to the product of the circumference of the base by the altitude.*

II. *The contents of a cylinder are equal to the product of the area of the base by the altitude.*

WRITTEN EXERCISES.

1. What is the convex surface of a cylinder, the diameter of whose base is 6 inches and whose altitude is 8 inches?

SOLUTION.—The circumference of the base equals 6×3.1416, which is 18.8496 inches; multiplying by the altitude, 8, we have 150.7968 square inches, the convex surface.

2. How many square feet of bark on a log 20 ft. long and 16 inches in diameter? *Ans.* 83.776 sq. ft.

3. A cistern is 12 feet deep and 4 feet in diameter; how many cubic feet does it contain? *Ans.* 150.7968 cu. ft.

4. What is the cost of digging a well 16 ft. deep and 6 ft. in diameter, at \$$1.87\frac{1}{2}$ a cubic yard? *Ans.* \$31.42 —.

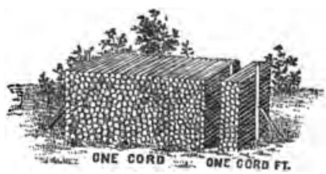
5. How much zinc will it take to line the sides of a cistern 10 ft. in diameter and $4\frac{1}{4}$ feet deep? *Ans.* $14.835\frac{1}{3}$ sq. yd.

6. Mr. Stokes put in his house a circular cistern 8 ft. in diameter and 6 ft. 6 in. deep; how many cubic feet of water did it hold? *Ans.* 326.7264 cu. ft.

WOOD MEASURE.

325. A **Cord** of wood is a pile 8 feet long, 4 feet wide, and 4 feet high. It contains 8 cord feet, or 128 cubic feet.

326. A **Cord Foot** is a part of this pile 1 foot long. It is thus 1 foot long, 4 feet wide, and 4 feet high, and contains 16 cubic feet.



Rule.—*The number of cords in a pile of wood is equal to the number of cubic feet reduced to cord feet and cords.*

Stone is often measured by the cord.

WRITTEN EXERCISES.

1. How many cords in a pile of wood 16 ft. long, 9 ft. high, and 11 ft. wide?

SOLUTION.—The number of cubic feet equals $16 \times 9 \times 11$, which equals 1584; dividing by 16 to reduce this to cord feet, we have 99 cord feet; dividing by 8 to reduce this to cords, we have 12 cd. 3 cd. ft.

2. How many cords of stone in a pile 36 ft. long, 12 ft. wide, and 7 ft. high? *Ans.* 23 cd. 80 cu. ft.

3. How many cords in a pile of wood 28 ft. 6 in. long, 6 ft. 8 in high, and 8 ft. wide? *Ans.* $11\frac{7}{8}$ cd.

4. A pile of wood containing $18\frac{3}{4}$ cords is 55 ft. long and 10 ft. wide; what is its height? *Ans.* $4\frac{4}{11}$ ft.

5. Required the cost of the wood that can be piled in a shed 20 ft. long, 10 ft. wide, and 8 ft. high, at $4.75 a cord. *Ans.* $59.37½.

6. What is the cost of a pile of stone 96 ft. long, 12 ft. wide, and 8 ft. high, at $3.75 a cord? *Ans.* $270.

BOARDS AND TIMBER.

327. **Boards** and **Timber** are usually estimated in square feet, called *board feet*, instead of in cubic feet.

A *standard board* in commerce is 1 inch thick, and its contents in board feet are the product of its length and breadth in feet. The same measurement applies if the board is less than 1 inch thick.

328. A *cubic foot* of lumber, therefore, contains 12 *board feet*. Hence, board feet may be reduced to cubic feet by dividing by 12, and cubic feet to board feet by multiplying by 12.

1. All *lumber* and *sawed* timber, as planks, scantling, joists, etc., are estimated in board feet.

2. Boards are quoted by the *hundred* or the *thousand*, meaning a *hundred square feet* or a *thousand square feet*. *Round* timber, as masts, etc., is estimated in cubic feet; *hewn* timber, as beams, etc., either in board or cubic feet.

Rule.—I. *To find the contents of a board, multiply the length in feet by the width in inches, and divide the product by* 12.

II. *To find the contents of a plank, joist, etc., multiply the length in feet by the width and thickness in inches, and divide the product by* 12.

1. If one of the dimensions is inches and the other two are feet, the product will be *board feet*.

2. When a board tapers regularly, the *mean width* is used, which is half the sum of the two ends.

WRITTEN EXERCISES.

1. Required the number of board feet in a board 15 feet long and 8 inches wide.

Solution.—Multiplying the length in feet by the width in inches, we have $15 \times 8 = 120$, and dividing by 12, we have 10 board feet, or *square feet*.

2. How much lumber in a board 24 ft. long, the ends being 16 and 12 inches respectively? *Ans.* 28 sq. ft.

3. How many square feet in 15 planks 15 ft. long, 18 inches wide, and 4 inches thick? *Ans.* 1350 sq. ft.

4. How much lumber in a stick of timber 35 feet long, 15 inches wide, and 12 inches thick? *Ans.* 525 sq. ft.

5. What must be the width of a board 16 ft. 6 in. long that it may contain $27\frac{1}{2}$ square feet? *Ans.* 20 inches.

6. What is the cost of 50 boards 15 ft. long, 18 in. wide, at \$2.25 per hundred square feet? *Ans.* \$25.31$\frac{1}{4}$.

7. What is the cost of 12 pieces of scantling 4 in. by 5 in. and 12 ft. long, at \$7.50 per thousand sq. ft.? *Ans.* \$1.80.

8. How many square feet of inch boards will it require, allowing for overlapping, to make a box 4 ft. long, 3 ft. 2 in. wide, and 2 ft. high, outside measure? *Ans.* 51 sq. ft.

NOTE.—Sides $= 2 \times 4 \times 2 = 16$ sq. ft.; ends $= 2 \times 3 \times 2 = 12$ sq. ft.; top and bottom, fitted inside, $= 2 \times 3\frac{5}{6} \times 3 = 23$ sq. ft.; surface $= 16 + 12 + 23$, or 51 sq. ft. Similarly, if top and bottom are not fitted inside.

STONEWORK AND MASONRY.

329. Masonry is usually estimated by the *perch*. In some places, however, it is estimated by the cubic foot.

330. A Perch of *stone* or of *masonry* is $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high; it contains $24\frac{3}{4}$ cubic feet. When stone is built into a wall 22 cubic feet make a perch, $2\frac{3}{4}$ cu. ft. being allowed for mortar and filling.

331. Excavations and **Embankments** are estimated by the *cubic yard*. A cubic yard of earth is called a *load*.

In estimating *labor*, bricklayers and masons measure the length of the wall on the outside. The corners are thus measured twice, but this is considered an allowance for the greater difficulty of building them. No allowance is made for windows and doors except by special contract, in which case it is customary to allow one-half the space actually required. In estimating *material*, allowance is made for doors, windows, and corners.

Rule.—*To find the number of perches in a piece of masonry, divide the number of cubic feet by* $24\frac{3}{4}$.

WRITTEN EXERCISES.

1. How many perches of masonry in a wall 50 ft. long, 5 ft. 6 in. high, and 18 inches thick?

SOLUTION.—Multiplying the length, breadth, and height together, we have $50 \times 1\frac{1}{2} \times 5\frac{1}{2}$, or $412\frac{1}{2}$ cu. ft., which, divided by $24\frac{3}{4}$, the number of cubic feet in a perch, equals $16\frac{2}{3}$ perches.

2. A man had a well dug in his yard, 6 ft. in diameter and 20 ft. deep; what did it cost at 50¢ a load? *Ans.* \$10.47+.

3. Required the cost of digging a cellar 45 ft. long, 32 ft. wide, and 6 ft. 6 in. deep, at 45¢ a load. *Ans.* \$156.

4. How much will it cost to fill in a street 540 feet long and 60 feet wide, averaging $3\frac{1}{2}$ ft. below grade, at \$.54 a cubic yard? *Ans.* \$2268.

5. How many perches ($24\frac{3}{4}$ cu. ft.) of stone, laid dry, will build a wall around a lot 100 ft. long and 80 ft. wide, the wall to be $4\frac{1}{2}$ ft. high and 2 ft. thick? *Ans.* 128 perches.

6. Required the cost of walling the cellar of a house 42 ft. long by 28 ft. wide, the wall being $7\frac{1}{2}$ ft. high and $1\frac{1}{2}$ ft. thick, the masonry costing \$4.40 a perch, no allowance being made for corners? *Ans.* \$280.

BRICKWORK.

332. Brickwork is generally estimated by the *thousand bricks*, but sometimes in *cubic feet.*

The average size of bricks is 8 in. $\times 4 \times 2$, but Philadelphia and Baltimore bricks are $8\frac{1}{4}$ in. $\times 4\frac{1}{8} \times 2\frac{3}{8}$; Maine bricks, $7\frac{1}{2}$ in. $\times 3\frac{3}{8} \times 2\frac{3}{8}$; North River bricks, 8 in. $\times 3\frac{1}{2} \times 2\frac{1}{4}$; and Milwaukee bricks, $8\frac{1}{2}$ in. $\times 4\frac{1}{8} \times 2\frac{3}{8}$.

333. To build one *square foot* of wall 1 brick, or 4 inches, thick, requires about 7 common bricks; 2 bricks, or 9 in., thick, 14 bricks; 3 bricks, or 13 in., thick, 21 bricks. Hence the following rule:

I. *To find the number of common bricks required for a wall, multiply the number of square feet in the wall by* 7, *if the wall is* 1 *brick thick; by* 14, *if* 2 *bricks thick; by* 21, *if* 3 *bricks thick.*

334. A close approximation to the number of bricks in a wall is given by the following rule:

II. *Increase the three dimensions of a brick by* $\frac{1}{4}$ *of an inch to allow for the mortar, and divide the contents of the bricks thus enlarged into the contents of the wall.*

NOTE.—In buildings, the corners are deducted for material, but not for labor. In paving walks, no allowance is made for mortar.

WRITTEN EXERCISES.

1. How many common bricks will it require to build the walls of a house 48 ft. long, 25 ft. wide, and 21 ft. high, the wall being 13 in. thick (21 bricks to sq. ft.), allowing 240 sq. ft. for doors and windows?

SOLUTION.—The outside surface of the walls equals $(48+25)\times 2\times 21 = 3066$ sq. ft. As we are estimating material, we must deduct the corners, which equal $\frac{13}{12}\times 21\times 4 = 91$ sq. ft. Hence the wall $= 3066 - (91+240) = 2735$ sq. ft., and multiplying by 21, the number of bricks in 1 square foot, we have 57,435 bricks.

2. How many common bricks will it require to build a house 36 ft. square, the wall being 24 ft. high and 3 bricks thick, if an allowance of 216 sq. ft. is made for windows and doors? *Ans.* 65,856 bricks.

The outside surface $= (36+36)\times 2\times 24 = 3456$ sq. ft.
The deduction $= 216 + \frac{13}{12}\times 4\times 24 = 320$ sq. ft.
No. of bricks $= (3456 - 320)\times 21 = 65{,}856$.

3. What will be the cost of the bricks in a house 40 ft. long, 35 ft. wide, and 24 ft. high, the walls being 13 in. thick, of common bricks, at $15 per M., allowing 250 sq. ft. for doors and windows? *Ans.* $1022.49.

4. What will be the cost of laying the bricks of a house 50 ft. long, 22 ft. wide, 23 ft. high, built with common brick,

the walls 13 in. thick, 394 sq. ft. being allowed for openings, the laying costing $2.50 ℔ M.? *Ans.* $153.20.

5. What will be the cost of paving a sidewalk 45 ft. long, 7 ft. wide, with Milwaukee bricks laid on edge, 60 to a square yard, the bricks costing $10 ℔ M., and the laying 45¢ a sq. yd.? *Ans.* $36.75.

6. I wish to pave a yard, 30 ft. by 16 ft., with common bricks laid flat, the bricks costing $8 ℔ M., and the workman charging 40 cents a square yard; what will the job cost me? *Ans.* 38.61\frac{1}{3}$.

CAPACITY OF CISTERNS, ETC.

335. The **Capacity of Cisterns,** etc. is usually expressed in *gallons* or *barrels.*

336. The **Standard Liquid Gallon** of the United States contains 231 cubic inches. A cubic foot of pure water weighs about 1000 ounces.

337. The **Barrel** of 31$\frac{1}{2}$ gallons, and the hogshead of 63 gallons, are used in measuring the capacity of cisterns, vats, tanks, etc. When used as the names of vessels these terms express no definite quantity.

Rule.—*To find the capacity of a cistern or vessel in gallons, divide the contents in cubic inches by* 231.

WRITTEN EXERCISES.

1. How many gallons of water will a tank 8 ft. long, 4 ft. wide, and 2 ft. 6 in. deep contain?

Solution.—The contents of the tank equal $8 \times 4 \times 2\frac{1}{2}$, which are 80 cubic feet; multiplying by 1728 to reduce to cubic inches, we have 138240 cu. in.; dividing by 231, the number of cubic inches in a gallon, we have 598$\frac{34}{77}$ gallons.

2. How many gallons of water are contained in a tank 18 ft. long, 5 ft. wide, and 4 ft. 9 in. deep? *Ans.* 3197$\frac{71}{77}$ gal.

3. Required the number of cubic feet in a cistern containing 83 hogsheads. *Ans.* $699\frac{1}{64}$ cu. ft.

4. A cistern 10 ft. square and 8 ft. deep will contain how many gallons of water? *Ans.* $5984\frac{8}{7}\frac{2}{7}$ gal.

5. A tank measuring 9 ft. square by 7 ft. deep will contain how many barrels of water? *Ans.* $134\frac{5}{7}\frac{9}{7}$ bbl.

6. A tank 11 ft. long, 7 ft. wide, and 9 ft. deep will contain how many hogsheads of water? *Ans.* $82\frac{2}{7}$ hhd.

7. A tank 4 yd. long, 2 yd. wide, and 9 ft. deep is half full of water; what is the weight of the water? *Ans.* 20250 lb.

8. How many hogsheads of water can be contained in a well whose diameter within the curb is $3\frac{1}{2}$ ft. and depth 10 feet? *Ans.* 11.424 hhd.

CAPACITY OF BINS, ETC.

338. The **Capacity of Bins**, etc. is usually expressed in *bushels.*

339. The **Standard Bushel** of the United States is a cylindrical measure $18\frac{1}{2}$ in. in diameter and 8 in. deep, containing 2150.42 cubic inches.

Grain, seeds, and small fruits are sold by *stricken measure.* Potatoes, corn in the ear, large fruits, coal, and other bulky articles are sold by *heaped measure.*

Rule.—I. *To find the capacity of a bin in bushels, divide the contents in cubic inches by* 2150.42.

II. *To find the cubic feet in a given number of bushels, multiply the number of bushels by* 2150.42 *and divide by* 1728.

WRITTEN EXERCISES.

1. Required the number of bushels in a bin 10 ft. long, 8 ft. wide, and 4 ft. deep.

SOLUTION.—The contents equal $10 \times 8 \times 4$, or 320 cubic feet, which equals 552,960 cubic inches; dividing by 2150.42, the number of cubic inches in a bushel, we have 257.14 + bushels.

2. A bin is 15 ft. long, 8 ft. wide, and 3 ft. deep; how many bushels will it hold? *Ans.* 289.28 + bu.

3. What is the width of a bin 25 ft. long and 4 ft. 9 in. deep, to contain 600 bushels of wheat? *Ans.* 6.29 — ft.

4. One division of an elevator is 40 ft. long, 25 ft. wide, and contains 3000 bu. of grain; what is its depth? *Ans.* 3.73+ft.

5. An ice-house 40 ft. long, 25 ft. wide, and 15 ft. high is filled with ice; how many tons are there if a cubic foot weighs $58\frac{1}{8}$ lb.? *Ans.* $435\frac{15}{16}$ tons.

APPROXIMATE MEASUREMENTS.

340. While the previous rule gives exact results, the following approximate values are often used, being more easily reckoned and sufficiently exact in most cases.

Since 2150.42 is to 1728 as 5 to 4, nearly, a bushel is nearly equal to $1\frac{1}{4}$ cu. ft. Therefore, for practical purposes, *$\frac{4}{5}$ of the number of cubic feet will equal the number of bushels, and $\frac{5}{4}$ of the number of bushels will equal the number of cubic feet.*

Notice also that $\frac{4}{5}$ of the number of bushels *stricken measure* equals the number of bushels *heaped measure.*

Coal.—Coal is bought and sold in large quantities by the *ton;* in small quantities by the *bushel,* 28 heaped bushels, or about 43.5 cu. ft., being considered equal to a ton.

Ordinary *anthracite coal* measures from 36 to 40 cu. ft. to the *ton; bituminous coal,* from 36 to 45 cu. ft. to the ton. Lehigh white-ash coal, egg size, measures about $34\frac{1}{2}$ cu. ft. to the ton; Schuylkill white-ash, 35 cu. ft., and pink, gray, or red ash, 36 cu. ft., to the ton.

Hay.—Hay, when loose or in loads, or upon a scaffold, measures about 500 cu. ft. to the ton; on a mow, 400 cu. ft.; and in a large well-settled stack, 10 cu. yd.

WRITTEN EXERCISES.

1. I have a box $4\frac{1}{2}$ ft. long, 3 ft. wide, and 2 ft. 6 in. deep; how many bushels of cranberries will it hold?

SOLUTION.—The contents equal $4\frac{1}{2} \times 3 \times 2\frac{1}{2} = 33\frac{3}{4}$ cu. ft. Then, since cranberries are sold by the "stricken bushel," the number of bushels will be $\frac{4}{5}$ of $33\frac{3}{4}$, or 27 bu.

2. A bin is 10 ft. long, 7 ft. wide, and 3 ft. 6 in. deep; how many bushels of shelled corn will it hold? *Ans.* 196 bu.

3. A bin 10 ft. long, 8 ft. wide, and 4 ft. deep is $\frac{3}{4}$ full of barley; what is it worth at 85¢ a bushel? *Ans.* $163.20.

4. What is the difference in the number of bushels in a bin 4 ft. by 5 ft. by 6 ft., calculated exactly or by taking $\frac{4}{5}$ of the cubic feet as bushels? *Ans.* .43 — bu.

5. A bin 8 ft. long, $4\frac{1}{2}$ ft. wide, and $3\frac{1}{2}$ ft. deep is filled with Schuylkill white-ash coal (35 cu. ft. in a ton); what is its value at $6.25 a ton? *Ans.* $22.50.

6. A shed 9 yd. long, $5\frac{3}{4}$ yd. wide, and 7 ft. high is half full of Lehigh white-ash coal ($34\frac{1}{2}$ cu. ft.); what is the value of the coal at $6.75 a ton? *Ans.* 318.93\frac{3}{4}$.

7. A hay-mow is 20 ft. long by 15 ft. wide and 12 ft. high; what is the value of the hay when it is filled, valued at $13 a ton, 400 cu. ft. to a ton? *Ans.* $117.

8. Mr. Warner sold a stack of hay 10 ft. long, 8 ft. wide, and 7 ft. high, at $12 a ton; what was the value of the hay, 10 cu. yd. to a ton? *Ans.* 24.88\frac{8}{9}$.

MISCELLANEOUS EXAMPLES

IN PRACTICAL MEASUREMENTS.

1. What is the circumference of the planet Venus, its diameter being about 7800 miles? *Ans.* 24,504.48 miles.

2. How much stair-carpet will be required for a flight of 15 steps, each 12 in. wide and 7 in. high? *Ans.* $7\frac{11}{12}$ yd.

3. How many planks laid crosswise, 1 ft. wide, will it take for a board-walk 1 mi. 80 rd. long and 5 ft. wide? *Ans.* 6600.

4. To dig a sewer $1\frac{1}{4}$ miles long, 4 ft. wide, and 6 ft. deep cost $2200; what was the price per load? *Ans.* $37\frac{1}{2}$¢.

5. If 4 persons can stand on one square yard of ground, how many people can stand in a public park 36 rods square? *Ans.* 156816.

6. How many paving-stones $1\frac{1}{2}$ ft. by 2 ft. will be required for a street 60 ft. wide and 1720 ft. long? *Ans.* 34400.

7. A bin 6 ft. long and 4 ft. wide contains 75 bushels of wheat; what is its depth? *Ans.* $3\frac{29}{32}$ ft.

8. A rectangular piece of land is 120 ft. wide by 150 ft. long; what part of an acre does it contain? *Ans.* $\frac{50}{121}$.

9. How much will it cost to build a straight road from one side of a Western township to the opposite side at $1.25 per rod? *Ans.* $2400.

10. How many square inches of gold-leaf would be required to cover the entire outside of a box 8 in. long, 5 in. wide, and 3 in. high? *Ans.* 158 sq. in.

11. In a Western township $\frac{3}{4}$ of the land is valued at $5 an acre, and the remainder at $9 an acre; what is the total valuation? *Ans.* $138,240.

12. A farm 1 mile square is divided into 4 square fields; how many acres in each field, and how many rods of fence will enclose them? *Ans.* 160 A.; 1920 rods.

13. A plot of ground 20 rods long and 16 rods wide is enclosed by a tight board fence 6 ft. high; how many square yards in the surface of the fence? *Ans.* 792 sq. yd.

14. How much will a granite block weigh which is 8 ft. long, 3 ft. 6 in. wide, and 2 ft. 8 in. thick, if 12 cu. ft. of granite weigh a ton? *Ans.* $6\frac{2}{9}$ T.

15. A railroad passes through 7194 ft. of a farm; what will be the cost of the right of way at $66 an acre if the strip taken is 50 ft. wide? *Ans.* $545.

16. A wire fence encloses a circular field 100 rods in diameter; what will be the area of a square field which this fence would enclose? *Ans.* 38 A. 88.53 P.

17. What will it cost to shingle the roof of a house, the rafters of which are 18 ft. 6 in. long and the ridge-pole 26 ft. long, at $7\frac{1}{2}$ per square? *Ans.* $72.15.

18. A has a mow 25 ft. long, 16 ft. wide, and 9 ft. deep; how many tons of hay (400 cu. ft.) does it hold? *Ans.* 9 tons.

19. What is the cost of flooring a three-story house, the floors being 56 ft. by 32 ft. and the plank $1\frac{1}{2}$ inches thick, at \$33 per thousand? *Ans.* \$266.112.

20. A tank 8 ft. long, 6 ft. wide, and $3\frac{1}{2}$ ft. deep can be emptied by a waste-pipe in 2 hours; how many gallons are discharged in 1 minute? *Ans.* $10\frac{26}{55}$ gallons.

21. From a quartz rock yielding silver at the rate of \$126.25 per ton a miner obtained \$85.65 worth; what was the weight of the rock? *Ans.* 13 cwt. $56\frac{84}{101}$ lb.

22. How much less will it cost to fence a field 72 rods square than a rectangular field three times as long and one-third as wide, if fencing cost \$2.50 a rod? *Ans.* \$480.

23. A street 36 ft. wide was paved with asphalt, at 25¢ per square yard; what did it cost to pave a "square" 32 rods long? *Ans.* \$528.

24. How many bundles of lath will be required for the walls and ceiling of a room 16 ft. long, 12 ft. wide, 10 ft. high, each bundle being estimated to cover 5 sq. yd.? *Ans.* $16\frac{32}{45}$.

25. I wish to cover my parlor, 25 ft. by 17 ft. 6 in., with carpet $\frac{3}{4}$ yd. wide; what will it cost me at $\$1.37\frac{1}{2}$ per yard, the strips running lengthwise, with $3\frac{1}{3}$ yd. waste? *Ans.* \$96.25.

26. A railroad tunnel is one-eighth of a mile long, averaging 30 ft. wide and 18 ft. high; what did the excavation cost, at \$1.25 a cubic yard? *Ans.* \$16,500.

27. How many freight-cars will be required to transport 72,000 bu. of wheat, 60 lb. each, 24,000 lb. being the weight allowed for a single car? *Ans.* 180 cars.

28. A circular flower-bed being divided into four equal parts by lines drawn from the centre, one quarter was planted with tulips; if the outer edge of the tulip-bed was 5 feet, what was its area? *Ans.* 7.9575 sq. ft.

EXAMPLES FOR REVIEW.

1. Draw a figure representing a garden 40 ft. long and 32 ft. wide, with a walk around it 4 ft. wide.

2. How many steps 2 ft. 3 in. long will it take to walk around the outside of the above garden? *Ans.* 64.

3. How many trees 8 feet apart will be required to plant on the outside border of the walk? *Ans.* 22.

4. How many square feet in the garden? How many square feet in the walk? *Ans.* 1280 sq. ft.; 640 sq. ft.

5. How many gold dollars will it take to make a pound Troy when each gold dollar weighs 25.8 grains? *Ans.* $223\frac{11}{43}$.

6. How many yards of carpet will cover a stairway 12 ft. high, each step 8 in. high and 10 in. wide? *Ans.* 9 yd.

7. If the first day of April is Thursday, what day of the week is the following Fourth of July? *Ans.* Sunday.

8. How many panels of fence 10 ft. long will it take to enclose a rectangular field 42 rods long and 38 rods wide? *Ans.* 264.

9. How many silver coins, each coin weighing $412\frac{1}{2}$ grains, can be coined from a bar of silver weighing 6 lb. 8 oz. Avoirdupois? *Ans.* 110 coins; 125 gr. rem.

10. A floor 30 ft. long, 18 ft. wide, has a painted border round it 3 ft. deep; how many square feet in the unpainted part? *Ans.* 288 sq. ft.

11. How much will it cost to cement the floor of a cellar 30 ft. long and 24 ft. wide, at $45\frac{1}{2}$¢ a sq. yd.? *Ans.* $36.40.

12. If a family uses 5 pints of milk a day, what will be their milk bill for the quarter ending June 30, at $7\frac{1}{2}$ cents a quart? *Ans.* $$17.06\frac{1}{4}$.

13. How many posts 6 feet apart will be required to enclose a half-acre lot of land 66 ft. wide? *Ans.* 132 posts.

14. What will it cost to plaster a room 14.5 ft. long, 12.25 ft. wide, and 10.5 ft. high, @ 25¢ a sq. yd.? *Ans.* $20.54.

15. A car-load of wheat, weighing 15 T. 25 cwt., was sold

for $325; what was the price per bushel, allowing 60 lb. to the bushel? *Ans.* 60¢.

16. If James goes to bed at 9.15 P. M. and rises at 7.30 A. M., how many hours is he out of bed in a day? *Ans.* 13 h. 45 min.

17. A lady left the city for the sea-shore June 20, at 4.30 P. M., and returned Sept. 12, at 10.45 A. M.; how long was she away? *Ans.* 83 da. 18 h. 15 min.

18. If a young lady weighs 144 lb. Av., how much will she weigh by Troy weight? *Ans.* 175 lb.

19. A farmer wishes to construct a bin 6 ft. square that will contain 120 bushels of grain; what will be the depth of the bin? *Ans.* 4.17— ft.

20. If a man smokes 6 cigars a day at the rate of 3 cigars for a quarter, what did his cigars for February, 1896, cost him? *Ans.* $14.50.

21. A farmer gave 158 dozen eggs, worth 32 cents a dozen, in part payment for 11 tons of coal, at $5.25 a ton; how much remains unpaid? *Ans.* $7.19.

22. A tub of butter, weighing 36 lb. 8 oz., lasted a family from Oct. 10, 1895, to March 4, 1896; how many ounces a day on the average did they use? *Ans.* 4 oz.

23. A train passes 15 telegraph-poles every minute; if the distance between the poles is 60 yards, at what rate per hour is the train going? *Ans.* $30\frac{1}{2}\frac{5}{2}$ miles.

24. A chest measures on the outside 20 in. by 12 in. by 8 in., and the boards are $\frac{3}{4}$ in. thick; what are the cubic contents of the chest? *Ans.* $1262\frac{5}{8}$ cu. in.

25. A cubic foot of ice weighs about 57.5 lb.; how many tons could be stored in an ice-house that is 80 ft. long by 40 ft. 6 in. wide and 20 ft. deep? *Ans.* 1863 T.

26. How many paper boxes 4 in. long, 3 in. wide, and 2 in. deep can be packed in a box 4 ft. long by 3 ft. wide and 1 ft. 6 in. deep? *Ans.* 1296.

INTRODUCTION TO PERCENTAGE.

ORAL EXERCISES.

1. A gain of $4 on $10 is a gain of how many dollars on *the hundred?*

SOLUTION.—If the gain on $10 is $4, on $100, which is 10 times $10, the gain is 10 times $4, which is $40.

2. A gain of $2 on $5 is a gain of how many dollars on *the hundred?*

3. What is the gain on a hundred when the gain is 3 on 20? 4 on 20? 5 on 25?

4. If the gain on $100 is $25, what is the gain on $4? On $12? On $20? On $24? On $5? On $15? On $30? On $40?

5. If the gain on $100 is $20, what is the gain on $1? On $12? On $24? On $36?

6. If the gain on $100 is $20, what part of the $100 equals the gain?

7. If the gain on $100 is $40, what part of the $100 equals the gain?

8. What is the gain on $48 at the *rate* of 25 on the 100?

9. What is the gain on $35 at the *rate* of 20 on the 100?

10. What is the gain on $40 at the *rate* of 10 on the *hundred?*

11. What is the gain on $200 at the *rate* of 20 on the *hundred?*

12. What is the gain on $360 at the *rate* of 15 on the *hundred?*

13. What is the *rate per hundred* at a gain of $8 on $40?

14. What is the *rate per hundred* at a gain of $15 on $60?

15. *Per cent.* means the same as *per hundred;* what, then, can we call the *rate per hundred?* *Ans. Rate per cent.*

16. A gain of $15 on $60 is a gain of what *per cent.?*

17. A loss of $12 on $60 is a loss of what *per cent.?*

18. What *per cent.* is a gain of 30 on 60? 4 on 20? 3 on 60? 4 on 80? 8 on 200?

19. What is 10 per cent. of 40? 25 per cent. of 24? 20 per cent. of 40? 50 per cent. of 48?

SOLUTION.—10 per cent. is at the rate of 10 on the 100, and since 10 is $\frac{1}{10}$ of 100, 10 per cent. of 40 is $\frac{1}{10}$ of 40, which is 4.

20. What is 50 per cent. of 28? 30 per cent. of 40? 60 per cent. of 35? 40 per cent. of 45? 20 per cent. of 60?

21. What shall we call the operations which treat of rate per cent.? *Ans. Percentage.*

SECTION VIII.

PERCENTAGE.

341. **Percentage** is the process of computation in which the basis of comparison is a *hundred.*

342. The **Term** *per cent.*—from *per*, by, and *centum*, a hundred—means *by* or *on the hundred.* Thus, 6 per cent. of any quantity means 6 of every hundred of the quantity.

343. The **Symbol** of percentage is %. The per cent. may also be indicated by a common fraction or a decimal. Thus, $6\% = \frac{6}{100} = .06$; $25\% = \frac{25}{100} = .25$.

344. The **Quantities** considered are the *Base*, the *Rate*, the *Percentage*, and the *Amount* or *Difference.*

345. The **Base** is the number on which the percentage is computed.

346. The **Rate** is the number of hundredths of the base which are taken.

347. The **Percentage** is the number which is a certain per cent. of the base.

348. The **Amount** is the sum of the base and percentage. The *Difference* is the difference of the base and percentage.

349. The *Amount* and *Difference* may both be embraced under the general term *Proceeds.*

In computation the rate is usually expressed as a decimal.

EXPRESSION OF THE RATE.

1. Express 5% as a decimal and common fraction.

SOLUTION.—Since 5 per cent. is 5 on a *hundred*, 5% of a quantity is .05 of it; or, as a common fraction, $\frac{5}{100}$ or $\frac{1}{20}$ of it.

OPERATION.

$5\% = .05 = \frac{5}{100} = \frac{1}{20}$

Express as a decimal and common fraction:

2. 4%.	8. 25%.	14. $12\frac{1}{2}$%.	20. $87\frac{1}{2}$%.
3. 6%.	9. 50%.	15. $16\frac{2}{3}$%.	21. $\frac{1}{4}$%.
4. 7%.	10. 75%.	16. $33\frac{1}{3}$%.	22. $\frac{1}{2}$%.
5. 8%.	11. $6\frac{1}{4}$%.	17. $37\frac{1}{2}$%.	23. $\frac{3}{4}$%.
6. 10%.	12. $8\frac{1}{3}$%.	18. $62\frac{1}{2}$%.	24. $\frac{5}{8}$%.
7. 20%.	13. $11\frac{1}{9}$%.	19. $83\frac{1}{3}$%.	25. $\frac{15}{16}$%.

Express in per cent. decimally:

1. $\frac{1}{2}$.	5. $\frac{3}{4}$.	9. $\frac{1}{3}$.	13. $\frac{1}{25}$.
2. $\frac{1}{4}$.	6. $\frac{1}{8}$.	10. $\frac{1}{6}$.	14. $\frac{1}{50}$.
3. $\frac{1}{5}$.	7. $\frac{1}{9}$.	11. $\frac{3}{8}$.	15. $\frac{1}{16}$.
4. $\frac{1}{10}$.	8. $\frac{1}{12}$.	12. $\frac{5}{8}$.	16. $\frac{37}{40}$.

350. Cases.—The subject of percentage is conveniently treated under three distinct cases:

1. Given the base and the rate, to find the percentage or proceeds.

2. Given the percentage or proceeds and the rate, to find the base.

3. Given the percentage or proceeds and the base, to find the rate.

CASE I.

351. Given the base and the rate, to find the percentage or the proceeds.

ORAL EXERCISES.

1. What is 20% of 150 yards?

SOLUTION.—20% of anything is $\frac{20}{100}$, or $\frac{1}{5}$ of it, and $\frac{1}{5}$ of 150 yards is 30 yards. Therefore, etc.

What is

2. 20% of 65 oz.?	**6.** 15% of 80 yd.?
3. 25% of 72 bu.?	**7.** 45% of 140 qt.?
4. 50% of 112 mi.?	**8.** $12\frac{1}{2}$% of 128 cows?
5. 75% of 108 tons?	**9.** $16\frac{2}{3}$% of 120 horses?

10. A grocer bought 120 dozen oranges, and 25% turned out to be bad; how many did he lose?

11. From a hogshead of wine, containing 108 gallons, $33\frac{1}{3}$% was drawn out; how many gallons remained?

12. A train of cars running 40 miles an hour slackens its speed 45%; what is the rate of running after the decrease?

13. A salesman received $36 a month, but at the beginning of the year his salary was raised $11\frac{1}{9}$%; what did he then receive a month?

14. In the 10th Problem, which is the base, which the rate, and which the percentage?

WRITTEN EXERCISES.

1. What is 5% of $246? What is the amount of $246, increased by 5% of itself?

SOLUTION.—5% of $246 equals .05 times $246, which, by multiplying, we find to be $12.30.

OPERATION.

$$\begin{array}{r} \$246 \\ .05 \\ \hline \$12.30 \end{array}$$

SOLUTION.—A number increased by 5%, or .05 times itself, equals 1.05 times itself; 1.05 times $246 equals $258.30.

OPERATION.

$$\begin{array}{r} \$246 \\ 1.05 \\ \hline \$258.30 \end{array}$$

Rule.—I. *To find the percentage, multiply the base by the rate.*

II. *To find the proceeds, multiply the base by 1 plus the rate, or by 1 minus the rate.*

1. When the rate gives a small common fraction, take such a part of the base as is indicated by this fraction.

2. The *amount* equals the base *plus* the percentage; the *difference* equals the base *minus* the percentage.

What is	*Ans.*		*Ans.*
2. 6% of 950?	57.	**9.** $33\frac{1}{3}$% of 1860 rd.?	620.
3. 8% of 625?	50.	**10.** $66\frac{2}{3}$% of $3720?	$2480.
4. 16% of 185?	$29\frac{3}{5}$.	**11.** $42\frac{6}{7}$% of 231 yd.?	99 yd.
5. 25% of 432?	108.	**12.** $45\frac{5}{11}$% of $165.44?	$75.20.
6. 35% of 740?	259.	**13.** $\frac{3}{4}$% of $18\frac{3}{4}$ bu.?	$\frac{9}{64}$ bu.
7. $12\frac{1}{2}$% of 512?	64.	**14.** $\frac{4}{5}$% of $640?	$5.12.
8. $16\frac{2}{3}$% of 1728?	288.	**15.** $\frac{7}{8}$% of $\frac{27}{28}$ lb. Av.?	$.13\frac{1}{2}$ oz.

16. Find 20% of 54 lb. $11\frac{1}{2}$ oz. Av. *Ans.* 10 lb. $15\frac{1}{10}$ oz.

17. Find $42\frac{1}{2}$% of 13 yd. 1 ft. 8 in. *Ans.* 5 yd. 2 ft. $3\frac{3}{5}$ in.

18. In a mixed school of 180 pupils $55\frac{5}{9}$% are girls; how many boys are there? *Ans.* 80.

19. Mr. Smith's salary is $2500 a year, of which he pays 12% for house-rent; what rent does he pay? *Ans.* $300.

20. How much pure silver is there in $412\frac{1}{2}$ oz. of U. S. silver coin if the coin contains 10% alloy? *Ans.* 371.25 oz.

21. If a piece of cloth shrinks $5\frac{3}{4}$% of its length in sponging, what will be the length after sponging of a piece of 40 yards? *Ans.* 37.7 yards.

22. A barrel of flour weighs 196 lb.; if the bread made from it weighs $33\frac{1}{3}$% more than the flour, what is the weight of the bread? *Ans.* $261\frac{1}{3}$ lb.

23. A train of cars was running at the rate of 36 miles an hour, when the speed was increased 25%; at what rate did the train then run? *Ans.* 45 miles.

24. If flaxseed contains 11% of oil, and a pint of oil weighs $\frac{3}{4}$ of a pound, how much linseed oil can be extracted from 2 cwt. 54 lb. of flaxseed? *Ans.* 4 gal. 2 qt. $1.25\frac{1}{3}$ pt.

25. A speculator bought 1820 acres of land, of which he sold 25% to Mr. Jones, and $66\frac{2}{3}$% of the remainder to Mr. Robinson; how much remained? *Ans.* 455 acres.

26. A lady's income is $3000 a year, of which she spends 15% the first quarter, 12% the second, $12\frac{1}{2}$% the third, and $6\frac{1}{4}$% the fourth; how much did she spend? *Ans.* $1372.50.

27. A book-keeper has a salary of $3500 a year; he spends 25% for board, 15% for clothes and books, and $12\frac{1}{2}$% for incidentals; what can he save in a year? *Ans.* $1662.50.

28. Mr. Brown owned $\frac{2}{3}$ of a factory worth $50,000, and sold $37\frac{1}{2}$% of his share to his partner; what part of the whole factory did he still own, and what was its value?

Ans. $\frac{5}{12}$; value, $$20,833\frac{1}{3}$.

CASE II.

352. Given, the percentage or proceeds and the rate, to find the base.

ORAL EXERCISES.

1. Sixteen is 25 per cent. of what number?

SOLUTION.—If 16 is 25 per cent. of some number, it is $\frac{25}{100}$, or $\frac{1}{4}$, of that number; if 16 is $\frac{1}{4}$ of some number, $\frac{4}{4}$, or the number, equals 4 times 16, or 64.

Find the number of which

2. 14 is 20%.

3. 17 is $16\frac{2}{3}$%.

4. 28 is $33\frac{1}{3}$%.

5. 2.7 is 50%.

6. 7 lb. is $6\frac{1}{4}$%.

7. 3 bu. is $12\frac{1}{2}$%.

8. 12 A. is $66\frac{2}{3}$%.

9. 5.5 rd. is 55%.

10. Thirty is 25% more than what number? 50% more than what number? 100% more than what number?

11. Sixty is 25% less than what number? 60 is 50% less than what number? Can 60 be 100% less than any number?

12. A certain village has a population of 5400, which is $12\frac{1}{2}$% more than it was three years ago; what is its yearly gain?

13. A gentleman gave his daughter $100 as a Christmas present, which is $62\frac{1}{2}$% of what he gave to his wife; what did he give his wife?

14. The average attendance for one quarter of a certain Sunday-school was 240 pupils, which was $11\frac{1}{9}$% less than the number registered: how many were registered?

WRITTEN EXERCISES.

1. 40 is 5% of what number? What number, increased by 20% of itself, equals 480?

SOLUTION.—If 40 is 5% of some number, then .05 times *some number* equals 40; if .05 times *some number* equals 40, the *number* equals $40 \div .05$, which is 800.

OPERATION.

The No. $\times$ .05 = 40

The No. = $40 \div .05 = 800$

SOLUTION.—A number increased by 20%, or .20, of itself, equals 1.20 times the number; and if 1.20 times a *number* equals 480, the *number* equals $480 \div 1.20$, or 400.

OPERATION.

The No. $\times$ 1.20 = 480

The No. = $480 \div 1.20 = 400$

Or, $100\% + 20\% = 120\%$.	Or, $20\% = \frac{1}{5}$; $\frac{5}{5} + \frac{1}{5} = \frac{6}{5}$.
120% of the number $= 480$	$\frac{6}{5}$ No. $= 480$
Hence 1% of the number $= 4$	$\frac{1}{5}$ No. $= 80$
And 100% of the number $= 400$	$\frac{5}{5}$ No. $= 400$

NOTE.—Teachers can use the solution which they prefer.

Rule.—I. *To find the base, divide the percentage by the rate.*

II. *To find the base, divide the amount by* 1 *plus the rate, or the difference by* 1 *minus the rate.*

Find the number of which

2. 24 is 20%. *Ans.* 120.

3. 32 is 25%. *Ans.* 128.

4. 220 lb. is 40%. *Ans.* 550 lb.

5. 723 A. is 30%. *Ans.* 2410 A.

6. $\frac{7}{24}$ is $33\frac{1}{3}\%$. *Ans.* $\frac{7}{8}$.

7. $18\frac{3}{4}$ is 75%. *Ans.* 25.

8. \$715 is $62\frac{1}{2}\%$. *Ans.* \$1144.

9. \$$675\frac{3}{4}$ is $12\frac{1}{2}\%$. *Ans.* \$5406.

What number increased by

10. 8% of itself $= 54$?

11. $12\frac{1}{2}\%$ of itself $= 72$?

12. 40% of itself $= 1617$?

13. $33\frac{1}{3}\%$ of itself $= 1728$?

14. 16% of itself $= \frac{58}{15}$?

15. $37\frac{1}{2}\%$ of itself $= \frac{11}{36}$?

16. 60% of itself $= \frac{8}{5}$?

17. $62\frac{1}{2}\%$ of itself $= 9100$?

What number diminished by

18. $8\frac{1}{3}\%$ of itself $= 55$?

19. $18\frac{3}{4}\%$ of itself $= 91$?

20. 25% of itself $= 72$?

21. 45% of itself $= 121$?

22. 75% of itself $= \frac{1}{25}$?

23. $87\frac{1}{2}\%$ of itself $= \frac{8}{81}$?

24. 15 A. 108 P. is $33\frac{1}{3}$ per cent. of how many acres of land? *Ans.* 47 A. 4 P.

25. 1 T. 1 cwt. 28 lb. is $16\frac{2}{3}$ per cent. more than what number? *Ans.* 18 cwt. 24 b.

26. A clerk spends 45% of his salary, and can thus save \$1100 a year; what is his salary? *Ans.* \$2000.

27. A bootblack earns \$5, which is 25% of the money he has already put in the savings-bank; how much had he in bank? *Ans.* \$20.

28. To pay a note for \$6750, I am obliged to draw $42\frac{6}{7}\%$ of my bank deposit; what did I have in bank? *Ans.* \$15,750.

29. The population of a city increased 30,000 in 10 years, which was a gain of 20%; what was the population at the beginning of the 10 years? *Ans.* 150,000.

30. A teacher spends 25% of his salary for board, 10% for clothes, 15% for books, and $12\frac{1}{2}\%$ for travelling expenses and incidentals; he saves \$375; what is his salary? *Ans.* \$1000.

31. A grain-dealer bought a quantity of flour, and sold 20% of it to Mr. Martin and $62\frac{1}{2}\%$ of the remainder to Mr. Mason; if he sold Mr. Mason 850 barrels, how much did he buy? *Ans.* 1700 barrels.

32. A lady drew $33\frac{1}{3}\%$ of her money from bank, and paid $37\frac{1}{2}\%$ of it for a house costing \$5700; how much money had she remaining in bank? *Ans.* \$30,400.

33. Mr. Green invested 80% of his money in railroad stock, but afterward sold 25% of his stock, and still had \$6000 invested; what was the whole amount of his money?

Ans. \$10,000.

CASE III.

353. Given the percentage or the proceeds and the base, to find the rate.

ORAL EXERCISES.

1. 15 is what per cent of 60?

SOLUTION.—60 is 100 per cent. of 60, and 15, which is $\frac{1}{4}$ of 60, is $\frac{1}{4}$ of 100 per cent., or 25 per cent. of 60.

What per cent.

2. Of 60 is 12?

3. Of 75 is 25?

4. Of \$16 are \$4?

5. Of 16 qt. are 3 qt.?

6. Of $\frac{3}{4}$ is $\frac{4}{5}$?

7. Of $\frac{3}{5}$ is $\frac{2}{3}$?

8. Of 75% is 15%?

9. Of 2.5% is 1.5%?

10. From a hogshead of ale, containing 80 gallons, 12 gallons leaked out; what per cent. was lost?

11. A base-ball club won 12 games and lost 8 games; what per cent. of its games did it win?

12. The standard for gold and silver coin in the United States is 9 parts pure to 1 of alloy; what % of pure metal is there?

13. A merchant having invested \$3000 finds, on withdrawing his money, that he has received \$3750; what % did he gain?

WRITTEN EXERCISES.

1. 24 is what per cent. of 96?

SOLUTION.—If 24 is some per cent. of 96, then 96 multiplied by *some rate* equals 24; if 96 multiplied by *some rate* equals 24, the *rate* equals 24 divided by 96, which is .25 or 25%.

OPERATION.
$96 \times \text{rate} = 24$
$\text{Rate} = 24 \div 96 = .25$

2. 280 yd. being increased by a certain per cent. of itself, equals 350 yd.; required the rate.

SOLUTION 1ST.	SOLUTION 2D.
$350 - 280 = 70 =$ the percentage.	$280 = 100\%$
Then, $280 \times \text{rate} = 70$	$1 = \frac{1}{280}$ of 100%
$\text{Rate} = 70 \div 280 = .25 = 25\%$	$70 = \frac{70}{280}$ or $\frac{1}{4}$ of $100\% = 25\%$

Rule.—I. *To find the rate, divide the percentage by the base.*

II. *To find the rate, divide the difference between the proceeds and base by the base.*

1. The rate may also be found by dividing the proceeds by the base and taking the difference between 1 and the quotient.
2. Teachers can use the method of analysis which they prefer.

What per cent. of

3. 450 is 90? *Ans.* 20%.	**7.** $\frac{5}{6}$ is $\frac{3}{4}$? *Ans.* 90%.
4. 840 is 210? *Ans.* 25%.	**8.** $\frac{8}{9}$ is $\frac{2}{5}$? *Ans.* 45%.
5. 900 is 360? *Ans.* 40%.	**9.** 64% is $5\frac{1}{3}$%? *Ans.* $8\frac{1}{3}$%.
6. 960 is 512? *Ans.* $53\frac{1}{3}$%.	**10.** 4.5% is $3\frac{3}{8}$%? *Ans.* 75%.

11. In a school with 250 pupils registered the average attendance is 175; what is the per cent. of attendance?
Ans. 70%.

12. If a miller takes 8 quarts of every bushel he grinds for toll, what per cent. does he take for toll? *Ans.* 25%.

13. A man walked 25 miles in a week, and rode 75 miles; what per cent. of the entire distance did he walk? *Ans.* 25%.

14. A merchant bought 468 yd. of cloth and sold 312 yd.; what per cent. of his cloth did he sell? *Ans.* $66\frac{2}{3}$%.

15. If the base is given as $28.20, and the percentage $4.70, what is the rate per cent.? *Ans.* $16\frac{2}{3}$%.

16. The gold dollar weighs 25.8 gr., and the alloy in it weighs 2.58 gr.; what % of the coin is alloy? *Ans.* 10%.

17. A cask containing 28 gal. 3 qt. lost 5 gal. 3 qt. by leakage; what per cent. leaked out? *Ans.* 20%.

18. A house worth $5000 rents for $600 a year; what per cent. of the value is the rent? *Ans.* 12%.

19. A regiment lost in a campaign 300 men out of 945; what was the per cent. of loss? *Ans.* $31\frac{47}{63}$%.

20. A merchant's liabilities are $15,760, and his assets $7289; what % of his debts can he pay? *Ans.* $46\frac{1}{4}$%.

21. A farmer sold 24 acres of land, which was 20% of what he had left; what per cent. of his farm did he sell? *Ans.* $16\frac{2}{3}$%.

22. An ice-dealer stored 7500 tons of ice, and after losing 15% of it he sold 60% of the remainder; what per cent. of the whole remained? *Ans.* 34%.

23. A lady put $864 in a savings-bank, which was 25% of all her money, and afterward deposited 45% of the rest of her money; what per cent. of all her money had she then in bank? *Ans.* $58\frac{3}{4}$%.

24. 25 per cent. of a regiment being sick, only 720 men were able to enter battle, of whom $\frac{1}{6}$ were killed; how many did the regiment number, and what per cent. of the whole number were killed? *Ans.* 960 men; $12\frac{1}{2}$%.

GENERAL FORMULAS.

354. These methods and rules may all be represented in general formulas as follows:

CASE I.	CASE II.
1. Base × rate = Percentage.	1. Percentage ÷ rate = base.
2. Base × (1 + rate) = Amount.	2. Amount ÷ (1 + rate) = base.
3. Base × (1 − rate) = Difference.	3. Difference ÷ (1 − rate) = base.

CASE III.

Percentage ÷ base = rate.
Amount ÷ base = 1 + rate.
Difference ÷ base = 1 − rate.

NOTE.—These formulas apply to all the cases in the practical applications, and may be used instead of the rules, or with them, as the teacher prefers.

APPLICATIONS OF PERCENTAGE.

355. The **Applications of Percentage** are extensive, owing to the great convenience of reckoning by the hundred in business transactions.

356. These Applications of Percentage are of two classes; those not involving time and those involving time. The following are the most important of these applications:

1st Class.	2d Class.
1. Profit and Loss.	1. Simple Interest.
2. Commission.	2. Annual Interest.
3. Stocks, Dividends, etc.	3. Promissory Notes.
4. Premium and Discount.	4. Partial Payments.
5. Brokerage.	5. True Discount.
6. Stock Investments.	6. Discounting and Banking.
7. Taxes.	7. Exchange.
8. Duties or Customs.	8. Compound Interest.
9. Insurance.	9. Annuities.

TRADE DISCOUNTS.

357. A **Trade Discount** is a deduction from the price of goods, the amount of a bill, etc.

358. The **List Price** of goods is the fixed or published price. The **Net Price** is the list price minus the discount.

There are often several successive discounts deducted from a bill. Thus, when a bill is marked "20 and 10% off," it means that 20% is to be deducted from the bill, and 10% from the remainder.

Sometimes one or more of the discounts are expressed as fractions; thus, "$\frac{1}{8}$ and 10% off."

Fixed price lists are printed in certain kinds of business, and the rates of discount are changed as the market prices vary.

WRITTEN EXERCISES.

1. A merchant sold a bill of goods amounting to $600, and marked it "10 and 5% off;" what was the net amount of the bill?

SOLUTION.—10% of $600 is $60, which taken from $600 leaves $540; and 5% of $540 is $27, which taken from $540 leaves $513.

OPERATION.

10% of $600 = $60
$600 − $60 = $540
5% of $540 = $27
$540 − $27 = $513

2. I sold a lot of envelopes marked $7.50 ℔ M., at 10 and 10% off; what was the price received? *Ans.* $6.075.

3. Some valentines marked $15 were sold at 25, 20, and 10% off; what was the selling price? *Ans.* $8.10.

4. Bought 2 doz. cast-steel hammers at 32.06\frac{1}{4}$ a dozen, at 25 and 5% off; what was the "list" price? *Ans.* $45.

5. Sold a lot of carriage-bolts @ $2.91 ℔ C., which was 10 and 5% on; what was their cost? *Ans.* $2.52.

6. I bought a quantity of Christmas cards, wholesale, at a discount of 50, 50, and 25%; what was the rate of discount? *Ans.* 81$\frac{1}{4}$%.

7. What deduction from price is 10 and 15% off? What deduction is 5, 10, and 15% off? *Ans.* 23$\frac{1}{2}$%; 27$\frac{13}{40}$%.

8. What is the difference between 10% on and 5 and 5% on? between 20 and 10% on and 20 and 10% off? *Ans.* $\frac{1}{4}$%; 60%.

9. Bought a bill of envelopes amounting to $75.20 on 30 days' credit at $\frac{1}{8}$ and 5% off, and 2% off for cash; what did I pay for them? *Ans.* $61.26.

10. Bought a bill of valentines amounting to $175.50 on 3 mo. credit at $\frac{1}{8}$, 10, and 5% off, and 2% for cash; what was the net cash amount of the bill? *Ans.* $122.54.

11. What is the difference in a bill of $750 between a discount of 30 off and a discount of 20 and 10% off? *Ans.* $15.

12. What is the difference between a discount of 40% and 10% taken 4 times? Between 40% and 20% taken twice? *Ans.* 5.61%; 4%.

PROFIT AND LOSS.

359. **Profit and Loss** are terms which denote the gain or loss in business transactions.

360. The **Quantities** considered are as follows:

1. The **Cost**, which is the *base.*

2. The **Rate** of profit or loss.

3. The **Profit** or **Loss**, which is the *percentage.*

4. The **Selling Price**, which is the *amount* or *difference.*

In marking goods it is customary to take one or more words or a phrase or sentence, consisting of ten different letters, and let each letter in succession represent one of the Arabic figures. The prices marked thus can be read only by those who have the key.

CASE I.

361. Given the cost and rate of profit or loss, to find the profit or loss or the selling price.

ORAL EXERCISES.

1. A lady paid $8 for a dress, and sold it at a loss of 25%; required the loss.

Solution.—At a loss of 25 per cent., $\frac{25}{100}$, or $\frac{1}{4}$, of the cost equals the loss. $\frac{1}{4}$ of $8 is $2. Therefore, etc.

2. A grocer bought coffee at 60 cents a pound, and sold it so as to gain 20%; what was the gain?

3. James paid $75 for a bicycle, but afterward sold it at a loss of 20%; for what did he sell it?

4. A lady bought a velvet coat for $90, and sold it at a gain of $16\frac{2}{3}$%; what was her gain?

5. If I buy Henrietta cloth at 40¢ a yard and sell it at an advance of $12\frac{1}{2}$%, what is my selling price?

6. A sofa was sold at auction for $20, and, having been repaired at a cost of 20%, was sold at an advance of $33\frac{1}{3}$% on its whole cost; what was the gain?

WRITTEN EXERCISES.

1. A house was bought for $5700, and sold at a gain of 12%; what was the gain?

Solution.—If the house was bought for $5700, and sold at a gain of 12%, the gain was .12 times $5700, which is $684.

OPERATION.

$$\begin{array}{r} \$5700 \\ .12 \\ \hline \$684.00 \end{array}$$

Rule.—*To find the profit or loss, multiply the cost by the rate.*

The selling price equals the cost multiplied by 1 plus the rate of profit, or by 1 minus the rate of loss.

2. A grain-dealer bought wheat at 65¢ a bushel, and sold it at a gain of $7\frac{1}{2}$%; what was the selling price? *Ans.* $69\frac{7}{8}$¢.

3. If a coal-dealer buys coal at $4.75 a ton and sells at 18% advance, what is his gain? *Ans.* $.85\frac{1}{2}$.

4. Paid $520 for a lot of dress-goods, and sold them at a gain of $15\frac{1}{2}$%; what did I receive for the goods? *Ans.* $600.60.

5. Mr. Wood weighed 175 lb., but by a spell of sickness he lost $14\frac{2}{7}$% of his weight; what is his weight? *Ans.* 150 lb.

6. A dealer in musical instruments buys pianos at 35% off, and sells at a gain of 30%; what does he get for a piano whose list price is $750? *Ans.* $633.75.

7. A train of cars was running 25 miles an hour, but, wish-

ing to make up lost time, the engineer increased the speed 25%; what was the rate then run? *Ans.* $31\frac{1}{4}$ miles.

8. A drover bought 80 cows at $20 a head; if 10 were killed by an accident, for how much must he sell the remainder per head to gain 25%? *Ans.* $$28\frac{4}{7}$.

9. A merchant bought 64 yd. of French chally at 45¢ a yard, and sold it at a gain of $33\frac{1}{3}$%; what did he gain by the transaction? *Ans.* $9.60.

10. I bought 60 yards of silesia @ 15¢, and marked it at a profit of 20%; what will be my profit if I sell $11\frac{1}{9}$% lower than the selling mark? *Ans.* $6\frac{2}{3}$%.

11. A gentleman bought a yacht for $4500, sold it at a gain of 25%, and the buyer sold it at a loss of 20%; what did the latter receive? *Ans.* $4500.

12. At the close of the season a merchant marks down a certain class of goods $12\frac{1}{2}$%; what will be the marks of those selling at 16¢, 20¢, 25¢, $37\frac{1}{2}$¢, and 40¢, taking the nearest half-cent? *Ans.* 14¢, $17\frac{1}{2}$¢, 22¢, 33¢, 35¢.

13. The firm of Knox & Irwin use as their key for marking goods, "Samuel Knox;" if they buy cashmere at 50¢ a yard, how must it be marked to sell at a gain of 25%? *Ans.* la $\frac{s}{a}$.

14. Mr. Colton had as his key "don't give up;" if he buys silk at $1.25 a yard, how must he mark it to fall 10% and yet gain 8%? *Ans.* d. gp.

CASE II.

362. Given the rate and the profit or loss, or the selling price, to find the cost.

ORAL EXERCISES.

1. A grocer sold flour for 50¢ a barrel less than cost, and thereby lost 10%; what did it cost him?

Solution.—If he lost 10 per cent., then $\frac{10}{100}$, or $\frac{1}{10}$ of the cost, equals the loss, which equals 50 cents; if $\frac{1}{10}$ of the cost equals 50 cents, $\frac{10}{10}$, or the whole cost, equals 10 times 50 cents, or 5 dollars.

2. A merchant charged 50 cents more for a trunk than it cost him, and thereby gained 10%; what was the cost?

3. Mr. Thompson sold a horse for $91, and thereby lost 35%; what did the horse cost him?

4. A drover bought a cow for 20% less than its value, and sold it for 30% more than its value; what % did he gain?

5. A picture-dealer sold two pictures for $150 each; on one he gained 25%, and on the other he lost 25%; what was his gain or loss?

WRITTEN EXERCISES.

1. A man gained $50 on a watch by selling it at a gain of 20%; what did the watch cost?

SOLUTION.—At a gain of 20%, .20 times the *cost* equals the gain, which is $50; if the *cost* multiplied by .20 equals $50, the *cost* equals $50 divided by .20, or $250.

OPERATION.
Cost × .20 = $50
Cost = $50 ÷ .20 = $250

Or, If 20% of the cost = $50	Or, 20% = $\frac{1}{5}$
1% of the cost = $\frac{1}{20}$ of $50 = 2\frac{1}{2}$	$\frac{1}{5}$ of cost = $50
100% of the cost = 2\frac{1}{2}$ × 100 = $250	$\frac{5}{5}$ of cost = $250

Rule.—*To find the cost, divide the profit or loss by the rate.*

The cost equals the selling price divided by 1 plus the rate of profit, or by 1 minus the rate of loss.

2. I sold velvet at $2.75 a yard, and gained 10%; what did the velvet cost me? *Ans.* $2.50.

3. A furrier sold a damaged sealskin coat for $175, and lost 12$\frac{1}{2}$%; what was the cost? *Ans.* $200.

4. A drover lost 2$\frac{1}{2}$% of his cattle by disease and 3$\frac{1}{2}$% by accident; his whole loss was 42; how many had he at first? *Ans.* 700.

5. I sell ginghams @ 15¢ and make 20%, and another lot @ 12¢ and lose 20%; what was their cost? *Ans.* 12$\frac{1}{2}$¢; 15¢.

6. On silks marked $2.50 I can fall 10% on my price and make 12$\frac{1}{2}$% on the cost; what was the cost? *Ans.* $2.00.

7. A manufacturer's capital produces an income of $5120 in a year at a gain of 16%; what would be his income at a gain of 24%? *Ans.* $7680.

8. Prof. Allen sells his library for $1000 less than it cost him, and thereby loses 20%; what would he have received if he had sold it at an advance of 20%? *Ans.* $6000.

9. A case of dry goods was somewhat damaged when opened; I marked them 76¢ a yard, so that I can abate 25%, and yet lose only 5%; what did they cost me? *Ans.* 60¢.

10. A merchant, having some goods marked 22¢, marked them down $9\frac{1}{11}$%, and still had a profit of 25%; what did they cost him? *Ans.* 16¢.

11. Mr. Walcott's key for marking goods is "Charleston;" if he has satin marked *a.ln*, and gains at that $16\frac{2}{3}$%, what was the cost? *Ans.* $3.00.

12. Mr. Winslow offered his house for sale at an advance of 20%, but afterward sold it for $6300, which was $12\frac{1}{2}$% less than his original offer; what was the first cost of the house? *Ans.* $6000.

CASE III.

363. Given the cost and the profit or loss or the selling price, to find the rate.

ORAL EXERCISES.

1. A lady bought a piece of land for $120, and sold it at a gain of $15; what was her gain per cent.?

SOLUTION.—If on $120 she gained $15, on $1 she gained $\frac{1}{120}$ of $15, which is $\$\frac{15}{120}$, or $\$\frac{1}{8}$, and on $100 she would gain 100 times $\$\frac{1}{8}$, which are $\$\frac{100}{8}$, or $\$12\frac{1}{2}$; hence the gain is $12\frac{1}{2}$ per cent.

2. If a grocer buys flour @ $4, and sells it at a gain of $1.60, what is his gain per cent.?

3. Sold a lot which cost $700 for $525; what was the loss per cent.?

4. What % is gained by selling goods at *double* their cost? What % is lost by selling them at *one-half* their cost?

5. If a miller takes $3\frac{1}{3}$ quarts out of every bushel he grinds for toll, what % does he take for toll?

6. If goods are bought at 20% below market price, and sold at 16% above that price, what is the gain %?

7. I bought a lot of goods at 5% below current prices, and sold them at 5% above those prices; what was my gain %?

8. A coal-dealer sold some coal for $18, and thereby cleared $\frac{1}{3}$ of this money; what would he have lost % if he had sold it for $12?

WRITTEN EXERCISES.

1. A man bought a watch for $80, and sold it at a loss of $8; what was the loss per cent.?

SOLUTION.—Since $80, the *base*, multiplied by the *rate* of loss, equals $8, the rate must equal $8 divided by $80, which is .10, or 10%.

OPERATION.
$80 × rate = $8
Rate = $8 ÷ $80 = .10

Rule.— *To find the rate, divide the profit or loss by the cost.*

The rate equals the difference between the cost and the selling price, divided by the cost.

2. If milk is bought at 3¢ a quart and sold at 5¢, what is the gain %? *Ans.* $66\frac{2}{3}$%.

3. If a grocer buys coffee for 50¢ a lb. and sells @ 65¢, what does he gain %? *Ans.* 30%.

4. A merchant sold damaged goods costing 50¢ a yard for 35¢; what was the loss per cent.? *Ans.* 30%.

5. Bought a lot of goods for 20% below the market price, and sold them for 20% above market price; what % did I gain? *Ans.* 50%.

6. A stationer bought paper at $2.50 a ream, and sold it at 16¢ a quire; what % did he gain? *Ans.* 28%.

7. If I buy at 20 and 10% off, and sell for 20 and 10% on, what % do I gain? *Ans.* $83\frac{1}{3}$%.

8. Bought Easter cards at 25, 20, and 10% off, and sold for 10 and 5% on; what per cent. did I gain? *Ans.* $113\frac{8}{9}$%.

9. A real-estate dealer, having a tract of land, sold 20% to A, and 75% of the remainder to B; what % of the whole remained? *Ans.* 20%.

10. Mr. Sansom bought 60 shares of bank-stock for $6600, and sold 40 shares for what they all cost; what was the gain %? *Ans.* 50%.

11. Miss Hartman sold her piano for $450, clearing 20% of this amount; what would have been her gain % if she had received $30 more? *Ans.* $33\frac{1}{3}$%.

12. A merchant marked a lot of goods *u. bn*, his key being "Republican;" if he bought them @ $3.60, what was his gain % at the marked price? *Ans.* 25%.

13. I paid $150 for a watch, and asked such a price that after falling $20 I still made 20% on the cost; what % did I abate from the asking price? *Ans.* 10%.

COMMISSION.

364. **Commission** is a percentage paid to a person who transacts business for another.

365. The person who transacts business for another is called a *Commission Merchant, Agent,* or *Broker.*

366. The merchandise sent to an agent to be sold is called a *Consignment.*

367. The person who sends the merchandise is called the *Consignor;* the person to whom the merchandise is sent is called the *Consignee.*

368. The **Net Proceeds** is the sum left after the commission and charges have been deducted from the amount of a sale or collection.

369. The **Entire Cost** is the sum obtained by adding the commission and charges to the amount of a purchase.

370. The **Quantities** considered are: 1. The *amount sold, bought,* etc.; 2. The *Rate of Commission;* 3. The *Commission;* 4. The *Entire Cost* or *Net Proceeds.*

371. The Base in commission is the actual *amount of the sale, purchase, collection,* or *exchange.*

An agent residing at a great distance from his employer is often called a *correspondent;* the person for whom an agent does business is called the *principal.*

CASE I.

372. Given the base and rate, to find the commission, or the net proceeds, or the entire cost.

1. An agent sold goods to the amount of $7650, his rate of commission being $3\frac{1}{2}\%$; what was his commission?

SOLUTION.—At the rate of $3\frac{1}{2}\%$ the commission was $.03\frac{1}{2}$ times $7650, which equals $267.75.

OPERATION.

$$\begin{array}{r} \$7650 \\ .03\frac{1}{2} \\ \hline \$267.75 \end{array}$$

WRITTEN EXERCISES.

2. An agent sold goods to the amount of $8725; what was his commission at $2\frac{1}{2}\%$? *Ans.* $218.12½.

3. An agent collects $3840 on a commission of $3\frac{1}{2}\%$; how much should he remit to his employer? *Ans.* $3705.60.

4. An auctioneer sold a house at auction for $5784, and the furniture for $3250.85; what was his commission at $2\frac{1}{4}$ per cent.? *Ans.* $203.28.

5. An agent sold 350 acres of land at $25 an acre on a commission of 5%; required the commission and the amount paid over. *Ans.* $437.50; $8312.50.

6. What would be the net proceeds of a consignment of 500 bbl. of flour, @ $4.25, allowing $2\frac{1}{4}\%$ commission, and paying storage for 15 days at 5¢ a barrel? *Ans.* $2052.18¾.

7. A debt of $1728 is compromised for 85%; what is the collector's commission at $4\frac{1}{2}\%$, and what are the proceeds remitted? *Ans.* $1402.70.

8. A factor sells $6742 worth of woolen goods, charging $2\frac{1}{2}\%$ commission and $1\frac{1}{4}\%$ for insuring payment; what amount will he remit to his principal? *Ans.* $6489.17½.

9. A tax-collector had a warrant for $35,870, $15,000 of which he collected at 1½%, and the remainder at 2%; what was the amount of his fees? *Ans.* $642.40.

10. A college made a contract with an architect to erect a library-building to cost $50,000, and allowed him ¾% for his plans and specifications and 2¼% for superintendence; when the accounts were settled he claimed $1550; how much did he overcharge? *Ans.* $50.

11. A factor charges 5¾% when he guarantees payment, and 4% when he does not. He receives a consignment of cotton which he sells for $9725, but $325 of the amount turns out a bad debt; which rate of commission on this transaction would be the most profitable for the agent?

Ans. The second, by $154.81¼.

CASE II.

373. Given the rate and the commission, or the net proceeds, or the entire cost, to find the base.

1. An agent receives $302.75 commission for buying a house, at the rate of 3½%; what was the cost of the house?

SOLUTION.—At a commission of 3½%, .03½ times the cost of the house equals the commission, which is $302.75; hence, the *cost* equals $302.75 divided by .03½, which we find is $8650.

OPERATION.

$$\frac{\$302.75}{.03\frac{1}{2}} = \$8650$$

2. An agent receives $5535 to be invested in goods after retaining his commission, 2½%; required the amount invested.

SOLUTION.—The sum to be invested, increased by 2½% of itself, equals 1.02½ times the sum, which equals $5535. If 1.02½ times the *sum* equals $5535, the sum equals $5535 divided by 1.02½, which we find is $5400.

OPERATION.

$$\frac{\$5535}{1.02\frac{1}{2}} = \$5400$$

WRITTEN EXERCISES.

3. An agent received $275 for collecting rents; if his commission was 5%, what was the amount collected? *Ans.* $5500.

4. A cotton-factor received \$7210 to be invested in cotton after deducting his commission of 3%; how much was his commission? *Ans.* \$210.

5. An agent buys a quantity of leather, paying \$20 for cartage, and charging $1\frac{1}{4}$% commission; the entire cost was \$7613.75; what was the commission? *Ans.* \$93.75.

6. A consignment of cotton was sold on commission, the agent charging $1\frac{3}{4}$% for the sale and $2\frac{1}{2}$% for guaranteeing payment; the commission was \$286.87$\frac{1}{2}$; what was the amount received for the goods? *Ans.* \$6750.

CASE III.

374. Given the base and the commission, or the net proceeds, or the entire cost, to find the rate.

1. An agent collects \$6300, and his commission was \$84; required the rate of commission.

SOLUTION.—The commission, \$84, equals the sum collected, \$6300, multiplied by the *rate;* hence, the *rate* equals \$84 divided by \$6300, which we find is .01$\frac{1}{3}$ or $1\frac{1}{3}$%.

OPERATION.

$$\frac{\$84}{\$6300} = .01\tfrac{1}{3}$$

WRITTEN EXERCISES.

2. An agent in Kansas sold some land for me, and, retaining his commission, \$180, remitted me \$3820; what was the rate of commission? *Ans.* $4\frac{1}{2}$%.

3. My agent bought 50 horses, @ \$150, paid \$25 for keeping and \$80 for transportation; he drew on me for \$7867.50; what was the rate of commission? *Ans.* $3\frac{1}{2}$%.

4. I sold a consignment of flour through an agent for \$19,400; my commission was \$970, and I paid the agent \$582; what was the rate of commission of each? *Ans.* My com. 5%; agent's, 3%.

5. A commission-merchant sold a consignment of wheat for \$15,000; he charged \$225 for commission and \$375 for guaranteeing payment; what was the rate of commission and of guaranty? *Ans.* Com., $1\frac{1}{2}$%; guar., $2\frac{1}{2}$%.

STOCKS AND DIVIDENDS.

375. A **Company** is an association of individuals for the transaction of business.

376. A **Corporation** is a company regulated in its operations by a general law or a special charter.

377. The capital invested in the business of a company is called the **Stock.** The owners of stock are called *Stockholders.*

378. The stock is divided into equal parts called **Shares.** A share is usually $50 or $100.

379. An **Installment** is a sum required of stockholders as a payment on their subscription.

380. A **Dividend** is a sum paid to stockholders out of the gains of the company.

381. An **Assessment** is a sum required of stockholders to meet the expenditures or losses of the company.

1. *Bonds* are written or printed obligations to pay certain sums of money at or before a specified time.

2. *State* or *United States Bonds* are bonds of a State or of the United States, payable at some future time, with interest at a fixed rate.

3. *Stocks* is a general name applied to the scrip or bonds of a corporation and to government bonds and public securities.

4. *Certificates of Stock* are the papers issued by a corporation to its stockholders as evidence of the number of shares belonging to each respectively.

382. The **Base** upon which dividends and assessments are estimated is the original or par value of the stock.

383. The **Quantities** considered are as follows: 1. The *Stock;* 2. The *Rate;* 3. The *Dividend* or *Assessment.*

NOTE.—In order to make these business subjects intelligible, teachers should show pupils the business papers, illustrate how a company is formed, its stock issued, its dividends declared and paid, etc. The subject is simple when pupils have a clear idea of the real business transactions referred to in the examples.

CASE I.

384. Given the stock and rate of dividend or assessment, to find the dividend or assessment.

1. A owns 300 shares, at $50 each, of the stock of a bank which declares a dividend of 6%; what is his dividend?

OPERATION.

SOLUTION.—A's capital is $50 × 300 = $15,000, and his dividend equals $15,000 × .06 = $900.

$50 × 300 = $15,000
$15,000 × .06 = $900

Principle.—*The dividend or assessment equals the par value of the stock multiplied by the rate.*

NOTE.—It is often convenient to find the result by multiplying the dividend or assessment on one share by the number of shares.

WRITTEN EXERCISES.

2. What is the amount of a 5% dividend on 65 shares of stock, par $50? *Ans.* $162.50.

3. A lady owns 75 shares of Penna. R. R. stock, par $50; if the company declares a semi-annual dividend of 3%, what does she receive? *Ans.* $112.50.

4. Mr. Garrett has 85 shares, at $50, in the Reading R. R. Co., which, on account of reorganization, calls for an assessment of $4\frac{1}{2}$%; what does he pay? *Ans.* $191.25.

5. A telephone company whose stock is $750,000 declares a semi-annual dividend of 5%; what is the amount of the dividend? *Ans.* $37,500.

6. I have 80 shares ($50) of stock in a bank which declares a dividend of 4%; what is my dividend, and how many shares of stock would it buy at par? *Ans.* 3 shares and $10 surplus.

7. Mr. Stanton owns 120 shares in an Electric Traction Co., par $50, which declares an extra dividend of 10%, payable in stock; how many shares will he then own? *Ans.* 132 shares.

8. An electric lighting company, having $500,000 capital, pays a dividend of $33.75 on 15 shares ($50), reserving as a surplus $7500; what were the net earnings? *Ans.* $30,000.

CASE II.

385. Given the rate and the dividend or assessment, to find the stock.

1. A bank divides \$7500 among the stockholders, being the amount of a 4% dividend; required the whole amount of stock.

SOLUTION.—If \$7500 is 4% of the stock, then .04 times the *stock* equals \$7500; hence, the *stock* equals \$7500 divided by .04, which is \$187,500.

OPERATION.

$$\frac{\$7500}{.04} = \$187,500$$

Principle.—*The par value equals the dividend or assessment divided by the rate.*

WRITTEN EXERCISES.

2. I received \$875 from a $3\frac{1}{2}\%$ dividend; what is the face value of my stock? *Ans.* \$25,000.

3. I receive \$742 as my share of a 7% dividend; how many shares at \$50 each do I own? *Ans.* 212 shares.

4. I paid an assessment of \$350, at $3\frac{1}{2}\%$, on insurance stock; how many shares, \$50 each, had I? *Ans.* 200 shares.

5. A trolley company divides \$15,000 among its stockholders, being a 5% dividend; what is C's stock if he owns $\frac{1}{10}$ of the whole? *Ans.* \$30,000.

6. An electric lighting company whose annual expenses are \$5780, and gross earnings \$8280, pays a dividend of 5%; required the face value of the stock. *Ans.* \$50,000.

7. A lady received \$500, payable in stock, as a 10% dividend; how many shares had she at first, and how many has she now, shares \$100 each? *Ans.* 50; 55.

8. Mr. Allen received 4 shares and \$40 in money as a 6% dividend; how many shares, valued at \$50 each, did he then own? *Ans.* 84 shares.

9. I received a stock dividend of 20% in the Pennsylvania R. R., and I then had 108 shares; how many shares, @ \$50 each, had I at first? *Ans.* 90 shares.

10. I received two stock dividends in a mining company, the first of 8% and the second of 10%; I then owned 297 shares, at $25 a share; how many had I at first? *Ans.* 250 shares.

CASE III.

386. Given the stock and dividend or assessment, to find the rate.

1. A mining company, whose stock is $500,000, clears $37,500 in a year; what rate of dividend can it declare?

SOLUTION.—Since the dividend is some per cent. of the stock, the base, $500,000, multiplied by the *rate*, equals $37,500; hence, the *rate* equals $37,500 divided by $500,000, which equals .07½.

OPERATION.

$$\frac{\$37,500}{\$500,000} = .07\tfrac{1}{2}$$

WRITTEN EXERCISES.

2. A company whose stock is $150,000 requires an assessment of $3750; what was the rate? *Ans.* 2½%.

3. Mr. A owns 85 shares of railroad stock, @ $50, and receives a dividend of $233.75; what was the rate? *Ans.* 5½%.

4. Mr. Thomson owns 80 shares ($100) in a bank whose capital is $250,000; he receives a dividend of $480; what is the rate of dividend and the whole dividend? *Ans.* 6%; $15,000.

5. A life and trust company, having a capital of $1,500,000, clears $50,000; what is the largest integral rate of dividend they can declare, and what is the surplus? *Ans.* 3%; $5000 surplus.

6. After receiving a stock dividend I had 80 shares ($100), and $64 toward another share; if I had 72 shares at first, what was the rate of dividend? *Ans.* 12%.

7. A lady had 200 shares in a Colorado mining company ($25), and received a stock dividend of 10 shares one year, and the next year of 16 shares and $20; what were the rates of dividend? *Ans.* 5%; 8%.

PREMIUM AND DISCOUNT.

387. The **Par Value** of stock is the value named in the certificate, called the *nominal value* or *face*.

388. The **Real Value,** or *Market Value,* of stock is the amount for which it will sell.

389. Stock is **Above Par,** or at a *premium* or *advance,* when it sells for more than its nominal value.

390. Stock is **Below Par,** or at a *discount,* when it sells for less than its nominal value.

The stock of a company will generally be above par when the company is doing a lucrative business, and below par when it is doing a poor business. The bonds of a town, city, etc. vary according to the confidence in their security, the fluctuations of the money market, etc.

391. A **Draft,** *Check,* or *Bill of Exchange* is a written order for the payment of money at some definite place.

392. The **Base** upon which premium and discount are estimated is the *par value.*

393. The **Quantities** considered are four: 1. The *Par Value;* 2. The *Rate;* 3. The *Premium* or *Discount;* 4. The *Real Value.*

The problems under this subject are solved without brokerage—the sales and exchanges being regarded as direct without the aid of a broker.

CASE I.

394. Given the par value and the rate of premium or discount, to find the premium or discount or real value.

1. A broker bought 25 shares of stock ($100) at 6% premium; required the premium and cost or real value.

OPERATION.

$$
\begin{array}{rl}
\$100 \times 25 = \$2500 & \text{par value.} \\
\underline{.06} & \\
\$150.00 & \text{premium.} \\
\underline{2500.00} & \\
\$2650.00 & \text{real value.}
\end{array}
$$

SOLUTION.—The par value of 25 shares at $100 each is $100 × 25 = $2500, and the premium at 6% is .06 times $2500, which is $150, and this, added to the par value, equals $2650, the real value.

Principle.—*The premium or discount equals the par value multiplied by the rate.*

WRITTEN EXERCISES.

2. I sold 54 shares of gas-stock ($100) at 3% discount; required the discount and the real value. *Ans.* $162; $5238.

3. When gold was at a premium of 16%, how much in greenbacks was required to buy $640 in gold? *Ans.* $742.40.

4. What is the market value of 65 shares of N. J. Central ($100) at 8% below par? *Ans.* $5980.

5. A broker bought 45 shares ($100) N. Y. Central at ½% discount, and sold them at 2¼% premium; what did he gain in the transaction? *Ans.* $123.75.

6. Mr. Reed bought $2000 Phila. & Erie 5's at 17½% premium, and paid $2000 in drafts, at 1¼% discount, and the balance in cash; how much cash did he pay? *Ans.* $375.

7. A banker exchanged 80 shares ($50) Lehigh Valley R. R., at a discount of 3½%, for 30 shares ($100) N. Y. Car Trust, at 2¼% premium, receiving the difference in cash; how much cash did he receive? *Ans.* $792.50.

CASE II.

395. Given the rate and the premium or discount, or the real value, to find the par value.

1. A broker sold some drafts at a discount of 3¼%, receiving $130 less than their face; what was their face value?

SOLUTION.—If the discount at 3¼% is $130, then .03¼ times the *par value* equals $130; hence, the *par value* equals $130 divided by .03¼, which we find is $4000.

OPERATION.
$130 ÷ .03¼ = $4000

Principle.—*The par value equals the premium or discount divided by the rate.*

WRITTEN EXERCISES.

2. I sold 50 shares of stock at a premium of 5¾%, and received $287.50 advance; what was the par value of a share? *Ans.* $100.

3. A broker sold some stocks at $6\frac{3}{4}\%$ premium, receiving \$337.50 above par; what was their par value? *Ans.* \$5000.

4. The premium on a village bond at $1\frac{1}{4}\%$ was \$3.75; required the face of the bond. *Ans.* \$300.

5. If Pennsylvania State 4's are at 15% premium, what is the par value of the amount that can be bought for \$4600? *Ans.* \$4000.

6. A lady paid \$3067.50 for a New York State bond, at $2\frac{1}{4}\%$ premium; required its face value. *Ans.* \$3000.

7. My broker sold stock bought at par at a premium of $2\frac{1}{8}\%$, and received \$102 advance; how many shares (\$50) did he sell? *Ans.* 96 shares.

8. Required the face of a draft, at $\frac{1}{2}\%$ discount, which will buy 50 shares West Jersey R. R. (\$50), selling at 5% premium. *Ans.* \$2638.19.

CASE III.

396. Given the par value and the real value, or the premium or discount, to find the rate.

1. A lady sold a draft for \$750, at a premium of \$26.25; what was the rate of premium?

SOLUTION.—Since the premium equals the par value multiplied by the rate, \$750 multiplied by the *rate*, equals \$26.25; hence, the *rate* equals \$26.25 divided by \$750, which we find is .03½.

OPERATION.

$$\frac{\$26.25}{\$750} = .03\tfrac{1}{2}$$

Principle.—*The rate equals the premium or discount divided by the par value.*

WRITTEN EXERCISES.

2. A man bought a draft, face value \$2500, at a discount of \$12.50; required the rate of discount. *Ans.* $\frac{1}{2}\%$.

3. A broker bought 150 shares of railroad stock (\$50) for \$7050; what was the rate of discount? *Ans.* 6%.

4. If the 150 shares mentioned in the previous example are sold for \$7312.50, what is the rate of discount and also the rate of gain? *Ans.* $2\frac{1}{2}\%$; $3\frac{34}{47}\%$.

5. During the Civil War a broker gave a draft for \$6250, at 4% discount, for \$5000 in gold; what was the premium on gold? *Ans.* 20%.

6. A banker purchased a \$5000 bond, premium $12\frac{1}{2}\%$, and sold it for \$6000; for what premium did he sell the bond, and what was the gain %? *Ans.* 20% premium; $6\frac{2}{3}\%$ gain.

BROKERAGE.

397. **Brokerage** is a percentage charged by brokers for the transaction of business.

398. A **Broker** is a person who buys or sells money, stocks, bills of exchange, real estate, etc. for others.

A *stock broker* is one who deals in stocks; he is generally called simply a *broker.*

399. The **Rate** of brokerage is usually $\frac{1}{4}\%$, and will be so understood when no other rate is mentioned. In New York the rate is $\frac{1}{8}\%$ on both bonds and stocks.

400. The **Base** upon which the commission for the purchase and sale of bonds and stocks is estimated is their *par value.*

401. The **Quantities** considered are: 1. The *Par value of the amount bought or sold;* 2. The *Rate of Brokerage;* 3. The *Brokerage;* 4. The *Market value of the stock;* 5. The *Entire Cost* or *Net Proceeds.*

1. Stocks are quoted either at the price of one share or at the price of \$100 of par value of the stock, whatever be the par value of a share. The former method is used in Philadelphia; the latter in New York.

2. Bonds are often named from the rate of interest they draw; thus, we have 4's, $4\frac{1}{2}$'s, 5's, etc. The time to run or date when due sometimes gives the name; as, 4's of '97.

CASE I.

402. Given the par value, the rate, and the market value, to find the brokerage, net proceeds, or entire cost.

1. A broker bought for me 24 shares Un. Cos. N. J. ($100), rate of brokerage $\frac{1}{4}\%$; required the brokerage.

SOLUTION.—The par value was $24 \times \$100$, or $2400; the brokerage was $.00\frac{1}{4}$ times $2400, which equals $6.

OPERATION.

$$\$100 \times 24 = \$2400$$
$$\$2400 \times .00\tfrac{1}{4} = \$6$$

Principle.—*The brokerage equals the par value multiplied by the rate.*

NOTE.—It is often shorter to multiply the brokerage on one share by the number of shares. When the par is $50, one-half the rate should be used in this method.

WRITTEN EXERCISES.

2. A broker bought for me 75 shares of bank stock ($100) at par, brokerage $\frac{1}{4}\%$; required the brokerage and the cost of the stock. *Ans.* $18.75.

3. My broker bought for me 150 shares of canal stock ($50), at $44\frac{1}{4}$; what did the stock cost me, brokerage $\frac{1}{4}$ per cent.? *Ans.* $6656.25.

4. A speculator sold, through his broker, 72 shares Mine-hill R. R. ($50), at $66\frac{3}{8}$; required the brokerage and net proceeds. *Ans.* Proceeds, $4770.

5. My broker bought, on my account, 45 shares Lake Shore R. R. ($100), at 112, and sold them at 120; what was his commission at $\frac{1}{8}\%$ and my profit? *Ans.* Profit, $348.75.

6. Sold 45 shares Norristown R. R. stock ($50) at 105, and bought $4000 Lehigh Valley 6's at $111\frac{1}{2}$; what balance shall I have in hand from the transaction, including brokerage in both cases? *Ans.* 249.37\frac{1}{2}$.

7. What are the proceeds of the following stocks sold through a broker: 250 shares Texas & Pacific, at $50\frac{5}{8}$ ($100); 300 shares Morris & Essex ($50), at $60\frac{3}{4}$; and 200 shares Union Pacific ($100), at $117\frac{3}{8}$, brokerage $\frac{1}{8}\%$? *Ans.* $54281.25.

CASE II.

403. Given the rate, the brokerage, or the net proceeds, or entire cost, and the market value, to find the par value.

1. My broker charged \$75 for selling some drafts for me, at the rate of $1\frac{1}{2}\%$; what amount of drafts did he sell?

SOLUTION.—At a rate of $1\frac{1}{2}\%$, .015 times the par value of the drafts equals the brokerage, which is \$75; hence, the *par value* equals \$75 divided by .015, which we find is \$5000.

OPERATION.

$$\frac{\$75}{.015} = \$5000$$

Principle.—*The par value equals the brokerage divided by the rate.*

WRITTEN EXERCISES.

2. A broker charged \$25 at $\frac{1}{4}\%$ for buying Pennsylvania R. R. (\$50); how many shares did he buy? *Ans.* 200 shares.

3. I paid my broker \$2347.50 for an investment in Reading 4's, at 78, including his commission; what was the par value of the bonds? *Ans.* \$3000.

4. I sent a New York broker a draft on Brown Brothers for \$4631.25, to pay for Illinois Central (\$100) at $92\frac{1}{2}$, and his commission at $\frac{1}{8}\%$; how many shares did he buy? *Ans.* 50 shares.

5. Through my broker I sold \$7000 Penna. State 4's at 115, and invested the proceeds in Northern Central R. R. (\$50), at $69\frac{1}{2}$; how many shares did he buy, brokerage on each transaction $\frac{1}{4}\%$? *Ans.* 115 shares; $\$25.62\frac{1}{2}$ surplus.

CASE III.

404. Given the par value, and the brokerage, or the net proceeds, or entire cost, and the market value, to find the rate.

1. A broker bought City of Pittsburg 7's, par value \$8500; his charge was \$21.25; what was the rate of brokerage?

SOLUTION.—The brokerage, \$21.25, equals the par value, \$8500, multiplied by the *rate;* hence, the *rate* equals \$21.25 divided by \$8500, which we find is $.00\frac{1}{4}$, or $\frac{1}{4}\%$.

OPERATION.

$$\frac{\$21.25}{\$8500} = .00\tfrac{1}{4}$$

Principle.—*The rate equals the brokerage divided by the par value.*

WRITTEN EXERCISES.

2. A broker buys for me 75 shares Philadelphia Traction stock ($50); the brokerage was $9.37½; what was the rate of brokerage? *Ans.* ¼%.

3. My broker having purchased, according to order, 20 shares of United Cos. of New Jersey ($100), at 236½, notifies me that the entire cost is $4735; what is the rate of brokerage? *Ans.* ¼%.

4. I send a broker in New York $10,815, for which he buys 60 shares in the Equitable Gaslight Co., at 180 ($100), retaining the brokerage, and remitting a balance of $7.50; what is the rate? *Ans.* ⅛%.

INCOME FROM INVESTMENTS.

405. Investments in stocks may be made either for interest on the money or for gain in selling.

406. The **Several Classes** of stocks, bonds, etc. are those of *Corporations, States,* and the *General Government.*

407. Bonds are distinguished as *Registered* and *Coupon Bonds.* The *Registered* bonds are payable to order, and cannot be transferred without being indorsed.

408. The **Coupon** bonds have certificates of interest called coupons attached to them. These coupons may be cut off and presented for payment when the interest is due.

409. The principal bonds of the United States, called Government Bonds, are the following:

The 4's of 1907, which are 4% bonds due in 1907. These bonds are both coupon and registered, and the interest is payable quarterly.

The currency 6's, issued to aid in constructing several railroads to the Pacific. There are several series of these bonds, maturing respectively in 1895, 1896, 1897, 1898, and 1899.

Government Bonds are so secure that they are much sought after for investment, and thus command a premium. This premium usually becomes smaller each year as the bonds approach maturity.

410. A **Mortgage** is a conditional conveyance of property as security for the payment of a debt.

1. Mortgages are usually given to secure the payment of an interest-bearing note. Should the interest not be promptly paid, the mortgage may be *foreclosed*, and the property is then sold by the sheriff to the highest bidder, and the mortgage paid off from the proceeds.

2. Property is usually not mortgaged beyond a certain part of its value, in order that the mortgagee may be secure from loss. A second mortgage is sometimes given on a property, but second mortgages are not always a good security, since in case of foreclosure the first mortgage has a prior claim.

411. A **Ground-Rent** is a fixed rent paid for ground, generally used for building purposes.

It is a common practice in some cities, when a person wishes to build one or more houses, instead of *buying* the ground required, to agree to pay the interest of its value as rent, the contract to continue in force as long as the rent is regularly paid.

NOTE.—In changing from one investment to another there is often a little more realized from the sale of the first than will procure an exact number of shares of the second. In such cases the income will be calculated on the number of shares, without noticing the surplus.

412. The **Quantities** considered are: 1. The *Amount Invested;* 2. The *Rate of Dividend or Interest;* 3. The *Income;* 4. The *Market Value of* \$100, *or of one share;* 5. The *Rate of Income.*

CASE I.

413. Given the amount of an investment, the market value, and the rate of dividend or interest, to find the income.

1. A lady invests \$5895 in 6% bonds, at 98, brokerage $\frac{1}{4}$%; what will be her annual income from them?

SOLUTION.—Since for \$1 of bonds you pay $98\frac{1}{4}$ cents, for \$5895 you can buy as many dollars' worth of bonds as \$.$98\frac{1}{4}$ is contained times in \$5895, or \$6000. The annual income on the bonds = \$6000 × .06, which equals \$360.

OPERATION.

$\$5895 \div .98\frac{1}{4} = \$6000$

$\$6000 \times .06 = \$360$

WRITTEN EXERCISES.

2. I sent my broker $5662.50 to invest in City of Pittsburg 7's, at 113, brokerage $\frac{1}{4}\%$; what annual income shall I receive? *Ans.* $350.

3. I invested $2556.25 in Pennsylvania R. R. stock ($50), at 51, dividend $5\frac{1}{2}\%$, brokerage $\frac{1}{4}\%$; what was my annual income from the investment? *Ans.* $137.50.

4. Mr. Wilson invested $5237.50 in Reading 6's at $104\frac{1}{2}$, interest payable semi-annually, brokerage $\frac{1}{4}\%$; what will be his semi-annual income from the investment? *Ans.* $150.

5. Mr. Jenkins bought on ground-rent a lot 150 ft. front by 210 ft. deep, valued at $56.25 per foot front; what would be the ground-rent per foot front at 6%? *Ans.* $$3.37\frac{1}{2}$.

6. A conveyancer sold a lot 50 ft. front and 150 ft. deep on ground-rent, redeemable on payment of $3000; what is the ground-rent at 6% per annum? *Ans.* $180.

7. I have $8500 6% bonds selling at 111; will my income be greater or less if I sell them and buy 7's at 116, brokerage on both transactions $\frac{1}{4}\%$? *Ans.* Greater by $50.

CASE II.

414. Given the income, the rate of dividend, and the market value, to find the amount invested.

1. When 6% railroad bonds are selling at 116, how much must be invested to produce an income of $750?

SOLUTION.—Since $1 of stock gives an income of $.06, to give an income of $750 it will require $750 ÷ .06, or $12500; $12500 of stock at 116% will cost $12500 × 1.16, or $14500.

OPERATION.
$750 ÷ .06 = $12,500
$12500 × 1.16 = $14,500

WRITTEN EXERCISES.

2. What sum must I invest in 5% bonds, at $89\frac{1}{2}$, to secure an annual income of $800, brokerage $\frac{1}{4}\%$? *Ans.* $14,360.

3. How much must a lady invest in railroad 7's, at 129, to yield her $350 a year, brokerage $\frac{1}{4}\%$? *Ans.* $6462.50.

4. A conveyancer buys a 6% ground-rent of $450 per annum at par; what does it cost him? *Ans.* $7500.

5. A dwelling-house subject to a ground-rent of $225 at 6% was sold for $6250; what was its value? *Ans.* $10000.

6. What sum must be invested in Reading 5's, selling at $119\frac{1}{2}$, to secure an income of $800, brokerage $\frac{1}{4}\%$? *Ans.* $19160.

7. What must be the market value of 4% bonds to realize 5% on the investment? Of 7's to realize 5%? *Ans.* 80%; 140%.

8. A man sold $3500 Government 6's of '98 at 108, and bought sufficient Am. Steamship Co. 6's at $102\frac{1}{2}$ to yield $210 income; how much had he left, brokerage $\frac{1}{4}\%$ on sale and purchase? *Ans.* $175.

CASE III.

415. Given the market value and the income or rate of dividend, to find the rate of interest on the investment.

1. If I buy 5% bonds at 98, what per cent. of income shall I realize?

SOLUTION.—$1 of stock will cost $.98, and pays $.05; if on $.98 the gain is $.05, on $1 it is as many per cent. as .05 ÷ .98, or $5\frac{5}{49}\%$.

OPERATION.
$.05 \div .98 = .05\frac{5}{49}$

WRITTEN EXERCISES.

2. If a broker buys Lehigh Valley 6's at 124, what rate of income do they pay? *Ans.* $4\frac{26}{31}\%$.

3. When 7.3 bonds are selling at 123, what per cent. will these bonds yield? *Ans.* 5.93+ %.

4. A lady bought a ground-rent of $78 per annum for $1400; what % does she realize on her money? *Ans.* $5\frac{4}{7}\%$.

5. Which is the better investment, 5% bonds at $99\frac{1}{2}$, or 4% bonds at $75\frac{3}{8}$? 6's at 130, or 4's at 110? *Ans.* 4's; 6's.

6. Mr. Johnson bought Reading R. R. 7's at $115\frac{3}{8}$, and afterward exchanged them for Lehigh Navigation 6's at $109\frac{1}{2}$; which investment gives the better income? *Ans.* The 7's.

7. Miss Atherton, desiring a permanent investment, was recommended to take 5's at 108, 6's at 124, or 7's at 129; which gives the better income? *Ans.* The 7's.

GENERAL TAXES.

416. A **Tax** is a sum of money assessed on persons or property for public purposes.

417. A **Property Tax** is a tax upon property. Property is of two kinds—*Real Estate* and *Personal Property.*

418. **Real Estate** is immovable property; as, land, buildings, etc. *Personal Property* is movable property; as, money, stock, furniture, etc.

419. A **Poll Tax** is a fixed sum assessed on each person or citizen of a community.

420. A list or schedule containing the names of persons taxed, the valuation of their property, and the amount of their taxes is called an **Assessment Roll.**

421. The officer who appraises the property and prepares the assessment roll is called an *Assessor.*

422. The **Quantities** to be considered are: 1. The *Taxable Property;* 2. The *Rate of Taxation;* 3. The *Amount of Tax.*

1. Real estate is often assessed by the proper officer for not more than one-half or two-thirds of its real value. The value of personal property may be given in by the owner under oath, or, if he neglects to do this, it may be valued by the officer.

2. After the taxes have been assessed, each person receives a notice of his taxation, stating the day of appeal, when he may appear before the proper officers and show reasons for correcting any mistakes that have been made.

WRITTEN EXERCISES.

1. The taxable property of a town is $576,000, and the rate of taxation $.006 on a dollar; what is the tax?

SOLUTION.—If the tax is $.006 on $1, on $576,000 it will be .006 times $576,000, or $3456.

OPERATION.

$576,000 × .006 = $3456

2. The real estate of a town is valued at $875,600, and the personal estate at $1,054,896; and there are 450 persons subject to a poll tax of $1.50 each; what is the whole tax, the rate being 6 mills on a dollar? *Ans.* $12,257.976.

423. Table.—In the assessment of taxes in a town, city, etc. a table is usually constructed by which the labor of calculation is greatly facilitated.

424. The following table is based on the rate of $.015 to the dollar. This table is used in Problems 3, 4, 5, 8:

Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
$1	.015	$10	.15	$100	$1.50	$1000	$15	$10,000	$150
2	.030	20	.30	200	3.00	2000	30	20,000	300
3	.045	30	.45	300	4.50	3000	45	30,000	450
4	.060	40	.60	400	6.00	4000	60	40,000	600
5	.075	50	.75	500	7.50	5000	75	50,000	750
6	.09	60	.90	600	9.00	6000	90	60,000	900
7	.105	70	1.05	700	10.50	7000	105	70,000	1050
8	.12	80	1.20	800	12.00	8000	120	80,000	1200
9	.135	90	1.35	900	13.50	9000	135	90,000	1350

3. Find by the table A's tax, whose property is $9540, and who pays a poll tax of $1.25.

SOLUTION.—We find from the table the tax on $9000 is $135, on $500 is $7.50, and on $40 is $.60; under these write $1.25, the poll tax; the sum will be the entire tax.

OPERATION.

Tax on $9000	=	$135
" 500	=	7.50
" 40	=	.60
" 1 poll	=	1.25
Whole tax	=	$144.35

4. Find Mr. Wilson's tax, whose property is assessed at $7890, and who pays a poll tax of $1.25. *Ans.* $119.60.

5. My property is assessed at \$10,745, and my sister's at \$7536; what is the aggregate of our taxes? *Ans.* \$274.215.

6. What is the assessed value of property taxed \$87.50, at the rate of 5 mills on a dollar? *Ans.* \$17,500.

7. Mr. Nelson's entire tax is \$285; he pays a poll tax of \$1.25; the rate is 5 mills on the dollar; what is the valuation of his property? *Ans.* \$56,750.

8. Mr. Winthrop's property is assessed at \$17,000; he pays a poll tax of \$1.25 and .1½% on the income from his occupation, assessed at \$1000; what was his entire tax? *Ans.* \$257.75.

9. I have \$25,000 on interest, and my tax for money on interest is \$42, at 1½ mills on the dollar; for how much money at interest am I overtaxed? *Ans.* \$3000.

10. The expense of building a town-hall was \$10,750, which was raised by a tax on the property-holders of the town of 3¼ mills on the dollar; the collector's commission was 2¼%; what was the valuation of the property? *Ans.* \$3,383,828.447+.

NOTE.—The collector's commission is included in the tax.

11. I paid one year .35% township tax, .2½% county tax, .54% school tax, and \$1.50 poll tax; my whole tax was \$856.50; what was the value of my property? *Ans.* \$75,000.

12. Mr. Carter owns a house and lot worth \$45,000, the tax rate is 2¼%, and his tax bill \$675; what per cent. of the actual value of the property is the assessed value? *Ans.* 66⅔%.

13. A tax of \$14,250 is to be assessed on a town; the real estate is valued at \$1,200,000 and the personal property at \$750,000; there are 400 polls, each of which is taxed \$1.50; what is the rate of taxation? *Ans.* \$.007.

14. The trustees of a school expended for salary of teacher, \$600; fuel, \$54.25; apparatus, \$30.75. The school fund amounted to \$150, and the rest of the expenses was paid by a rate bill; if the entire attendance was 8320 days, what was A's bill, who sent three pupils 90 days each? *Ans.* \$17.36.

DUTIES OR CUSTOMS.

425. **Duties** are taxes levied by government upon imported goods. Duties are of two kinds, *ad valorem* and *specific*.

426. An **Ad Valorem** duty is a certain percentage assessed on the cost of the goods in the country from which they were imported.

427. A **Specific Duty** is a certain sum assessed on goods without regard to their cost.

428. **Tare** is an allowance for the weight of the box, cask, or covering containing the goods. Allowance is also made for *leakage* and *breakage*.

429. **Gross Weight** or **Value** is the weight or value of the goods before any deductions have been made.

430. **Net Weight** or **Value** is the weight or value of the goods after all allowances have been deducted.

431. An **Invoice**, or a *Manifest*, is an inventory or list of the goods and the prices at which they were purchased.

1. By the present tariff most duties of the United States are *ad valorem*, but some duties are specific, and some articles are charged both a specific and an *ad valorem* duty. The duty is reckoned on the actual cost at the place of purchase or manufacture, increased by all charges for transportation previous to final shipment.

2. All merchandise imported from foreign ports or places must be consigned in the manifest, invoice, or bill of lading to some person or firm at the port of importation, by whom it must be duly entered—either for immediate consumption or for warehouse.

3. Merchandise not intended for immediate consumption may be deposited in U. S. bonded warehouses, and remain there not longer than three years, the owner being at liberty to withdraw it at any time upon payment of the duties and charges for storage.

4. In custom-house business the *long* ton of 2240 lb., cwt. of 112 lb., and qr. of 28 lb. are used.

WRITTEN EXERCISES.

1. What is the specific duty, at $2\frac{1}{2}$¢ a lb., on 85 boxes of raisins, each containing 24 lb., tare $5\frac{1}{4}$ lb. a box?

SOLUTION.—We find the number of pounds to be 2040; multiplying $5\frac{1}{4}$ lb., the tare on 1 box, by 85, we have the whole tare, $446\frac{1}{4}$ lb., which, subtracted from 2040 lb., leaves the net weight, $1593\frac{3}{4}$ lb. The duty on 1594 lb. (calling the $\frac{3}{4}$ lb. 1 lb.) is $39.85.

OPERATION.

$85 \times 24 = 2040$ lb.
$5\frac{1}{4} \times 85 = 446\frac{1}{4}$ lb. tare
$2040 - 446\frac{1}{4} = 1593\frac{3}{4}$ lb.
$1594 \times 2\frac{1}{2}¢ = \$39.85$

Rules.—I. *For ad valorem duties, multiply the cost of the goods by the rate of duty.*

II. *For specific duties, deduct first the allowances, and compute the duty on the remainder.*

NOTE.—In reckoning duties, whole dollars, pounds, gallons, etc. are used as the base, fractions less than $\frac{1}{2}$ being rejected, and more than $\frac{1}{2}$ being reckoned as 1. Duties are payable in gold.

2. What is the duty on 375 yards of English worsted, invoiced at $2.25 a yard, at 20% ad valorem? *Ans.* $168.80.

3. I received from Havre an invoice of 80 dozen bottles of champagne, costing $10.50 per dozen; what is the duty at $6 a dozen, leakage 5%? *Ans.* $456.

4. Bailey & Co. received an invoice of Brussels laces costing 3600 francs in Brussels, charges 141.64 francs; what was the duty at 30%? *Ans.* $216.60.

5. What is the duty, at 35% ad valorem, on a bale of linen containing 56 webs, each $37\frac{1}{2}$ yd. long, 30 inches wide, invoiced at 27¢ per square yard? *Ans.* $165.55.

6. Required the duty on 45 pieces of woolen bunting, each 38 yd. long, 42 in. wide, and costing 55¢ per yard, the rates of duty being 20¢ per square yard and 35% ad valorem? *Ans.* $728.35.

7. I imported 267 pieces of colored cotton goods, 30 inches wide, having 37 yards in each piece, @ 7¢; what will be the duty if there is a specific duty of $5\frac{1}{2}¢$ a square yard and an ad valorem duty of 20%? *Ans.* $$591.21\frac{1}{2}$.

PROPERTY INSURANCE.

432. Insurance is a contract of indemnity for loss or damage within a given time. It is of two kinds: *Property Insurance and Personal Insurance.*

433. Property Insurance is security against loss by fire, transportation, etc. Insuring anything is called "taking a risk."

434. Property Insurance includes: *Fire Insurance, Marine Insurance, Transit Insurance*, and *Stock Insurance.*

1. *Transit Insurance* is security against loss by transportation by land or by both land and water; *Stock Insurance* is indemnity for loss of cattle, etc.

2. *Personal Insurance* includes *Life Insurance, Accident Insurance*, and *Health Insurance.*

3. *Accident Insurance* is indemnity for casualties to travellers and others; *Health Insurance* secures a weekly allowance in case of sickness.

435. Fire Insurance is security against loss by fire; *Marine Insurance* is security against loss by navigation.

436. The **Insurer**, or *Underwriter*, is the party or company taking the risk. The *Insured* or *Assured* is the party protected.

1. The *Policy* is the written agreement or contract between the insurers and the insured.

2. The *Premium* is the sum charged for insurance; it is a certain rate per cent. of the amount insured.

3. The *Sum Covered* by insurance is the amount insured on a property.

4. The *Base* is the amount insured on a property. The *Rate* varies with the risk.

The *Rate* of insurance is quoted at so many cents on the $100, or as so much per cent. Policies are renewed annually or at stated periods, and the premium is paid in advance. Risks are usually *rated* per

annum. The rate for more than 1 year is determined by the following table:

The rate for			2 yr. is	$1\frac{1}{2}$	times the annual rate.				
"	"	"	3 "	" 2	"	"	"	"	
"	"	"	4 "	" $2\frac{1}{2}$	"	"	"	"	
"	"	"	5 "	" 3	"	"	"	"	
"	"	"	7 "	" $3\frac{1}{2}$	"	"	"	"	

1. Insurance is generally done by ***stock companies***. When an ***individual*** takes a risk, it is called an "out-door" business. A ***Mutual Insurance*** Company is one in which the profits and losses are shared by those who are insured.

2. To prevent fraud, companies will seldom insure the full value of property. In cases of loss the ***underwriters*** may either replace the property insured or pay its value. Only the amount of actual loss can be recovered, and often claims are ***adjusted*** for a part of the amount insured.

437. The Quantities considered are: 1. The *Amount Insured;* 2. The *Rate of Insurance;* 3. The *Premium;* 4. The *Valuation of the Property.*

438. Given the amount insured and the rate, to find the premium.

1. I insured my house for $4000 at $1\frac{1}{4}\%$; required the amount of the premium.

SOLUTION.—The premium on $4000 at $1\frac{1}{4}\%$ is $.01\frac{1}{4}$ times $4000, which we find to be $50.

OPERATION.

$$\begin{array}{r} \$4000 \\ .01\frac{1}{4} \\ \hline \$50.00 \end{array}$$

Principle.—*The premium equals the amount insured multiplied by the rate.*

1. To find the amount insured, ***divide the premium by the rate;*** to find the rate, ***divide the premium by the amount insured.***

2. To find what amount must be insured to cover the premium in case of loss, ***divide the valuation of the property by 1 minus the rate.***

WRITTEN EXERCISES.

2. Mr. Smith insured a factory valued at $10,500 for $7000 at $1\frac{1}{4}\%$; what premium did he pay? *Ans.* $87.50.

3. I insured my house and furniture for $4500 for 5 years at $3\frac{3}{4}\%$; required the premium. *Ans.* $168.75.

4. Professor Sanford insured his house for \$5000, his library for \$1500, and his furniture for \$2000; what is the cost of insuring at $1\frac{1}{2}\%$, the policy costing \$1.50? *Ans.* \$129.

5. The premium paid for insuring $\frac{2}{3}$ of the value of a house for 3 years, at $2\frac{1}{2}\%$, was \$100; what was the value of the house? *Ans.* \$6000.

6. I insured my house, worth \$8000, for $\frac{3}{4}$ of its value at $2\frac{1}{4}\%$, for 5 years, so as to include the premium if burned; required the amount insured. *Ans.* \$6138.11.

7. Mr. Stone effected an insurance of \$12,000 on a hotel, paying \$91.25, including \$1.25, the cost of the policy; what was the rate of insurance? *Ans.* $\frac{3}{4}\%$.

8. Insured my house for \$6000 for 3 years at an annual rate of $\frac{7}{8}\%$; what premium did I pay? *Ans.* \$105.

Note.—For 3 years the premium would be 2 times the annual rate. See table under Art. 436.

9. Copeland & Co. paid $\$196.87\frac{1}{2}$ for a 7-year policy on their stable, the annual rate being $\frac{3}{4}\%$; what is the amount of the policy? *Ans.* \$7500.

10. I insured \$15,000 on my factory and \$10,000 on the machinery for a period of 5 years, paying \$658.75, including \$2.50, the cost of the policy; what would the premium be rated at annually? *Ans.* $\frac{7}{8}\%$.

11. Bingham & Hollis insured a vessel in the Ætna Insurance Co. against fire for \$80,000, at $\frac{3}{4}\%$, and also had a marine insurance of $\frac{1}{4}$ as much on the cargo, at $1\frac{3}{8}\%$; $\frac{1}{2}$ of the cargo was thrown overboard in a storm, and the vessel on her way home was destroyed by fire; what was the loss to the underwriters? *Ans.* \$89,125.

12. A store in Chicago valued at \$50,000, and a stock of goods worth \$75,000, were insured for $\frac{3}{4}$ of their value at $1\frac{1}{4}\%$; what was the loss of the owners, and what the loss to the underwriters, if the store and goods were destroyed in the great fire of 1872? *Ans.* $\$32,421.87\frac{1}{2}$; $\$92,578.12\frac{1}{2}$.

LIFE INSURANCE.

439. Life Insurance is a contract by which a company agrees to pay a certain sum of money to a person when he attains a certain age, or to his heirs at his death.

440. Life insurance companies are either **Stock** or **Mutual.**

1. In *Stock Companies* the capital is subscribed as in any other corporation, and all profits or losses are divided among the stockholders.
2. In *Mutual Companies* the profits or losses are divided among the insured. The profits may go to decrease the premiums or increase the policies.
3. In *Assessment Companies* no regular premium is paid, but whenever a loss occurs each person insured is assessed for his share of the loss.

441. The **Policies** of Life Insurance most frequently used are the following:

1. *Life Policies,* payable at the death of the insured, premium payable annually during life or in one, five, or ten annual payments.
2. *Endowment Policies,* payable to the insured at the end of a certain number of years, or to his heirs if he dies sooner, premium payable either annually during the continuance of the policy or in one or more annual payments.

NOTE.—Insurance companies use tables in their calculations showing the amount of premium that must be paid on $1000 at different ages. In the following examples the rates are taken directly from these tables.

WRITTEN EXERCISES.

1. At 40 years of age, what must I pay annually for a life policy of $5000, the premium being $31.50 per $1000?

SOLUTION.—The premium on $1000 is $31.50; hence, on $5000 it will be 5.000 times $31.50, which is $157.50.

OPERATION.

$\$31.50 \times 5.000 = \$157.50$

Principle.—*The premium equals the premium on* $1000 *multiplied by the amount of the policy divided by* 1000.

2. What is the annual premium on a life policy of $6000, the rate being $33.90 on $1000? *Ans.* $203.40.

3. Mr. Jones insured his life for $15,000, at an annual cost of $397.50; what was the rate on $1000? *Ans.* $26.50.

4. A lady insures her life for $8000, at an annual payment of $29.30 per $1000; if she lives 15 years, what amount will she have paid in premiums? *Ans.* $3516.

5. If a man whose life is insured for $20,000, at an annual premium of $24.80 per $1000, dies after making 8 payments, how much more than the amount of the premium will be paid to his heirs? *Ans.* $16,032.

6. Mr. Cornwell took out a policy in the N. E. Mutual Life Ins. Co. for $30,000, paying a premium of $22.70 per $1000; he made 20 payments, and 12% of the premium was returned as dividends; how much did the insurance actually cost him? *Ans.* $11,985.60.

7. Mr. Westlake took out an endowment policy for $10,000, payable in 20 years, the premium being $47.45 per $1000; if he lives to receive the endowment, will he have paid more or less than if he had taken a policy of the same amount 5 years later for 15 years at $67.15 per $1000? *Ans.* $582.50 less.

REVIEW PROBLEMS.

1. The list price of a lot of ginghams is 20¢ a yard; if I buy them at 20% off and sell them at 25% on, what is my gain per yard? *Ans.* 9¢.

2. Mr. Watson bought mining stock ($50) at 9% premium, and sold it at a loss of $5 on a share; at what rate was it sold? *Ans.* 1% disc.

3. Bought French merinos at $1.15 a yard, and marked them *cln*, my key being "Charleston;" what was my gain % if I sold the goods at the marked price? *Ans.* $30\frac{10}{23}$%.

4. An agent sold 150 barrels of flour, charging $2\frac{1}{2}$% commission and $2\frac{1}{4}$% guaranty; the net proceeds due the consignor were $1143; for how much was the flour sold per barrel? *Ans.* $8.

SIMPLE INTEREST.

442. **Interest** is money charged for the use of money for a certain time.

443. The **Principal** is the sum for which interest is charged. Interest is reckoned as a percentage of the principal.

444. The **Rate** of interest is the interest on $1 for a certain time. The usual time is one year.

445. The **Time** is the period during which the money is on interest.

446. The **Amount** is the sum of the principal and interest.

447. **Simple Interest** is interest on the principal only. **Compound Interest** is interest also on the interest.

448. **Legal Interest** is interest at the rate fixed by law. It varied in different States in 1895 as follows:

STATES.	RATE %.		STATES.	RATE %.		STATES.	RATE %.	
Alabama	8	8	Kentucky	6	6	North Dakota	7	12
Alaska	8	10	Louisiana	5	8	Ohio	6	8
Arizona	7	*	Maine	6	*	Oklahoma	7	12
Arkansas	6	10	Maryland	6	6	Oregon	8	10
California	7	*	Massachusetts	6	*	Pennsylvania	6	6
Colorado	8	*	Michigan	6	8	Rhode Island	6	*
Connecticut	6	6	Minnesota	7	10	South Carolina	7	8
Delaware	6	6	Mississippi	6	10	South Dakota	7	12
Dist. Columbia	6	10	Missouri	6	8	Tennessee	6	6
Florida	8	10	Montana	10	*	Texas	6	10
Georgia	7	8	Nebraska	7	10	Utah	8	*
Idaho	10	18	Nevada	7	*	Vermont	6	6
Illinois	5	7	New Hampshire	6	6	Virginia	6	6
Indiana	6	8	New Jersey	6	6	Washington	8	*
Indian Ter.	6	10	New Mexico	6	12	West Virginia	6	6
Iowa	6	8	New York	6	6	Wisconsin	6	10
Kansas	6	10	North Carolina	6	8	Wyoming	12	*

The first column gives the legal rate; second column, the rate that may be agreed upon; the * indicates no limit to the rate.

449. **Usury** is a rate of interest greater than the law allows. Various penalties are attached to taking usury.

450. The **Quantities** are five: 1. The *Principal;* 2. The *Interest;* 3. The *Rate;* 4. The *Time;* 5. The *Amount.*

1. In notes, contracts, accounts, mortgages, etc., when no rate is specified, the legal rate is understood.

2. In computing interest it is customary to reckon a month as $\frac{1}{12}$ of a year, and a day as $\frac{1}{30}$ of a month. In dealing with the U. S. Government each day is $\frac{1}{365}$ of a year.

THE METHOD FOR YEARS.

ORAL EXERCISES.

1. What is the interest of $60 for 2 yr. 6 mo. at 6%?

SOLUTION.—6 months equal $\frac{6}{12}$, or $\frac{1}{2}$ of a year, which with 2 yr. equals $2\frac{1}{2}$, or $\frac{5}{2}$ years. At 6 per cent. for 1 yr., $\frac{6}{100}$ of the principal equals the interest, and for $2\frac{1}{2}$, or $\frac{5}{2}$ yr., $\frac{5}{2}$ times $\frac{6}{100}$, or $\frac{15}{100}$ or $\frac{3}{20}$ of the principal, equals the interest; $\frac{3}{20}$ of $60 equals $9.

What is the interest of

2. $40 for 5 yr. at 5%?

3. $60 for 5 yr. at 4%?

4. $80 for 4 yr. at 5%?

5. $600 for 2 yr. 3 mo. at 8%?

6. $250 for 4 yr. 8 mo. at 6%?

7. $220 for 3 yr. 9 mo. at 8%?

8. $330 for 7 yr. 6 mo. at 4%?

9. $400 for 3 yr. 7 mo. 6 da. at 5%?

10. What is the amount of $50 for 3 yr. 6 mo. at 8 per cent.?

REMARK.—We find that $\frac{28}{100}$, or $\frac{7}{25}$ of the principal, equals the interest; hence, $\frac{32}{25}$ of the principal equals the amount; $\frac{32}{25}$ of $50 equals $64.

11. What is the amount of $300 for 7 yr. 10 mo. at 6 per cent.?

12. What is the amount of $400 for 5 yr. 4 mo. at 6 per cent.?

WRITTEN EXERCISES.

1. What is the interest of $3600 for 4 yr. 7 mo. 15 da. at 6%?

SOLUTION.—By reduction, we find that 4 yr. 7 mo. 15 da. equals $4\frac{5}{8}$ yr. At 6%, the interest for 1 year equals .06 times $3600, which is $216; if the interest for 1 year is $216, the interest for $4\frac{5}{8}$ yr. is $4\frac{5}{8}$ times $216, which, by multiplying, we find is $999. Hence the following

OPERATION.

$$\begin{array}{r} \$3600 \\ .06 \\ \hline \$216.00 \\ 4\frac{5}{8} \\ \hline \$999.00 \end{array}$$

Rule.—I. *To find the interest, multiply the principal by the rate, and that product by the time expressed in years.*

II. *To find the amount, add the interest to the principal.*

Required the interest

2. Of \$250 for 4 yr. 3 mo. at 6%. *Ans.* \$63.75.
3. Of \$680 for 5 yr. 4 mo. at 8%. *Ans.* \$290.13$\frac{1}{3}$.
4. Of \$860 for 5 yr. 9 mo. at 5%. *Ans.* \$247.25.
5. Of \$960 for 8 yr. 5 mo. at 7%. *Ans.* \$565.60.
6. Of \$590 for 3 yr. 10 mo. at 8%. *Ans.* \$180.93$\frac{1}{3}$.
7. Of \$1089 for 4 yr. 7 mo. 6 da. at 4%. *Ans.* \$200.376.
8. Of \$1040 for 3 yr. 3 mo. 9 da. at 5%. *Ans.* \$170.30.

THE SIX PER CENT. METHOD.

451. The **Six Per Cent. Method** is so called because the process is based upon that rate.

1. What is the interest of \$480 for 8 yr. 10 mo. 18 da. at 6%?

OPERATION.

$$\begin{array}{r} 8\times.06=\$0.48 \\ \frac{1}{2}\times 10= \quad .05 \\ \frac{1}{6}\times 18= \quad .003 \\ \hline \$0.533 \\ 480 \\ \hline \$255.84 \end{array}$$

SOLUTION.—The Int. of \$1 for 1 yr. is \$0.06, and for 8 yr. it is 8 times \$0.06, or \$0.48.

The Int. of \$1 for 1 mo., or $\frac{1}{12}$ of a year, is $\frac{1}{12}$ of 6¢, or $\frac{1}{2}$ of a cent, and the Int. for 10 mo. is 10 times $\frac{1}{2}$ of a cent, or \$0.05.

The Int. of \$1 for 1 mo., or 30 da., is $\frac{1}{2}$ of a cent, or 5 mills, and for 1 day it is $\frac{1}{30}$ of 5 mills, or $\frac{1}{6}$ of a mill, and for 18 days it is 18 times $\frac{1}{6}$ of a mill, or \$0.003.

Adding these results, we have \$0.533 as the Int. of \$1 for 8 yr. 10 mo. and 18 da., and on \$480 it is 480 times \$0.533, or \$255.84.

Rule.—I. *Multiply the rate, .06, by the number of years; take $\frac{1}{2}$ of the number of months as cents, and $\frac{1}{6}$ of the number of days as mills; their sum will be the interest of \$1 for the given time at 6%.*

II. *Multiply this sum by the principal, and the product will be the interest of the principal at 6%. For any other rate, take as many sixths of this interest as the rate is of six.*

1. Another method is to reduce the years to months, and take *half* the number of *months* for *cents*, etc., as before.

2. Another method is to take the *number* of *months* as *cents*, and *one-third* of the number of *days* as *mills*, and multiply the sum by *half the principal.*

3. The method for days, popularly expressed, is, "*Multiply dollars by days, and divide by 6000.*"

WRITTEN EXERCISES.

Required the interest of

2. \$450 for 4 yr. 8 mo. 12 da. at 6%. *Ans.* \$126.90.
3. \$996 for 6 yr. 4 mo. 15 da. at 6%. *Ans.* \$380.97.
4. \$765 for 8 yr. 7 mo. 6 da. at 6%. *Ans.* \$394.74.
5. \$84.75 for 8 yr. 9 mo. 24 da. at 7%. *Ans.* \$52.30+.
6. \$23.75 for 7 yr. 7 mo. 21 da. at 5%. *Ans.* \$9.07+.
7. \$.475 for 11 yr. 5 mo. 14 da. at 8%. *Ans.* \$.43½+
8. \$147.37½ for 4 yr. 11 mo. 13 da. at 7%. *Ans.* \$51.094
9. \$752.87½ for 9 yr. 8 mo. 10 da. at 9%. *Ans.* \$656.88+
10. \$387.18¾ for 10 yr. 7 mo. 7 da. at 10%. *Ans.* \$410.53–.
11. \$850.10 for 5 yr. 5 mo. 5 da. at 6¾%. *Ans.* \$311.61+.
12. \$890.83⅓ for 8 yr. 9 mo. 17 da. at 7½%. *Ans.* \$587.76+.

THE 60-DAY METHOD.

452. At 6% a year the rate for 2 mo., or 60 da., is 1%; hence, *for* 60 *da.,* $\frac{1}{100}$ *of the principal equals the interest.* From this we have the following method, called *the 60-Day Method.*

Rule.—*Point off two places in the principal for the interest for* 60 *days, and take multiples or aliquot parts of this interest for any other number of days.*

1. What is the interest of \$240 at 6% for 66 da.? for 96 da.?

SOLUTION.—Pointing off two places, we have \$2.40, the Int. for 60 da.; then take $\frac{1}{10}$ of \$2.40, we have \$0.24, the Int. for 6 da.; then the sum of these interests, or \$2.64, is the interest for 66 da.

OPERATION.

\$2.40	= Int. for	60 da.
.24	= " "	6 da.
\$2.64	= " "	66 da.

SOLUTION.—Pointing off two places, we have \$2.40, the Int. for 60 da.; take ½ of \$2.40 for the Int. of 30 da., and $\frac{1}{10}$ of \$2.40 for the Int. for 6 da.; their sum is the Int. for 96 da.

OPERATION.

\$2.40	= Int. for	60 da.
1.20	= " "	30 da.
.24	= " "	6 da.
\$3.84	= " "	96 da.

2. Find the Int. of \$720, at 6%, for 6 mo. 12 da. For 128 da.

OPERATION.

\$7	20	= Int. for 60 da.
21	60	= " " 6 mo. (3×2 mo.).
1	20	= " " 10 da. ($\frac{1}{6}$ of 60 da.).
	24	= " " 2 da. ($\frac{1}{5}$ of 10 da.).
\$23.04		= Int. for 6 mo. 12 da.

OPERATION.

\$7	20	= Int. for 60 da.
14	40	= " " 120 da.
	72	= " " 6 da.
	24	= " " 2 da.
\$15.36		= " " 128 da.

WRITTEN EXERCISES.

Required the interest

3. Of \$1800 for 6 mo. at 6%. *Ans.* \$54.
4. Of \$450 for 9 mo. at 6%. *Ans.* \$20.25.
5. Of \$750 for 3 mo. 10 da. at 6%. *Ans.* \$12.50.
6. Of \$1260 for 33 da. at 6%. *Ans.* \$6.93.
7. Of \$760 for 63 da. at 6%. *Ans.* \$7.98.
8. Of \$150 for 135 da. at 6%. *Ans.* \$3.37½.
9. Of \$720 for 5 mo. 22 da. at 6%. *Ans.* \$20.64.
10. Of \$1250 for 4 mo. 29 da. at 7%. *Ans.* \$36.21+.
11. Of \$325 for 1 yr. 7 mo. 12 da. at 7%. *Ans.* \$36.78—.
12. Of \$875 for 2 yr. 3 mo. 24 da. at 8%. *Ans.* \$162.17—.
13. Of \$621.50 for 3 yr. 7 mo. 15 da. at 5%. *Ans.* \$112.65—.
14. Of \$360.50 for 5 yr. 4 mo. 16 da. at 4½%. *Ans.* \$87.24+.
15. On a 3 mo. note for \$2400, dated March 21, 1894, at 6%, reckoning exact time.

EXACT TIME.

In March,	10 da.
In April,	30 da.
In May,	31 da.
In June,	21 da.
	92 da.

OPERATION.

\$24	00	= Int. for 60 da.
12	00	= " " 30 da.
	80	= " " 2 da.
\$36.80		

16. On a 3 mo. note for \$4500, dated Feb. 20, 1896, at 6%, reckoning exact time. *Ans.* \$67.50.

17. Of \$950, from April 14 to Sept. 21, at 6%, exact number of days. *Ans.* \$25.33⅓.

18. Of $480, from June 16 to Nov. 10, at 7%, exact number of days. *Ans.* $13.72.

19. On a note for $640, dated Oct. 9, 1894, and due Jan. 15, 1895, at 6%, allowing 3 days of grace. *Ans.* 10.77\frac{1}{3}$.

20. On a note for $1250, dated Nov. 16, 1894, and due Mar. 25, 1895, at 6%, allowing 3 days of grace. *Ans.* $27.50.

NOTE.—For Method by Cancellation, see Supplement.

METHOD OF EXACT INTEREST.

453. Exact Interest is interest obtained by reckoning 365 days to the year.

454. Exact Interest is reckoned by the United States Government, and is growing in favor with business-men.

455. Bankers and business-men often use *Interest Tables*, which are sometimes calculated to exact interest.

1. What is the exact interest of $875 from October 15 to February 12 at 7%?

SOLUTION.—From Oct. 15 to Feb. 12 there are 120 days; the interest of $875 for 1 year of 365 days, at 7%, is $61.25, and for 120 days it is $\frac{120}{365}$, or $\frac{24}{73}$ of $61.25, which is $20.14 –.

OPERATION.

$$\begin{array}{r} \$875 \\ .07 \\ \hline \$61.25 \\ 24 \\ \hline 73)\overline{1470.00} \\ \hline 20.13\frac{51}{73} \end{array}$$

Rule.—*Find the interest for the integral number of years; then multiply the interest for one year by the exact number of days and divide by* 365; *the sum of the two results will be the exact interest.*

NOTE.—The exact interest for any number of days less than one year may also be found by deducting from the common interest $\frac{1}{73}$ of itself.

WRITTEN EXERCISES.

2. What is the interest, at 4%, of $484.50, from Jan. 15th 1894, to July 18th, 1896? *Ans.* $48.58.

3. What is the amount of $540, on interest at 6%, from June 12th, 1887, to Oct. 30th, 1896? *Ans.* $844.03—.

4. A had \$1800 on interest from May 20th, 1892, to Sept. 5th, 1897; what was the interest at $5\frac{1}{2}\%$? *Ans.* \$524.29+.

5. Required the amount of \$1500.75, on interest at 5% from June 29th, 1890, to Jan. 11th, 1900. *Ans.* \$2216.38+.

6. Prove that $\frac{1}{73}$ deducted from common interest will give exact interest.

INTEREST ON PROMISSORY NOTES.

456. A **Promissory Note** is a written promise to pay a certain sum of money on demand or at a specified time.

457. The **Maker** of a note is the party who signs it.

458. The **Payee** is the party to whom it is made payable. The *Holder* is the one who owns it.

459. The **Face** of a note is the sum whose payment is promised.

The face is written in words in the body of the note to avoid error or fraud. The cents are usually written in figures as hundredths of a dollar.

FORMS OF NOTES.

\$450.75. NEW YORK, *Aug.* 10, 1895.

Two months after date, I promise to pay Mary Martin, or order, Four Hundred Fifty $\frac{75}{100}$ Dollars, for value received.

JAMES JOHNSON.

\$468.25. PHILADELPHIA, *Dec.* 21, 1894.

For value received, three months after date, I promise to pay Joseph Stanton, or order, Four Hundred Sixty-eight $\frac{25}{100}$ Dollars, without defalcation.

THOMAS RICHARDS.

460. In the above promissory notes let the pupil point out the *maker*, the *payee*, the *face*, the *date*, etc.

1. A note should contain the words "value received," otherwise the holder may be required to prove that value was received. In business language a note is said to be "made in favor of" the payee.

2. If a note reads "with interest," it draws interest from date; otherwise it draws interest from the time of maturity until paid. A note

may draw interest from a particular time after date if so specified in the note.

461. A **Negotiable Note** is a note that can be transferred from one party to another. A note not thus transferable is called a *Non-negotiable Note.*

1. A note is negotiable when it is made payable to the "bearer" or to the "order" of the payee. In order to transfer it to a second party the payee writes his name on the back of it, which is called *indorsing* the note. The person who writes his name on the back of a note as security for its payment is called the *Indorser.*

2. The words "without defalcation" are required in Pennsylvania to make a note negotiable; in New Jersey, "without defalcation or discount;" in Missouri, "negotiable and payable without defalcation or discount."

462. In many of the States a note *matures,* or is legally due, *three days* after the time specified in the note. These three days are called *Days of Grace.*

1. In several of the States, as New York, Pennsylvania, New Jersey, Illinois, etc., days of grace are abolished by law.

2. If a note falls due on Sunday or a legal holiday, it is to be paid in most States on the business-day preceding. In Pennsylvania it is to be paid the next business-day following.

3. When the time of a note is stated in months, calendar months are meant. A 4 mo. note, dated Oct. 15, would mature Feb. 15; but if dated Oct. 29, 30, or 31, it would mature on the last day of February.

463. A **Protest** is a written notice by a *notary public* to the indorsers that the note has not been paid.

1. A protest must be made out on the day the note matures, and be sent to the indorser immediately, to *hold him responsible.* In Pennsylvania a note is not to be protested on Saturday, but on the next business-day following.

2. The neglect to protest a note on maturity releases an indorser from all obligation to pay it unless the words "waiving demand and notice" appear above the indorser's signature.

464. The **Principal Kinds** of notes are the *Time Note, Joint Note,* and *Joint and Several Note.*

1. A *Time Note* is one made payable at a specified time; when no time of payment is specified, the note is due on *demand.* A *Joint Note* is a note signed by two or more persons, who are jointly liable for its

payment. A *Joint* and *Several Note* is a note signed by several persons, who are both jointly and singly liable for its payment.

2. In raising money on notes it is customary to have one or more responsible persons write their names on the back of the note as security for its payment. In case of the refusal of the maker to pay the note when due, each indorser is liable for the whole amount of the note in the order of signing, unless he writes above his name the words "without recourse," or unless there is an agreement between two or more indorsers to share the loss between them.

3. When the maker fails to pay a note, it is usual for the holder to make his demand on the last *liable* indorser, who pays the note and then gets the amount from the preceding indorser, and so on, up to the first indorser. The holder, however, has the option of collecting the amount from *any* liable indorser, and when so collected all *subsequent* indorsers are released, the indorser who pays becomes the holder, and may collect from any *prior* liable indorser, and so on up to the first.

WRITTEN EXERCISES.

1. TIME NOTE.

$375. PHILADELPHIA, *Nov.* 18, 1894.

Three months after date, I promise to pay William Stevens, or order, Three Hundred Seventy-five Dollars, with interest, for value received, without defalcation. EDWARD CARTER.

What will be due on this note at maturity?

EXACT TIME.	
Nov.	12 da.
Dec.	31 da.
Jan.	31 da.
Feb.	18 da.
Time,	92 da.

OPERATION.

3	75	= Int. for 60 da.
1	875	= " " 30 da.
	125	= " " 2 da.
	5.750	
	375	
	$380.75,	Amount due.

2. JOINT NOTE.

$800. NEW YORK, *Jan.* 7, 1895.

For value received, we promise to pay Philip Garret, or order, on demand, Eight Hundred Dollars, with interest, at 6 per cent. THOMAS CHURCHILL,
CHARLES CHASE.

What will be due on this note July 1, 1895? *Ans.* $823.33.

3. JOINT AND SEVERAL NOTE.

$640.75. St. Louis, Mo., *Feb.* 21, 1895.

Four months after date, we jointly and severally promise to pay George F. Bissell, or order, Six Hundred Forty $\frac{75}{100}$ *Dollars, with interest, for value received, negotiable and payable without defalcation or discount.* George Ellis,
Edward Preston.

What will be due on this note at maturity? *Ans.* $653.56.

4. COMPANY NOTE PAYABLE AT A BANK.

$560. New York, *March* 17, 1895.

Sixty days after date, for value received, we promise to pay James Ferris, or order, at the Chemical Bank, Five Hundred Sixty Dollars, with interest. Funk, Wagnalls & Co.

What will be due on this note at maturity? *Ans.* $565.60.

5. INDIVIDUAL NOTE PAYABLE AT A BANK.

$347.25. Newark, N. J., *Dec.* 29, 1895.

Six months after date, I promise to pay John Morris, or order, at the Third National Bank, Three Hundred Forty-seven $\frac{25}{100}$ *Dollars, with interest, for value received, without defalcation or discount.* George Andrews.

What will be due on this note at maturity? *Ans.* $357.84.

WRITING NOTES.

6. Write a negotiable note for $650.28, making yourself the payee and E. M. Hayward the maker; legal rate of interest.

7. Write a non-negotiable note for $412.65, making Charles Morton the payee and yourself the maker, payable on demand, with legal interest.

8. Write a note from the following data: Face, $500; negotiable; time, ninety days; payee, Alfred Foster; maker, Howard Truman; rate of interest, 6%; place, Lancaster, Pa.

PARTIAL PAYMENTS.

465. Partial Payments are payments in part of a note or other obligation.

The amounts and dates of the payments are usually written upon the back of the note or obligation. The person holding the obligation signs his name to this statement as a receipt.

466. An acknowledgment of a payment written on the back of the note or obligation, stating the amount and date of the payment, is called an *Indorsement.*

The term *Indorsement* is used in different business papers; in each case, however, it means a *writing on the back*, from the Latin *dorsum*, the back.

1. The writing of the name on the back of a check, draft, note, etc. is called a *General Indorsement*, or an *indorsement in blank.*

2. A *Special Indorsement* directs the obligation to be paid to some particular person or to his order.

467. The Supreme Court of the United States, and nearly all the States, adopt the following rule for partial payments, called

THE UNITED STATES RULE.

I. *Find the amount of the principal to the time of the first payment; if the payment equals or exceeds the interest, subtract the payment from the amount and treat the remainder as a new principal.*

II. *If the payment is less than the interest, find the amount of the same principal to the time when the sum of the payments shall equal or exceed the interest due, and subtract the sum of the payments from the amount.*

III. *Proceed in the same manner with the remaining payments until the time of settlement.*

1. This rule is founded upon the decision of Chancellor Kent. The principle is, that neither interest nor payment shall draw interest. It has been adopted by nearly all the States—New Hampshire, Vermont, and Connecticut being the principal exceptions.

2. The time between the dates is generally found by compound subtraction.

1. $500. NEWARK, N. J., *April* 10, 1893.

Two years after date, for value received, I promise to pay James Hill, or order, Five Hundred Dollars, with interest, without defalcation or discount. WILLIAM NEWHALL.

Indorsements: Jan. 1, 1894, $100; Nov. 15, 1894, $150.

How much was due April 10, 1895?

OPERATION.

Principal	$500.00
Interest to Jan. 1, 1894, 8 mo. 21 da.	21.75
Amount	521.75
First payment to be deducted	100.00
Remainder for new principal	421.75
Interest to Nov. 15, 1894, 10 mo. 14 da.	22.072
Amount	443.822
Second payment to be deducted	150.000
Remainder for new principal	293.822
Interest to April 10, 1895, 4 mo. 25 da.	7.101
Amount due at time of settlement	$300.923

2. $750. PHILADELPHIA, *May* 10, 1892.

Three years after date, for value received, I promise to pay Thomas Ellis, or order, Seven Hundred Fifty dollars, with interest, without defalcation. SAMUEL MIDDLETON.

Indorsements: Jan. 15, 1893, $124.75; Sept. 12, 1893, $20; Dec. 16, 1894, $216.80. How much remained due May 10, 1895?

OPERATION.

Principal		$750.000
Interest to Jan. 15, 1893, 8 mo. 5 da.		30.625
Amount		780.625
First payment to be deducted		124.75
Remainder for new principal		655.875
Interest to Sept. 12, 1893, 7 mo. 27 da.		25.907
Second payment less than interest due	$20.00	
Int. on $655.875 to Dec. 16, 1894, 1 yr. 3 mo. 4 da.		49.628
Amount		731.410
Third payment to be added to second	$216.80	236.800
Remainder for new principal		494.610
Interest to May 10, 1895, 4 mo. 24 da.		11.870
Amount due on settlement		$506.48

NOTE.—The interest may be calculated from the time of the first payment to that of the third, if preferred.

3. $600. CHICAGO, *Jan.* 11, 1892.

For value received, on demand, I promise to pay Philip Butler, or order, Six Hundred Dollars, with interest.

CHARLES MORTON.

Indorsements: Dec. 18, 1892, \$60; March 31, 1893, \$85; Sept. 21, 1893, \$67.20.

The note was paid March 1, 1894; what was then due? *Ans.* \$445.29.

NOTE.—The legal rate in Illinois is 5%; see Table, p. 283.

4. $3000. INDIANAPOLIS, IND., *Oct.* 20, 1890.

Three months after date, I promise to pay William Dennison, or order, Three Thousand Dollars, for value received, with interest from date. MORRIS PRITCHARD.

Indorsements: June 15, 1892, \$500; Aug. 18, 1893; \$750; Sept. 9, 1894, \$400.

How much was due Oct. 20, 1895? *Ans.* \$2120.08.

5. A note for \$4500 was given May 1, 1893, and was indorsed as follows: Oct. 17, 1893, \$250; Feb. 18, 1894, \$345; July 10, 1894, \$217.50; what was due May 1, 1895, interest at 7%? *Ans.* \$4280.06.

6. A note for \$7460 was given June 11, 1891; indorsed Dec. 8, 1891, \$200; March 25, 1893, \$500; June 15, 1894, \$225; Sept. 7, 1894, \$150; what was due at settlement, Jan. 1, 1895, interest 4%? *Ans.* \$7431.27.

MERCANTILE RULE.

468. Merchants often find the balance due on notes, payable within a year, on which partial payments have been made, by the following rule:

I. *Find the amount of the principal to the time of settlement, and also the amount of each payment to the time of settlement.*

II. *To find the balance due, subtract the sum of the amounts of the payments from the amount of the principal.*

1. As the periods in these notes are all short, the interest should be calculated for the exact number of days.

2. The method of EXACT INTEREST is sometimes used, but in the following problems 360 days are reckoned to a year.

WRITTEN EXERCISES.

1. $8500. HARRISBURG, *June* 18, 1893.

Thirty days after date, for value received, I promise to pay Henry Landis, or order, Eight Thousand Five Hundred Dollars, without defalcation. ANDREW HERR.

Indorsements: July 30, $150; Sept. 12, $300; Oct. 9, $500; Dec. 15, $700.

What is due June 15, 1894?

OPERATION.

Amount of

$8500 from July 18, 1893, to June 15, 1894,	332 da.		$8970.33
150 " " 30, " " " "	320 da.	$158.00	
300 " Sept. 12, " " " "	276 da.	313.80	
500 " Oct. 9, " " " "	249 da.	520.75	
700 " Dec. 15, " " " "	182 da.	721.23	1713.78
Balance due June 15, 1894,			$7256.55

NOTE.—This note does not begin to draw interest till July 18.

2. A note for $3500, given Oct. 9, 1892, has the following indorsements: Jan. 1, 1893, $200; Feb. 21, $300; April 17, $400; June 16, $350; Sept. 12, $500; what was due Oct. 1, 1893, at 5%? *Ans.* $1890.91.

3. A note for $4564.25, dated April 16, 1891, has the following indorsements: June 1, $345; Aug. 20, $742.50; Oct. 15, $324.25; Dec. 10, $600; Jan. 15, 1892, $560; what was due April 16, 1892? *Ans.* $2186.92.

4. A note for $6000, at 60 days, dated Aug. 26, 1894, has the following indorsements: Oct. 31, $200; Jan. 18, 1895, $400; April 20, 1895, $500; July 10, $150; what was due Aug. 1, 1895? *Ans.* $4998.73.

NOTES.—1. Notice in Ex. 3 that 1892 is a leap year.

2. For Connecticut, Vermont, and New Hampshire rules, see *Brooks's Higher Arithmetic.*

PROBLEMS IN SIMPLE INTEREST.

469. In **Simple Interest** there are three cases besides the finding of the interest on a sum of money. These will now be considered.

470. Given the time, the rate, and the interest or the amount, to find the principal.

ORAL EXERCISES.

1. What principal will in 4 yr. 2 mo., at 6%, give $70 interest?

SOLUTION.—We find that $\frac{1}{4}$ of the principal equals the interest, which is $70; if $\frac{1}{4}$ of the principal is $70, $\frac{4}{4}$, or the principal, equals 4 times $70, or $280.

What principal will give an interest of

2. $72 in 6 yr. at 4%?

3. $70 in 7 yr. at 5%?

4. $27 in 9 yr. at 6%?

5. $21 in 7 yr. 6 mo. at 7%?

6. $54 in 2 yr. 3 mo. at 8%?

7. $53 in 8 yr. 10 mo. at 6%?

8. How much money must a person borrow at 6% that the annual interest shall be $120?

9. How much money has William on interest, supposing he receives $480 for 5 yr. 4 mo. at 6%?

WRITTEN EXERCISES.

1. What principal will in 3 yr. 6 mo., at 6%, give $136.50 interest?

SOLUTION.—We find the interest of $1 for 3 yr. 6 mo., at 6%, is $.21. If $1 gives an interest of $.21, to give $136.50 interest it will require as many dollars as $.21 are contained times in $136.50, which is $650. Hence the following

OPERATION.

3 yr. 6 mo. = 42 mo.

$42 \div 2 = .21$

$$\frac{\$136.50}{.21} = \$650$$

Rule.—*Divide the given interest by the interest of* $1 *for the given rate and time, or divide the amount by the amount of* $1.

What principal will

2. Give $476 interest in 4 yr. 8 mo. at 6%? *Ans.* $1700.

3. Give $72.80 interest in 10 yr. 10 mo. at 7%? *Ans.* $96.

4. Amount to $1200.50 in 8 yr. 4 mo. at 8%? *Ans.* $720.30.

5. Give $575.40 in 5 yr. 8 mo. 15 da. at 5%? *Ans.* $2016.

6. Am't to $2299.75 in 7 yr. 7 mo. 12 da. at 7%? *Ans.* $1500.

7. Give $988.40 in 5 yr. 10 mo. 18 da. at 7%? *Ans.* $2400.

8. The sum of A's and B's money on interest for 3 yr. 7 mo. at 6%, gives $3440 interest; how much money has each, if 3 times B's equals A's? *Ans.* $12000; $4000.

471. Given the principal, the rate, and the interest or the amount, to find the time.

ORAL EXERCISES.

1. In what time will $400 at 6% give $144 interest?

SOLUTION.—At 6 per cent. for *one* year, $\frac{6}{100}$ of the principal equals the interest; $\frac{6}{100}$ of $400 is $24; if it require *one* year for $400 to gain $24, to gain $144 it will require as many years as $24 are contained times in $144, which are 6 years.

In what time will

2. $200 at 4% give $48 int.?
3. $150 at 5% give $30 int.?
4. $300 at 8% give $72 int.?
5. $60 at 7% give $21 int.?
6. $75 at 6% am't to $111?
7. $50 at 9% am't to $86?

8. In what time will a principal at 10% gain 2 times itself? 3 times itself? 4 times itself?

9. In what time will a principal double itself at 5%? at 6%? at 8%? at 12½%? Treble itself at 5%? at 10%? at 20%?

10. The amount of a principal for a certain time at 6% is $260, and for the same time at 9% is $290; required the principal and the time.

WRITTEN EXERCISES.

1. In what time will $650 give $136.50 interest at 6%?

SOLUTION.—The interest of $650, at 6%, for *one* year is $39. If in *one* year the principal gives $39 interest, to give $136.50 interest it will require as many times 1 year as $39 is contained times in $136.50, which is 3½ yr., or 3 yr. 6 mo. Hence we have the following

OPERATION.

$$\begin{array}{r} \$650 \\ .06 \\ \hline 39.00 \end{array} \text{ Int. 1 yr.}$$

$$\frac{136.50}{39.00} = 3\tfrac{1}{2}\text{ yr.} = 3\text{ yr. }6\text{ mo.}$$

Rule.—*Divide the given interest by the interest of the principal at the given rate for* ONE *year.*

NOTE.—When the amount is given, subtract the principal from the amount to find the interest, and then proceed as before.

In what time will

2. \$840, at 6%, give \$184.80 interest? *Ans.* 3 yr. 8 mo.

3. \$350, at 6%, amount to \$462? *Ans.* 5 yr. 4 mo.

4. \$1080, at 5%, give \$124.20 int.? *Ans.* 2 yr. 3 mo. 18 da.

5. \$37.50, at $7\frac{1}{2}$%, give \$13.50 int.? *Ans.* 4 yr. 9 mo. 18 da.

6. \$18.20, at $5\frac{3}{4}$%, give \$10.23 int.? *Ans.* 9 yr. 9 mo. 9 da.

7. \$1800, at $4\frac{1}{2}$%, am't to \$3047.40? *Ans.* 15 yr. 4 mo. 24 da.

8. The amount of a certain principal in a certain time, at 6%, is \$964.80, and the amount for the same time at 10% is \$1128; required the principal and the time.

Ans. Prin., \$720; Time, 5 yr. 8 mo.

SUG.—The difference of the amounts equals the interest at 4%.

9. A certain sum of money on interest amounts, at 8%, for a certain time, to \$1540, and at 12%, for the same time, to \$1810; required the principal and time.

Ans. Prin., \$1000; Time, 6 yr. 9 mo.

472. Given the principal, the time, and the interest or the amount, to find the rate.

ORAL EXERCISES.

1. At what rate will \$75 in 4 years give \$27 interest?

SOLUTION.—For 4 years at *one* per cent., $\frac{4}{100}$ or $\frac{1}{25}$ of the principal equals the interest; $\frac{1}{25}$ of \$75 equals \$3; if \$75 in 4 years at *one* per cent. gains \$3, to gain \$27 it will require as many times 1 per cent. as \$3 is contained times in \$27, which are 9 per cent.

At what per cent. will

2. \$60 in 5 yr. give \$18 int.?

3. \$50 in 6 yr. give \$15 int.?

4. \$80 in 7 yr. give \$28 int.?

5. \$12 in 5 yr. give \$15 am't?

6. \$90 in 5 yr. give \$117 am't?

7. \$50 in 3 yr. 8 mo. give \$61 am't?

8. At what rate will a principal gain 2 times itself in 30 years? 4 times itself? 5 times itself?

9. At what rate will a principal double itself in 10 years? in 12 yr.? in 20 yr.? in 25 yr.?

10. At what rate will a principal treble itself in 20 years? in 25 yr.? in 40 yr.? in 80 yr.?

11. The amount of a certain principal for 5 years at a certain rate per cent. is $390, and for 8 yr. is $444; required the principal and the rate per cent.

WRITTEN EXERCISES.

1. At what rate will $650 give $136.50 interest in 3 yr. 6 mo.?

SOLUTION.—We find the interest of $650 for 3 yr. 6 mo. at *one* per cent. is $22.75. If the principal in the given time, at *one* per cent., gives $22.75 interest, to give $136.50 interest it will require as many times 1 per cent. as $22.75 is contained times in $136.50, which is 6%. Hence we have the following

OPERATION.

$650
.01
6.50
$3\frac{1}{2}$
$22.75 = Int. at 1%
$136.50 ÷ $22.75 = 6

Rule.—*Divide the given interest by the interest of the principal for the given time at* ONE *per cent.*

NOTE.—When the amount is given, subtract the principal from the amount to find the interest, and proceed as before.

At what rate will

2. $280 in 4 yr. 6 mo. give $63 interest? *Ans.* 5%.

3. $864 in 5 yr. 4 mo. give $368.64 interest? *Ans.* 8%.

4. $87.60 in 4 yr. 3 mo. 12 da. give $22.5132 int.? *Ans.* 6%.

5. $3975 in 6 yr. 7 mo. 20 da. give 2375.06\frac{1}{4}$ int.? *Ans.* 9%.

6. $15.60 in 9 yr. 10 mo. 15 da. give $12.324 int.? *Ans.* 8%.

7. The amount of a certain principal for 7 yr. at a certain rate is $4470, and for 11 yr. at the same rate it is $5310; required the principal and the rate. *Ans.* $3000; 7%.

8. The amount of a certain principal for 6 yr. at a certain rate per cent. is $3775, and for 18 yr. $6325; required the principal and rate. *Ans.* $2500; 8$\frac{1}{2}$%.

DISCOUNT AND PRESENT WORTH.

473. Discount is an allowance made for the payment of money before it becomes due.

474. The **Present Worth** of a debt payable at a future time without interest is such a sum as, being on interest for the time at a certain rate, will amount to the debt.

475. The **True Discount** is the difference between the amount of the debt and the present worth.

1. The *true discount* is the *interest* on the present worth for the time between the payment of the debt and the time it becomes due.
2. The present worth corresponds to the principal, the discount to the interest, and the debt to the amount; hence the different cases may be solved as in Interest.

1. What is the present worth of $620, payable 4 years hence, money being worth 6%?

SOLUTION.—The amount of $1 for 4 years at 6% is $1.24; hence the present worth of $1.24 is $1, and the present worth of $620 is as many times $1 as $1.24 is contained times in $620, which is $500. Hence the following

OPERATION.
$\$0.06 \times 4 = \$0.24$
Amount $= \$1.24$
$\$620 \div 1.24 = \$500$

Rule.—I. *To find the present worth, divide the given sum or debt by the amount of $1 for the given rate and time.*

II. *To find the discount, subtract the present worth from the given sum or debt.*

NOTE.—When several payments are made without interest, find the present worth of each separately, and take their sum.

WRITTEN EXERCISES.

2. What is the present worth of $484, payable 3 yr. 6 mo. hence, money worth 6%? *Ans.* $400.

3. What is the discount of $829, payable 2 yr. 7 mo. 18 da. hence, money worth 4%? *Ans.* $79.

4. What is the present worth of $1000, payable in 9 mo., money worth 6%? 7%? 8%? *Ans.* $956.94; $950.12; $943.40.

5. Mr. Johnson bought goods for \$3000 on 3 mo. credit; what discount should he receive if he pays cash, money worth 6%? *Ans.* \$44.34.

6. Mr. Watson, wishing to sell his house, offers it for \$5720 on 3 years' credit, or \$5000 cash; which would be most profitable for the buyer, money worth 4%? *Ans.* The latter, \$107.14.

7. I gave a note for \$1500, payable in 3 yr. 3 mo., but at the end of 9 mo. I wish to pay it; what should the holder receive, money worth 5%? *Ans.* \$1333.33⅓.

BANK DISCOUNT.

476. A **Bank** is an incorporated institution, established for the purpose of receiving and loaning money or furnishing a paper circulation.

477. The money received by a bank for safe-keeping is called its *Deposits*, and the person placing money in a bank is called a *Depositor*.

478. A **Check** is an order on a bank, given by one of its depositors, to pay a certain amount to some person, or to his order, or to bearer.

479. Banks loan money on notes, mortgages, and other securities. Much of their business, however, consists in discounting notes, or paying them before they are due.

1. A person wishing to borrow money at a bank presents a note, either made or indorsed by himself, payable at a certain time, and receives for it a sum equal to its face, *less* the interest for the time the note has to run. This amount, called *discount*, is withheld by the bank in consideration of advancing money on the note prior to its maturity.

2. In Pennsylvania, Delaware, Maryland, Missouri, and the District of Columbia, the *day of discount* and *day of payment* are both reckoned. A 60-day note in Pennsylvania would be discounted for 61 days; in the others named (grace being allowed) for 64 days.

3. Business-men often discount notes by deducting the interest for a given time, with or without grace, as may be agreed upon. The rate is fixed by agreement, and may be other than the legal rate.

480. **Bank Discount** is the interest on the face of the note for the time from the day of discount to the day of payment.

481. The **Proceeds** or **Avails** of a note is the sum received for it when discounted, and equals the face (or amount) of the note less the discount.

482. The **Term of Discount** is the number of days from the time of discounting a note to the time of its maturity.

483. The difference between bank discount and true discount may be shown as follows:

If I take my note to the bank, promising to pay $106 at the end of 1 year, to get it cashed, by the method of true discount I would receive $100; but by the method of bank discount, not counting days of grace, I would receive $106 minus the interest of $106 for 1 year—that is, $106 − $6.36 = $99.64.

484. To find the discount, proceeds, or face from the other three elements.

1. What is the present worth or proceeds of a note for $450, due in 90 days, discounted at a bank at 6 per cent.?

SOLUTION.—The interest on $450 for 90 da., at 6%, is $6.75, which is the discount. Subtracting $6.75 from $450, we have $443.25, the proceeds.

OPERATION.

Int. on $1 = $.015
$450 × .015 = $6.75
$450 − $6.75 = $443.25

Rule.—I. *Find the interest on the face of the note for the time and rate, for the discount.*

II. *Subtract the discount from the face to find the present worth.*

1. The discount of an *interest-bearing* note is computed on the *amount* of the note at its maturity.

2. Banks compute interest for the *actual number of days* a note has to run, whether a note is drawn for months or days.

3. Days of grace are not reckoned in the following problems except in the 11th. In Exs. 10 and 11 both the day of discount and the day of payment are reckoned.

WRITTEN EXERCISES.

2. A note for $350, due in 90 days, was discounted at a bank at 6%; what was the discount? *Ans.* $5.25.

3. A note for $540, due in 60 days, was discounted at a bank, at 5%; required the proceeds. *Ans.* $535.50.

4. I wish to borrow $2000 from a bank for 90 days; for what must I give my note, discount 6%? *Ans.* $2030.46.

5. I presented a 60-da. note at a bank, which, discounted at 6%, netted me $5000; what was its face? *Ans.* $5050.50.

6. A broker buys a 60-da. note for $20 less than its face; what was the face, discount 6%? *Ans.* $2000.

7. A 3-mo. note for $600, dated June 1, was discounted July 10, at 5%; what was the discount? *Ans.* $4.42 —.

8. A 4-mo. note for $2000, dated Oct. 20, was discounted Dec. 16, at 8%; required the proceeds. *Ans.* $1970.67 —.

9. Find the difference between the true and the bank discount of $950 for 3 yr. 3 mo. at 6%. *Ans.* $30.23.

485. Find the results of the following notes:

10. $400. PHILADELPHIA, *Jan.* 5, 1895.

Ninety days after date, I promise to pay Charles Garrett, or order, Four Hundred Dollars, at the Girard Bank, for value received, without defalcation. JOHN WATERMAN.

Discounted, Jan. 10, at 6%.

Ans. April 5; 86 da.; dis., $5.73; proceeds, $394.27.

11. 354\frac{25}{100}$. WASHINGTON, D. C., *April* 3, 1895.

Six months after date, for value received, I promise to pay Edward Strong, or order, Three Hundred Fifty-four $\frac{25}{100}$ *Dollars, at the First National Bank.* JOHN MCPHERSON.

Discounted June 3, at 6%.

Ans. Oct. 3|6; 126 da.; dis., $7.44; proceeds, $346.81.

EXCHANGE.

486. Exchange is the method of making payments in distant places by means of *Drafts* or *Bills of Exchange.*

Suppose A of New York owes B of Chicago $800 for grain, and C of Chicago owes D of New York $800 for dry-goods. Then C of Chicago may go to B and pay him $800 for an order upon A, which he remits to D, and D collects the money from A. Both debts will thus be cancelled without sending any money, B receiving his $800 from C, and D his $800 from A.

The business of exchange is usually carried on by means of banks and bankers, who buy and sell drafts for any amount required. Exchange in the United States is mostly through the banks of the principal business centres.

487. A **Draft** or **Bill of Exchange** is a written order for the payment of money. In domestic exchange a bill is usually called a *Draft.*

1. The person who signs the order is called the *Maker* or *Drawer.*
2. The person requested to pay is called the *Drawee.*
3. The person to whom the money is to be paid is the *Payee.*
4. The person who has possession of the bill is the *Owner* or *Holder.*

488. The **Acceptance** of a draft is the promise of the Drawee, when presented, to pay it at maturity.

489. When the Drawee *accepts* the draft, he writes across the face, "Accepted," with the date and his signature; the bill is then called an *Acceptance,* and is of the character of a promissory note.

1. If a bill is protested for non-acceptance, the maker is under obligations to pay it immediately, although the time specified in it has not expired. When a bill reads "Acceptance waived," it is not subject to protest until maturity.

2. Bills of exchange are entitled to "days of grace" according to the custom of the place where the draft is payable, unless a particular day is named. In New York, Pennsylvania, etc. no grace is allowed, and in most States no grace is allowed on sight drafts.

490. There are two kinds of exchange—*Domestic Exchange* and *Foreign Exchange.*

DOMESTIC EXCHANGE.

491. **Domestic or Inland Exchange** is the exchange between two places in the same country.

492. The **Rate of Exchange** is the rate per cent. which is reckoned upon a draft.

The *Course of Exchange* is the current price paid in one place for bills of exchange upon another. The *brokerage* is usually included in the quotation of exchange.

493. Exchange is at *par* when a draft or bill sells for its face; at a *premium* when it sells for more than its face; and at a *discount* when it sells for less than its face.

1. The rate of exchange between two places depends upon the "course of trade." If the trade between New York and Chicago is equal, exchange is at par. If New York owes Chicago, the demand in New York for drafts on Chicago is greater than the demand in Chicago for drafts on New York, hence the drafts are at a *premium* in New York. But if Chicago owes New York, the demand for drafts is less in New York than in Chicago; hence drafts in New York on Chicago are at a *discount*.

2. The reason why the banks in New York should charge a premium is, that they must be at the expense of actually sending money to the Chicago banks or be charged with interest on their unpaid balance; the reason why New York banks will sell at a discount in the second case is that they are willing to sell for less than the face of a draft in order to get the money owed them in Chicago immediately.

494. A **Sight Draft** is one payable "at sight," or on its presentation. A *Time Draft* is one payable at a specified time after sight or date.

495. The **Forms** and **Use** of drafts may be seen by the following examples and explanations:

NATIONAL UNION BANK OF MARYLAND,
$5000. BALTIMORE, MD., *Jan.* 16, 1895.

At sight, pay to the order of Thomas Brown, Five Thousand Dollars, value received. WM. H. WELLS,

To the PHILADELPHIA NATIONAL BANK, *Cashier.*
PHILADELPHIA, PA.

EXPLANATION.—Suppose Thomas Brown of Baltimore owes John Farnum & Co. of Philadelphia \$5000; he goes into a bank in Baltimore and gets the above draft. He then writes on the back of the draft, "Pay to the order of John Farnum & Co.," signing his name, and forwards it to John Farnum & Co. in Philadelphia, who take it to the Philadelphia National Bank, and, writing the name of their firm on the back, receive the money.

MERCHANTS' NATIONAL BANK,
\$3400. CINCINNATI, O., *June* 20, 1891.

At ten days' sight, pay to the order of Hilton, Hughes & Co., Three Thousand Four Hundred Dollars, value received, and charge to account of C. A. STEVENS,
To the MERCHANTS' NATIONAL BANK, *Cashier.*
NEW YORK.

EXPLANATION.—Suppose that James Elliott of Cincinnati, wishing to pay a debt of \$3400 to Hilton, Hughes & Co. of New York, buys the above draft on the Merchants' National Bank of New York. He forwards it to Hilton, Hughes & Co., who, having indorsed it, will present it at the bank. The "ten days' sight" means ten days after acceptance. It should be presented to the bank upon which it is drawn as soon as received, when the cashier writes upon it "Accepted," with the date of acceptance, and signs his name as cashier. This makes the bank liable for its payment, and is an agreement to pay it after ten days.

496. To find the cost or the face of a bill of exchange at sight or on time.

1. What must I pay in Chicago for a draft of \$500 on Savannah, exchange being $\frac{1}{2}$ per cent. discount?

SOLUTION.—Since exchange on Savannah is $\frac{1}{2}\%$ discount, the cost of \$1 is \$1 − $\frac{1}{2}$ ct. = \$.99$\frac{1}{2}$, and the cost of \$500 is 500 times \$.99$\frac{1}{2}$, or \$497.50. Hence for sight exchange we have the following

OPERATION.

$$\begin{array}{rl} \$1.000 & \\ .005 & = \text{rate of exchange.} \\ \hline .995 & = \text{cost of } \$1. \\ 500 & \\ \hline \$497.500, & \textit{Ans.} \end{array}$$

Rule.—*Find the cost of* \$1 *by adding the rate to* \$1 *when at a premium, or subtracting it when at a discount, and multiply the result by the face of the draft.*

2. What must be paid in Detroit for a draft of \$3000 on Boston at 30 days, exchange being $\frac{1}{4}\%$ premium?

SOLUTION.—The draft, being on time, should be purchased at a discount. The discount of \$1, at the legal rate in Michigan, for 30 + 3, or 33 da., is \$.0055, which, subtracted from \$1, equals \$.9945, the cost of \$1 of the draft at par; but the premium is $\frac{1}{4}$%, hence, adding \$.0025, we find the actual cost of \$1 of the draft to be \$.9970, and multiplying this by \$3000, we have \$2991, the entire cost. Hence, for time exchange the following

OPERATION.

\$1.0000	
.0055	= discount for 33 da.
.9945	= cost of \$1 at par.
.0025	= rate of exchange.
.9970	= cost of \$1 of draft.
3000	
\$2991	= whole cost.

Rule.—*From \$1 subtract the bank discount of \$1 for the time and rate where the draft is purchased; to this result add the rate of exchange when at a premium, and subtract it when at a discount, and multiply the result by the face of the draft.*

WRITTEN EXERCISES.

3. Find the cost of a Boston draft on Chicago for \$2500 at 75¢ discount per \$1000. *Ans.* \$2498.125.

4. D. C. Heath & Co. of Boston owe a bill in Chicago of \$4375; what must they pay for a draft on Chicago, exchange $\frac{3}{8}$% premium? *Ans.* \$4391.41 —.

5. Required the cost in New Orleans of a draft on Philadelphia for \$5000, payable 60 days after sight, at $\frac{1}{4}$% premium, rate 5%, with grace. *Ans.* \$4968.75.

6. A Baltimore firm received flour from Minneapolis costing \$3500; what must they pay for a draft at 15 days' sight, with grace, exchange $\frac{1}{8}$% discount? *Ans.* \$3484.54 +.

7. A San Francisco merchant bought goods in New York valued at \$5284; what will be the cost of a 3 mo. draft for the amount on New York at $\frac{3}{4}$% premium? *Ans.* \$5231.16.

8. What is the face of a sight draft that can be bought for \$2587, exchange $\frac{1}{2}$% discount?

Cost of \$1 = \$1.00 − \$.005 = \$0.995.
Face of draft = \$2587 ÷ \$.995 = \$2600.

9. I paid $2029 for a 30-da. draft on Baltimore, interest 6%, premium 2%; what was the face of the draft?

Cost of $1 = 1 - .0055 + .02 = $1.0145.

Face of draft = 2029 ÷ 1.0145 = $2000.

10. What is the face of a 90-day draft on Philadelphia, bought for $3500, at 6%, exchange 1½% premium? *Ans.* $3500.

11. I received from Chicago a check for $40.20, which cost ¾% to have cashed; what should have been the face of the check that I might have realized $40.20? *Ans.* $40.50.

FOREIGN EXCHANGE.

497. Foreign Exchange is the exchange that takes place between different countries.

1. Three bills of the same tenor and date are usually drawn, each containing a condition that it shall continue payable only while the others are unpaid. They are called a *Set of Exchange.*

2. To prevent loss or delay each bill of a set is sent by a different mail. When one bill of the set has been paid the others are void.

3. *Bills of Exchange* are usually made payable either 3 days after sight or 60 days after sight. The latter are quoted at a lower rate on account of the discount.

4. Most of the dealings in foreign exchange are with the following financial centres: London, Paris, Antwerp, Geneva, Amsterdam, Hamburg, Frankfort, Bremen, Berlin, and Vienna.

498. A Letter of Credit is a letter from a banking-house in one country to one or more of their correspondents in other countries, by which the person in whose favor the letter is written may draw money at different times and in different places, the total amount not exceeding a certain sum specified in the letter.

1. A bill of exchange is payable at a certain place, at a certain time, and for a certain amount, while a letter of credit is payable at different places, at different times, and for different amounts.

2. In giving quotations of foreign exchange no reference is made to the par value, but the premium or discount is included in the figures given.

499. The **Money of Account** of any country consists of the denominations of the money of that country in which accounts are kept.

1. What is the cost of a bill of exchange on London for £225 at $4.87¾ to the pound sterling?

	OPERATION.
SOLUTION.—If £1 costs $4.87¾, £225 cost 225 times $4.87¾, which is $1097.43¾.	$4.87¾
	225
	$1097.43¾

Rule.—*Find the cost of a unit of the currency in which the bill is given, and multiply the face by it for the cost or divide the cost by it for the face.*

WRITTEN EXERCISES.

2. A banker sold a bill of exchange on Hamburg, face 7200 marks, at 94⅛ cents = 4 marks; what was the amount received? *Ans.* $1694.25.

3. What will be the face of a bill of exchange on Paris that can be bought for $4822.762, exchange 5.18⅜ francs to $1? *Ans.* 25000 fr.—.

4. What is the face of a bill of exchange on Bremen that was purchased for $19818.75, exchange being quoted at 94⅜¢ = 4 marks? *Ans.* 84000 marks.

5. What will be the cost in New York of the following draft, exchange at 60 days being $4.89? *Ans.* $1467.

Exchange for £300. NEW YORK, *April* 20, 1894.

Sixty days after sight of this First of Exchange (second and third unpaid) pay to the order of John Smith, Three Hundred Pounds Sterling, for value received, and charge the same to account of BROWN BROTHERS & CO.

TO BROWN, SHIPLEY & CO., LONDON, ENGLAND.

6. What will it cost to remit 9750 francs to Antwerp if 19.5¢ = 1 franc? *Ans.* $1901.25.

INTRODUCTION TO PROPORTION.

ORAL EXERCISES.

1. What is the relation of 8 to 4? *Ans.* 8 is *two times* 4.

2. What is the relation of 12 to 3? Of 16 to 4? Of 18 to 6? Of 20 to 5? Of 24 to 6? Of 30 to 5?

3. What is the relation of 3 to 6? Of 4 to 12? Of 6 to 24? Of 7 to 35? Of 8 to 57? Of 9 to 62?

4. The measure of the relation of two numbers is called their *ratio.*

5. What is the ratio of 12 to 4? *Ans.* The ratio of 12 to 4 is *three.*

6. What is the ratio of 18 to 9? Of 25 to 5? Of 48 to 8? Of 63 to 7? Of 64 to 4? Of 70 to 10? Of 80 to 8?

7. What is the ratio of 3 to 6? *Ans.* The ratio of 3 to 6 is *one-half.*

8. What is the ratio of 4 to 12? Of 3 to 18? Of 4 to 30? Of 9 to 108? Of 11 to 132? Of 12 to 144?

9. What is the ratio of $\frac{1}{2}$ to $\frac{1}{3}$? Of $\frac{1}{5}$ to $\frac{1}{6}$? Of $\frac{2}{3}$ to $\frac{3}{4}$? Of .5 to .25? Of .2 to .04? Of .03 to .12?

10. The ratio of two numbers may be expressed by writing the colon between them; thus, 8 : 4 denotes the ratio of 8 to 4.

11. Required the value of 12 : 6; of 28 : 7; of 42 : 6; of 24 : 12; of 12 : 24.

12. How does the ratio of 8 to 4 compare with the ratio of 12 to 6? *Ans.* They are equal.

13. What number has the same ratio to 12 that 18 has to 6?

14. What number has the same ratio to 20 that 40 has to 10?

15. The ratio of 9 to 36 is the same as the ratio of 15 to what number?

16. 25 is to 5 as 40 to what number? 24 is to 12 as 15 is to what number?

17. When we express the ratio of two numbers equal to the ratio of two other numbers, as, 24 is to 4, as 36 is to 6, we have a *proportion.*

18. The equality of two ratios may be expressed by writing the symbol = between them; thus, 8 : 4 = 12 : 6.

19. How many numbers do we have in a proportion? How many ratios? Are the ratios equal or unequal?

20. Write the proportion 16 to 8 as 24 is to 12; also 15 is to 45 as 18 is to 54.

SECTION IX.

RATIO AND PROPORTION.

500. **Ratio** is the measure of the relation of two similar quantities; thus, the ratio of 8 to 4 is 2.

501. The **Symbol** of ratio is the colon (:); thus, 8 : 4 signifies the ratio of 8 to 4. Ratio is also expressed by writing the numbers in the form of a fraction; thus, $\frac{8}{4}$.

502. The **Terms** of a ratio are the two numbers compared, called respectively the *antecedent* and the *consequent.*

503. The **Antecedent** is the number compared with the consequent. Thus, in the ratio 8 : 4, 8 is the antecedent.

504. The **Consequent** is the number with which the antecedent is compared. Thus, in 8 : 4, 4 is the consequent.

505. A **Ratio** is found by dividing the antecedent by the consequent. Thus, in 8 : 4 the ratio is $\frac{8}{4}$, or 2.

506. A **Simple Ratio** is the ratio of two numbers, as 6 : 3. A *Compound Ratio* is the product of two or more simple ratios; as, $(3 : 4)\times(5 : 6)$, or $\frac{3}{4}\times\frac{5}{6}$.

507. A **Compound Ratio** is usually expressed by writing the simple ratios one under another; thus, $\left\{\begin{matrix}3 : 4\\ 5 : 6\end{matrix}\right\}$.

508. Ratio exists only between similar quantities, and is always an abstract number.

1. The symbol of ratio (:) is supposed to be a modification of the symbol of division.

2. Ratio is usually defined as the *relation* of two numbers. This is indefinite, for the ratio is the *measure* of the relation.

PRINCIPLES.

1. *The ratio equals the quotient of the antecedent divided by the consequent.*

Thus, if the antecedent is represented by a, and the consequent by c, and the ratio by r, we have $a \div c = r$, or $\frac{a}{c} = r$.

2. *The antecedent is equal to the product of the consequent and ratio.*

For, since $\frac{a}{c}=r$, multiplying by c, we have $a=c\times r$.

3. *The consequent is equal to the quotient of the antecedent divided by the ratio.*

For, since $\frac{a}{c}=r$, $a=c\times r$, from which we see that $c=\frac{a}{r}$.

ORAL AND WRITTEN EXERCISES.

What is the ratio of

1. 8 to 2?	*Ans.* 4.	5. \$288 to \$648?	*Ans.* $\frac{4}{9}$.
2. 18 to 3?	*Ans.* 6.	6. £144 : £256?	*Ans.* $\frac{9}{16}$.
3. 75 to 12?	*Ans.* $6\frac{1}{4}$.	7. $\frac{1}{2} : \frac{5}{8}$? $\frac{2}{3} : \frac{5}{6}$?	*Ans.* $\frac{4}{5}$.
4. 496 to 31?	*Ans.* 16.	8. $\frac{25}{16} : \frac{15}{12}$? $2\frac{3}{4} : 2\frac{1}{5}$?	*Ans.* $1\frac{1}{4}$.

9. What is the value of the compound ratio $\left\{\begin{matrix} 3 : 6 \\ 4 : 12 \end{matrix}\right\}$.

Solution.—This compound ratio equals $(3 : 6)\times(4 : 12)$, which equals $\frac{3}{6}\times\frac{4}{12}=\frac{1}{6}$.

10. What is the value of the ratio $\left\{\begin{matrix} 5 : 3 \\ 4 : 8 \end{matrix}\right\}$? *Ans.* $\frac{5}{6}$.

11. What is the value of the ratio $\left\{\begin{matrix} 3 : 2 \\ 8 : 6 \end{matrix}\right\}$? *Ans.* 2.

12. The antecedent is 15, the consequent 5; what is the ratio? *Ans.* 3.

13. The consequent is 7 and ratio 10; what is the antecedent? *Ans.* 70.

14. The antecedent is 56 and ratio 4; what is the consequent? *Ans.* 14.

15. The consequent is $\frac{18}{20}$ and ratio $\frac{25}{36}$; what is the antecedent? *Ans.* $\frac{5}{8}$.

16. The antecedent is $\frac{27}{40}$ and ratio $\frac{25}{32}$; what is the consequent? *Ans.* $\frac{108}{125}$.

17. Can you express the ratio between \$24 and 6 lb.? Why not?

18. The antecedents of a ratio are 7 and 8, and the consequents 12 and 18; what is the ratio? *Ans.* $\frac{7}{27}$.

SIMPLE PROPORTION.

509. A **Proportion** is the expression of equality between equal ratios, the terms of the ratios being indicated.

Thus, the ratios 8 : 4 and 12 : 6 being equal, they will form a proportion.

510. A **Proportion** is *written* by placing the sign = or the double colon (: :) between the two ratios.

Thus, $8 : 4 = 12 : 6$, or $8 : 4 :: 12 : 6$.

511. A **Proportion** is *read* in two ways; thus, $8 : 4 :: 6 : 3$ is read "the ratio of 8 to 4 equals the ratio of 6 to 3," or "8 is to 4 as 6 is to 3."

512. The **Terms** of a proportion are the four numbers used in the comparison. The first and fourth terms are the *Extremes;* the second and third are the *Means.*

513. The **Couplets** are the two ratios compared. The *first couplet* consists of the first and second terms; the *second couplet* consists of the third and fourth terms.

514. A **Simple Proportion** is the expression of the equality of two simple ratios.

515. The **Principles** of proportion are the truths relating to a proportion. They enable us to find any one term when the other three are given.

PRINCIPLES.

1. *In every proportion the product of the means equals the product of the extremes.*

In any proportion, as $6 : 3 = 8 : 4$, we have $\frac{6}{3} = \frac{8}{4}$, and multiplying these equals by 4 and 3, we have $6 \times 4 = 8 \times 3$; that is, the product of the two means, 8 and 3, equals the product of the two extremes, 6 and 4.

2. *Either extreme equals the product of the means divided by the other extreme.*

For, from the proportion $6 : 3 = 8 : 4$ we have $6 \times 4 = 3 \times 8$; hence, $6 = 3 \times 8 \div 4$, or $4 = 3 \times 8 \div 6$. Therefore, etc.

3. *Either mean equals the product of the extremes divided by the other mean.*

For, from the proportion $6 : 3 = 8 : 4$ we have $6 \times 4 = 3 \times 8$; hence, $3 = 6 \times 4 \div 8$, or $8 = 6 \times 4 \div 3$. Therefore, etc.

1. Let the pupils be required to demonstrate these principles by using symbols of any numbers; that is, by letters. French authors usually represent the unknown term by x; the same is done in this work.

2. Principle 1 may be demonstrated by showing that in a proportion we have 2d term × ratio : 2d term : : 4th term × ratio : 4th term; in which we see the factors in the means are the same as the factors in the extremes.

INTRODUCTORY EXERCISES.

1. Write a proportion and point out the different terms and couplets. Write a proportion and show that the ratios are equal.

2. If we multiply the antecedent of one couplet, what must we do to the other couplet to make the ratios equal?

3. If we divide the antecedent of one couplet, what must we do to the other couplet to make the ratios equal?

4. Write a proportion and illustrate Prin. 1; Prin. 2; Prin. 3.

5. Show that if we change the two means one for the other, or the two extremes, the four numbers will still form a proportion.

6. Take some proportion, and show that we may invert the terms of the couplets and the four terms will still be in proportion.

WRITTEN EXERCISES.

Find the terms denoted by x in each of the following proportions:

1. $x : 12 = 5 : 20$. *Ans.* 3.

2. $x : 9 = 13 : 39$. *Ans.* 3.

3. $5 : 15 = 9 : x$. *Ans.* 27.

4. $15 : 45 = 16 : x$. *Ans.* 48.

5. $32 : x = 21 : 63$. *Ans.* 96.

6. $x : \frac{5}{9} : : \frac{3}{5} : \frac{7}{8}$. *Ans.* $\frac{8}{21}$.

7. $9 : 7.2 : : 3.5 : x$. *Ans.* 2.8.

8. $4.5 : x : : 8\frac{1}{2} : 55\frac{1}{4}$. *Ans.* $29\frac{1}{4}$.

9. $72 : 27 : : x : 15$. *Ans.* 40.

10. \$8 : \$20 : : x : 75. *Ans.* 30.

11. \$21 : \$84 : : 65 lb. : x. *Ans.* 260 lb.

APPLICATION OF SIMPLE PROPORTION.

516. **Simple Proportion** is employed for the solution of problems in which three of four quantities are given, so related that the fourth may be determined from them by equality of the ratios.

517. Principle.—*The required quantity must bear the same relation to a given quantity of the same kind that one of the remaining quantities does to the other.*

518. From this principle we can form a proportion containing one unknown quantity, and find the unknown term by the principles of proportion.

NOTE.—Proportion was formerly called the "Rule of Three." Some of the old arithmeticians thought so highly of it that they called it "The Golden Rule of Three."

1. What will 25 tons of hay cost if 6 tons cost \$36?

OPERATION.

$$\begin{array}{lcccc} & \$ & & \text{T.} & \text{T.} \\ \text{Cost of 25 T.} : & 36 & = & 25 : & 6 \end{array}$$

$$\text{Cost of 25 T.} = \frac{25 \times 36}{6} = \$150$$

SOLUTION.—It is evident that the cost of 25 T. bears the same relation to the cost of 6 T. as 25 T. bears to 6 T.; hence, we have the proportion, *cost of* 25 T. *is to* \$36 *as* 25 T. *is to* 6 T.; from which, by Prin. 2, we have the cost of 25 T. $= \frac{25 \times 36}{6} =$ \$150. Hence the

Rule.—I. *Write the required quantity for the first term and the similar known quantity for the second term.*

II. *Place the other two quantities for the third and fourth terms, so that the two ratios will be equal.*

III. *Find the first term by dividing the product of the second and third terms by the fourth.*

OPERATION.

$$\begin{array}{lcl} \text{T.} \ \ \text{T.} & & \$ \\ 6 : 25 & = & 36 : \text{cost of 25 T.} \end{array}$$

$$\text{Cost of 25 T.} = \frac{25 \times 36}{6} = 150$$

SOLUTION 2D.—It is evident that the relation of 6 T. to 25 T. is the same as the relation of the cost of 6 T. to the cost of 25 T.; hence, we have the proportion 6 T. is to 25 T. as \$36 is to the *cost of* 25 T., from which, by Prin. 2, we have the *cost of* 25 T. equals \$150.

1. The author believes that the simplest method of using proportion is to put the *unknown* quantity in the *first term.* He gives the old method also for teachers who prefer it. The above rule may be readily changed to correspond to it.

2. Pupils should be required to put the unknown quantity, which they may represent by x, in different terms, that they may thoroughly understand the subject.

WRITTEN EXERCISES.

2. If 15 yards of silk cost $22.50, what will 65 yards cost? *Ans.* $97.50.

3. If 56 yards of cloth cost $182, how many yards can be bought for $156? *Ans.* 48 yd.

4. If $100 gains $4 in a year, what will $450 gain in a year? *Ans.* $18.

5. If I can buy 80 horses for $10,000, how many can I buy for $37,500? *Ans.* 300.

6. What is the time by rail from Philadelphia to New York, 90 m., at the rate of 5 miles in 7 min. 30 sec.? *Ans.* $2\frac{1}{4}$ h.

7. A merchant gains $350 by selling $7000 worth of goods; what amount must he sell to gain $5000? *Ans.* $100,000.

8. If 5 A. 120 P. of land cost $718.75, what will 25 A. 40 P. cost? *Ans.* $3156.25.

9. A bankrupt's debts are $4800 and assets $2000; what will be received for a claim of $1500? *Ans.* $625.

10. If 35 oxen eat 45 acres of grass in a month, how many oxen would 540 acres keep the same time? *Ans.* 420 oxen.

11. If the tax on property valued at $4800 is $88.80, what will be the tax on property valued at $7600? *Ans.* $140.60.

12. If a pole 4 ft. 6 in. long throws a shadow of 5 ft. 3 in., what will be the shadow of a tree 54 ft. high? *Ans.* 63 ft.

13. If a pole 10 ft. long casts a shadow of 8 ft. 4 in., what is the height of a steeple whose shadow is 240 feet long? *Ans.* 288 ft.

14. A grocer has a false balance which gives 15 oz. to the pound; what does he gain by it in selling sugar for which he receives $74.40? *Ans.* $4.65.

15. If a man spends $276 in the three summer months, how much will he spend in a year at the same rate per day? *Ans.* $1095.

16. If 12 men build a wall in 24 days, how long will it take 60 men to build it at the same rate? *Ans.* $4\frac{4}{5}$ days.

NOTE.—Here it is evident that the time in which 60 men do it is to 24 days, the time in which 12 men do it, as 12 men is to 60 men.

17. If 28 men mow a field of grass in 12 days, how many men will be required to mow it in 8 days? *Ans.* 42 men.

18. If 17 men can mow a field in 9 days, how long would it take to mow half of it if 5 men refuse to labor? *Ans.* $6\frac{3}{8}$ da.

19. If it requires 54 yards of carpet 1 yd. wide to cover a floor, how many yards of carpet $\frac{3}{4}$ yd. wide will cover the same floor? *Ans.* 72 yd.

20. If a man performs a journey in 25 days of 8 hours each, how many days, of 10 hours each, would it take him to perform the same journey? *Ans.* 20 days.

21. A merchant borrows $1800 from a friend, and keeps it 1 yr. 6 mo.; how long should he loan $1000 to return the favor? *Ans.* 2 yr. 8 mo. 12 da.

22. Two cog-wheels, one having 30 and the other 25 cogs, run together; in how many revolutions of the larger wheel will the smaller gain 15 revolutions? *Ans.* 75.

23. If a 3-cent loaf weighs 9 ounces when flour is $6 a barrel, what should it weigh when flour is $4 a barrel? *Ans.* $13\frac{1}{2}$ oz.

24. A house is worth $2700, and the price of the house is to that of the lot on which it stands as 9 to 11; what is the value of the lot? *Ans.* $3300.

25. If 500 laborers can build an embankment in 36 days, how many more days would be required if the number of men is diminished by 50? *Ans.* 4 days more.

26. A can do a piece of work in 8 days; B can do it in 10 days; what should A receive per day if B's wages are $2 per day? *Ans.* $2\frac{1}{2}$.

27. A has grain which cost 55¢ a bushel, and B has flour at $4.25 a barrel; if in exchange A puts his grain at 65¢ a bushel, what should B charge for his flour? *Ans.* $5.02\frac{3}{11}$.

28. A garrison of 4000 men has "hard tack" enough for 8 weeks, allowing each man 16 oz. a day; but 14,000 lb. having been spoiled, what must be each man's allowance that the provision may last the 8 weeks? *Ans.* 15 oz.

29. How large a reinforcement could the above garrison receive if none of the provision was spoiled, but each man's allowance was reduced to $14\frac{2}{9}$ oz.? *Ans.* 500.

COMPOUND PROPORTION.

519. A **Compound Proportion** is a proportion in which one or both ratios are compound.

520. Thus, $\left\{\begin{matrix} 2:4 \\ 5:15 \end{matrix}\right\} = 6:36$ and $\left\{\begin{matrix} 4:12 \\ 7:14 \end{matrix}\right\} = \left\{\begin{matrix} 5:10 \\ 6:18 \end{matrix}\right\}$, are examples of compound proportion.

521. Compound Proportion is used in the solution of problems in which the required term depends on a compound ratio.

522. In simple proportion the unknown quantity depends upon *one pair* of similar quantities; in compound proportion it depends upon *two or more pairs* of similar quantities.

523. The method of **Compound Proportion** is now seldom used in arithmetical operations.

524. The simplest method of solving the problems that belong to **Compound Proportion** is that of *Analysis.*

WRITTEN EXERCISES.

1. If 6 men can earn $90 in 5 days, how much can 8 men earn in 9 days?

ANALYSIS.—If 6 men earn $90 in 5 da., 1 man will earn $\frac{1}{6}$ of $90 and 8 men will earn $\frac{8}{6}$ of $90. If 8 men earn $\frac{8}{6}$ of $90 in 5 da., in 1 day they will earn $\frac{1}{5}$ of $\frac{8}{6}$ of $90, and in 9 da. they will earn $\frac{9}{5}$ of $\frac{8}{6}$ of $90, which, by cancelling and multiplying, equals $216.

OPERATION.

$$\frac{\overset{3}{\cancel{9}}}{\cancel{5}} \times \frac{\overset{4}{\cancel{8}}}{\underset{\cancel{2}}{\cancel{6}}} \times \$\overset{18}{\cancel{90}} = \$216$$

525. The analysis may be abbreviated as follows:

ANALYSIS.—If 6 men earn \$90, 8 men will earn $\frac{8}{6}$ of \$90. If they earn it in 5 da., in 9 da. they will earn $\frac{9}{5}$ of $\frac{8}{6}$ of \$90 = \$216.

2. If it costs \$42 to carpet a floor 24 feet long and 15 feet wide, what will it cost to carpet a floor 30 feet long and 18 feet wide? *Ans.* \$63.

3. If a railroad charges \$15 for carrying 12000 lb. 300 miles, what should be the charge for carrying 50000 lb. 250 miles? *Ans.* \$52$\frac{1}{12}$.

4. If it costs \$1.45 for burning 5 gas-burners 5 hours every evening for 6 days, how many burners may be used 4 hours every evening for 10 days, at a cost of \$6.96? *Ans.* 18.

5. If 10 men in 48 days build a wall 140 rd. long, 6 ft. high, and 4 ft. thick, how many men can in 54 days build a wall 180 rd. long, 7 ft. high, and 4$\frac{1}{2}$ ft. thick? *Ans.* 15 men.

6. If 15 carpenters build a house in 60 days, working 10 hours a day, in how many days would 30 carpenters build it, working 8 hours a day? *Ans.* 37$\frac{1}{2}$ days.

7. If 25 horses can eat a certain quantity of grain in 40 days, in what time will twice as much grain be consumed if 7 horses are added when the grain is $\frac{3}{4}$ eaten? *Ans.* 74$\frac{3}{8}$ days.

8. If 240 loaves of bread, weighing 6 oz. each, cost \$12 when flour is \$6 a barrel, what cost 100 loaves of 7 oz. each when flour is worth \$4.50 a barrel? *Ans.* \$4.37$\frac{1}{2}$.

9. If 22 men can dig a ditch 2500 ft. long, 5 ft. wide, and 3 ft. deep in 25 days of 10 hours each, in how many days of 11 hours each will 132 men dig a ditch 3600 ft. long, 6 ft. wide, and 4 ft. deep? *Ans.* 8$\frac{8}{11}$ days.

10. If 7 compositors in 17 days of 10 hours each set up 35 sheets of 16 pages each, 51 lines on a page and 45 letters in a line, in how many days, 12 hours long, can 6 compositors set up in the same type 27 sheets, 24 pages each, 45 lines in a page, 48 letters in a line? *Ans.* 18 days.

PARTNERSHIP.

526. Partnership is the association of two or more persons for the transaction of business.

527. The association is termed a *Firm, House*, or *Company*, and the persons associated are termed *Partners*.

528. The **Capital** of a firm is the money or property invested by the partners.

529. The **Resources** or **Assets** of a firm are its property of any kind, together with the amounts due it.

530. The **Liabilities** are its debts. The excess of resources over liabilities is called the *Net Capital.*

Partners are of three kinds, *General, Limited,* and *Special.*

General Partners risk their whole property in the business; *Limited and Special Partners* risk only the amount of capital they agree to contribute. Partners whose names do not appear are sometimes called *Silent Partners.*

531. Partnership is divided into *Simple* and *Compound Partnership* for convenience of treatment.

SIMPLE PARTNERSHIP.

532. In **Simple Partnership** the shares of the partners are employed for equal periods of time.

1. A, B, and C formed a partnership; A put in \$1000, B put in \$1200, and C put in \$1400; they gained \$900; what was each one's share of the gain?

SOLUTION.—The entire capital is \$3600. Since A put in \$1000, he furnished $\frac{1000}{3600}$, or $\frac{5}{18}$ of the capital, and hence should have $\frac{5}{18}$ of \$900, or \$250. B furnished $\frac{1200}{3600}$, or $\frac{1}{3}$ of the capital, and should have $\frac{1}{3}$ of \$900, or \$300, etc.

OPERATION.

$$
\begin{array}{rl}
\$1000 & \frac{1000}{3600} = \frac{5}{18} = \text{A's share.} \\
1200 & \frac{1200}{3600} = \frac{1}{3} = \text{B's share.} \\
\underline{1400} & \frac{1400}{3600} = \frac{7}{18} = \text{C's share.} \\
\text{Stock} = \$3600 &
\end{array}
$$

$\frac{5}{18}$ of \$900 = \$250 = A's share.

$\frac{1}{3}$ of \$900 = \$300 = B's share.

$\frac{7}{18}$ of \$900 = \$350 = C's share.

Rule.—*Divide the gain or loss among the partners in proportion to their shares of the stock.*

WRITTEN EXERCISES.

2. A, B, and C form a partnership for raising oranges: A puts in \$560, B \$640, and C \$800; they gain \$500; what does each receive? *Ans.* A, \$140; B, \$160; C, \$200.

3. Three men enter into partnership with a capital of \$15,000; A contributes $\frac{1}{2}$ of the capital, B $\frac{1}{3}$ of the capital, and C the remainder; what does each receive if they gain \$7500? *Ans.* A, \$3750; B, \$2500; C, \$1250.

4. A, B, and C go into the grain business with a capital of \$12,000; at the end of a year A's share of the profits is \$1250, B's \$1000, and C's \$750; what was each one's capital? *Ans.* A's \$5000; B's \$4000; C's \$3000.

5. A man leaves by his will \$5000 to his wife, \$4000 to his son, and \$3500 to his daughter; after his death his estate was found to amount to only \$10,000; what did each receive? *Ans.* Wife, \$4000; son, \$3200; daughter, \$2800.

6. A shipping-firm gained in one year \$4000; A's stock was \$7000, B's \$4500, and C's gain was \$406.25; what were A's and B's gain and C's stock?

Ans. C's stock, \$1300; A's gain, \$2187.50; B's, \$1406.25.

COMPOUND PARTNERSHIP.

533. In **Compound Partnership** the capitals of the partners are employed for different periods of time.

1. Two persons enter into partnership and gain \$560; A put in \$400 for 12 mo., and B \$800 for 8 mo.; what was each man's share of the gain?

SOLUTION.—\$400 for 12 months is equivalent to \$4800 for 1 mo., and \$800 for 8 mo. is equivalent to \$6400 for 1 mo.; hence, the entire capital is equivalent to \$11200 for 1 mo. The rest of the solution may be given as in Simple Partnership.

OPERATION.

\$400 × 12 = \$4800 for 1 mo. = A's.
\$800 × 8 = \$6400 for 1 mo. = B's.
\$11200 for 1 mo.
$\frac{4800}{11200} = \frac{3}{7}$ = A's share of capital.
$\frac{6400}{11200} = \frac{4}{7}$ = B's share of capital.
$\frac{3}{7}$ of \$560 = \$240, A's gain.
$\frac{4}{7}$ of \$560 = \$320, B's gain.

Rule.—*Multiply each partner's capital by the time it was employed, and divide the gain or loss in proportion to these products.*

WRITTEN EXERCISES.

2. Three persons formed a partnership; A had $800 in trade for 9 mo., B $600 for 10 mo., and C $500 for a year; they gained $960; what was each man's share of the gain? *Ans.* A's, $360; B's, $300; C's, $300.

3. Three wood-choppers agree to cut the wood on a certain tract for $150: the first worked 15 days of 10 hours each, the second 25 days of 6 hours each, and the third 30 days of 5 hours each; what does each receive? *Ans.* $50.

4. Four men rented a pasture for $75.60; the first put in 4 horses for 6 weeks, the second 3 horses for 10 weeks, the third 5 horses for 8 weeks, and the fourth 2 horses for 16 weeks; what should each pay? *Ans.* $14.40; $18; $24; $19.20.

5. Three drovers hire a pasture for $144 for 6 mo.; the first put in 40 cows, the second 60, and the third 80; at the end of 4 mo. the first sold $\frac{1}{2}$ of his, the second $\frac{1}{3}$ of his, and the third $\frac{1}{4}$ of his; what should each pay? *Ans.* $30; $48; $66.

6. Mr. Warner went into business with $5000 capital; at the end of 4 months he took in Mr. Granville with $8000 capital, and at the end of 6 months Mr. Randolph with $3000 capital; at the end of the year they had gained $1420; what was the share of each? *Ans.* $600; $640; $180.

7. Two persons, A and B, were in partnership 2 years; A at first put in $4000 and B $3000; at the end of 9 mo. A took out $1200 and B put in $600; they lost in 2 yr. $3816; what was each one's share of the loss? *Ans.* A's, $1872; B's, $1944.

8. A and B went into the hardware business, A's capital being to B's as 3 to 4; at the end of 6 months A withdraws $\frac{1}{3}$ of his capital and B $\frac{1}{4}$ of his; during the year they gain $1440; what was each man's share of the gain? *Ans.* A's, $600; B's, $840.

EQUATION OF PAYMENTS.

534. **Equation of Payments** is the process of finding the mean or equitable time for paying several sums due at different times.

535. The **Term of Credit** is the time allowed for the payment of a debt.

536. The **Average Term of Credit** is the time to elapse before several debts due at different times may in equity be paid together.

537. The **Equated Time** is the date at which several debts due at different times may be paid in one sum.

538. The **Focal Date** is the date from which we begin the reckoning in averaging an account.

CASE I.

539. **To find the average term of credit when the terms of credit begin at the same time.**

1. A merchant owes at a bank $250, due in 5 months, and $450, due in 3 months; what is the average term of credit?

OPERATION.

$$\begin{array}{ll} 250 \times 5 = 1250 & \\ 450 \times 3 = 1350 & \\ \hline 700 &)2600(3\frac{5}{7} \end{array}$$

SOLUTION.—A credit on $250 for 5 months is regarded as equivalent to a credit on $1 for 1250 months, and a credit on $450 for 3 months is equivalent to a credit on $1 for 1350 months, and adding, we have the same as a credit on $1 for 2600 months; if $1 has a credit of 2600 months, $700 would have a credit of $\frac{1}{700}$ of 2600 months, which is $3\frac{5}{7}$ months. Hence the

Rule.—*Multiply each debt by its term of credit, and divide the sum of the products by the sum of the debts; the quotient will be the average term of credit.*

1. Cents in any of the payments may be rejected when less than 50, and reckoned at $1 when more than 50. The fraction of a day in the answer is also rejected when less than $\frac{1}{2}$, and reckoned as 1 day if more than $\frac{1}{2}$.

2. The time may also be found by dividing the sum of the interest on the payments, using any rate, by the interest on the sum of the payments for 1 month or 1 day, according to the unit of time used in the calculation. This method is preferred by some accountants.

WRITTEN EXERCISES.

2. Henry Bowman owes the H. B. Claflin Co. \$4800, $\frac{1}{3}$ due in 3 mo., $\frac{1}{4}$ in 4 mo., and the remainder in 6 mo.; required the average term of credit. *Ans.* $4\frac{1}{3}$ mo.

3. T. B. Morgan owes \$1800, $\frac{1}{3}$ of which is due in 3 mo., \$600 in 6 mo., and the remainder in $8\frac{1}{4}$ mo.; required the average term of credit. *Ans.* $5\frac{3}{4}$ mo.

4. A. M. Stewart owes \$500 due in 4 mo., \$600 due in 6 mo., \$700 due in 8 mo., and \$1000 due in 9 mo.; what is the average term of credit? *Ans.* $7\frac{3}{14}$ mo.

5. I bought merchandise April 1, 1894, as follows: \$4500 for cash, \$1800 on 4 mo., and \$1200 on 6 mo.; what is the equated time of payment? *Ans.* May 29.

6. Mr. Woodruff bought a house, agreeing to pay $\frac{1}{2}$ in 6 mo., $\frac{1}{4}$ in 8 mo., and the remainder in 1 year; required the average term of credit. *Ans.* 8 mo.

REMARK.—Since the result will be the same whatever the sum owed, we may assume \$1 as the capital, and proceed as before.

CASE II.

540. To find the equated time when the terms of credit begin at different dates.

1. I purchased of A. S. Gardner & Co. the following bill of goods:

Mar. 10,	a bill	amounting to	\$800 on 2 mo.	credit.
Mar. 15,	"	"	\$600 on 3 mo.	"
Apr. 20,	"	"	\$700 on 3 mo.	"

Now, if I wish to make one payment of this bill, at what time in equity will it become due?

OPERATION.

May 10,	$\$800 \times 00 =$	00000
June 15,	$\$600 \times 36 =$	21600
July 20,	$\$700 \times 71 =$	49700
	2100	)71300(34 –

SOLUTION.—From the time the first is due to the time the second is due is 36 da, and to the time the third is due is 71 da.; hence, estimating from the time the first is due, the second has a credit of 36 da., and the third a credit of 71 da., and the first has a credit of no days.

We then average it as in Case I., and find the term of credit to be nearly 34 da. from May 10, the time at which the first debt is due; hence the equated time of payment is June 13. From the above we derive the following

Rule.—I. *Select the date at which the first debt becomes due, and multiply each debt by its term of credit reckoned from the date selected.*

II. *Divide the sum of the products by the sum of the debts, and the quotient will be the average term of credit, estimated from the date selected.*

When the *earliest* date is not the *first* of the month, it is often more convenient to take the *first* of the month as the standard date.

WRITTEN EXERCISES.

2. Bought goods on 60 days' credit as follows: Oct. 15, 1894, $200; Nov. 20, $360; Dec. 12, $155; required the equated time of payment of this bill. *Ans.* Jan. 14.

3. Bought goods on 4 mo. credit as follows: Jan. 12, 1893, $300; Feb. 15, $500; Apr. 4, $800; required the equated time of payment. *Ans.* July 4.

4. Sold goods on 6 mo. credit as follows: Feb. 16, 1893, $300; Mar. 12, $250; April 5, $350; June 1, $600; what is the equated time for the payment of this bill? *Ans.* Oct. 14.

5. Bought goods of Strawbridge & Clothier on 90 days' credit as follows: Mar. 4, $176.82; Apr. 7, $438.36; May 1, $632.65; June 1, $237.16; required the equated time for the payment of the bill. *Ans.* July 21.

6. F. A. Leggett & Co. bought of Lord & Taylor several bills of goods, as follows:

June 3, a bill of $375 on 30 days' credit.
" 28, " " $450 " 60 " "
July 16, " " $840 " 4 months' credit.
Sept. 12, " " $250 " 90 days' credit.

What is the equated time of payment? *Ans.* Oct. 5.

CASE III.

541. When a debt due at some future time has received partial payments, to find when the remainder should be paid.

1. A merchant bought goods to the amount of $5000 on a credit of 6 mo.; 3 mo. before it was due he paid $800, and 2 mo. before it was due he paid $1200; how long after the expiration of the 6 mo. may the balance remain unpaid?

SOLUTION.—A credit on $800 for 3 mo. is equivalent to a credit on $1 for 2400 mo.; a credit on $1200 for 2 mo. is equivalent to a credit on $1 for 2400 mo.; and adding, we have a credit on $1 for 4800 mo.; hence, $3000, the sum which remains unpaid, should have a credit of $\frac{1}{3000}$ of 4800 mo., which is $1\frac{3}{5}$ mo. Hence

OPERATION.

$$\begin{array}{rl} 800 \times 3 = & 2400 \\ \underline{1200} \times 2 = & \underline{2400} \\ 2000 & 4800 \end{array}$$

$$\frac{4800}{3000} = 1\tfrac{3}{5}.$$

Rule.—*Multiply each payment by the time it was paid before it was due, and divide the sum of the products by the sum remaining unpaid.*

WRITTEN EXERCISES.

2. Mr. Jones borrowed $3500 for 6 mo.: 4 mo. before it was due he paid $800, and 3 mo. before due he paid $1500; at what time in equity should the remainder be paid? *Ans.* $6\frac{5}{12}$ mo.

3. I lent Mr. Wilson $2000 for 9 mo., $\frac{1}{2}$ of which he paid in 4 mo., and $\frac{1}{2}$ of the remainder in 6 mo.; how long in equity may the remainder remain unpaid? *Ans.* 13 mo. after due.

4. I borrowed of Mr. Carpenter $500 for 3 mo., $700 for 5 mo., and $1000 for 6 mo.; at the end of 4 mo. I paid him $1600; at what time in equity should the remainder be paid? *Ans.* $7\frac{2}{3}$ mo. after borrowing.

5. A milliner bought of Sharpless & Co. a bill of $350 on a credit of 20 days, and $480 on a credit of 30 days; at the end of 16 days she paid $400, and at the end of 24 days she paid $250; when, in equity, should the balance of the bill be paid? *Ans.* 50 days.

SECTION X.

INVOLUTION AND EVOLUTION.

INVOLUTION.

542. **Involution** is the process of finding any power of a number.

543. **A Power** of a number is the product arising from using the number several times as a factor. The number itself is called the *first power.*

544. **The Second Power** of a number is the product obtained by using the number twice as a factor. Thus, 16 is the second power of 4, since $4 \times 4 = 16$.

545. **The Third Power** of a number is the product obtained by using the number three times as a factor. Thus, 64 is the third power of 4, since $4 \times 4 \times 4 = 64$.

546. **The Fourth Power** of a number is the product obtained by using the number four times as a factor; the *Fifth Power,* five times as a factor, etc.

547. **The Degree** of a power is denoted by a small figure, called an *exponent,* placed at the right and a little above the number. Thus, 5^2 represents the 2d power of 5; 6^3, the third power of 6, etc.

548. **The Exponent** indicates how many times the number is used as a factor. Thus, 8^3 denotes that 8 is used as a factor three times; that is, $8 \times 8 \times 8$, which equals 512.

The second power of a number is called its *square,* because the area of a square equals the product of its two equal sides. The third power of a number is called its *cube,* because the product of the three equal sides of a cube gives its contents.

PRINCIPLES.

1. *A power of a number is obtained by using the number as a factor as many times as there are units in the degree.*

2. *The product of any two powers of a number equals a power of the number denoted by the sum of the exponents.*

Thus, if we multiply the cube of a number by the 4th power of the number, we shall evidently have the number used seven times as a factor, or the 7th power of the number; thus, $5^3 \times 5^4 = (5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5) = 5^7$, and the same may be shown in any other case.

3. *A power of a number raised to any power equals a power of the number denoted by the product of the exponents.*

Thus, if we *square* the *cube* of a number, we shall evidently use the number as a factor *two* times *three* times, or *six* times; thus, $(5^3)^2 = 5^3 \times 5^3$, which, by Prin. 2, equals 5^6, and the same may be shown in any other case.

NOTE.—By means of this principle we can abbreviate the operation of involution; thus, we can raise a number to the 6th power by squaring its cube, or to the 12th power by squaring its 6th power, or cubing its 4th power, etc.

ORAL EXERCISES.

1. The square of 4 equals 4 used how often as a factor?

2. The cube of 5 equals 5 used how often as a factor?

3. How often is 8 used as a factor in finding the 5th power of 8?

4. How often is 5 used as a factor in the cube of the square of 5?

5. What power of 6 is 6^2 multiplied by 6^3?

6. If we multiply 7^3 by 7^4, what power of 7 shall we have?

7. What power of 5 is equal to 5^3 multiplied by 5^5?

8. What power of 2 is the square of the square of 2^3?

9. If the square of a number equals 6 times that number, what is the number?

10. What number multiplied by 8 gives ½ of the square of the number for a product?

11. What number multiplied by 16 gives ¼ of the square of the number for a product?

WRITTEN EXERCISES.

1. Find the square of 15.

	OPERATION.
SOLUTION.—To find the square of 15 we multiply 15 by itself, and we have 225. To find the cube of 15 we would multiply 225 by 15.	15 15 225

2. Square 16. *Ans.* 256.
3. Square 24. *Ans.* 576.
4. Square 56. *Ans.* 3136.
5. Square 105. *Ans.* 11025.
6. Cube 12. *Ans.* 1728.
7. Cube 36. *Ans.* 46656.
8. Cube 63. *Ans.* 250047.
9. Cube 99. *Ans.* 970299.

Find the value of

10. 65^2. *Ans.* 4225.
11. 32^3. *Ans.* 32768.
12. 27^4. *Ans.* 531441.
13. $(\frac{16}{17})^2$. *Ans.* $\frac{256}{289}$.
14. $9^3 \times 9^4$. *Ans.* 9^7.
15. $8^4 \times 8^5$. *Ans.* 8^9.
16. $15^3 \times 15^5$. *Ans.* 15^8.
17. $(\frac{1}{2})^2 \times (\frac{1}{2})^3$. *Ans.* $\frac{1}{32}$.
18. $(6\frac{1}{4})^3$. *Ans.* $244\frac{9}{64}$.
19. $(3.8)^4$. *Ans.* 208.5136.
20. $(6.25)^3$. *Ans.* 244.140625.
21. $(10.5)^4$. *Ans.* 12155.0625.
22. $3^2 \times 3^3 \times 3^4$. *Ans.* 3^9.
23. $(\frac{7}{8})^3 \times (\frac{7}{8})^4$. *Ans.* $(\frac{7}{8})^7$.
24. $(2.5)^4 \times (2.5)^6$. *Ans.* $(2.5)^{10}$.
25. $(3.3)^2 \times (3.3)^3$. *Ans.* $(3.3)^5$.

SQUARING NUMBERS.

549. There are **Two Methods** of explaining the principles of squaring numbers, called the *Analytic or Algebraic*, and the *Synthetic* or *Geometrical.*

1. The object of these methods is to find the law of forming the square, and thus prepare for corresponding methods of explaining Evolution.

2. Teachers who prefer the geometrical method will find it in the Supplement.

NOTE.—Teachers who prefer the geometrical method of explaining evolution may omit involution by the analytic method.

1. Find the square of 45 analytically.

OPERATION.

$$\begin{array}{rl} 45 = & 40+5 \\ \underline{45} = & \underline{40+5} \\ 225 = & 5\times 40+5^2 \\ \underline{180} = & \underline{40^2 + 5\times 40} \\ 2025 = & 40^2+2\times(5\times 40)+5^2 \end{array}$$

ANALYTICAL SOL.—Forty-five equals 40 plus 5, or 4 tens plus 5 units. Writing this as $40+5$, and commencing at units to square, we have 5 times 5 equals 5^2, 5 times 40 equals 5×20, 40 times 5 equal 5×40, 40 times 40 equals 40^2, and adding, we have $40^2+2\times(5\times 40)+5^2$; hence, the square of 45 equals the *square of the tens, plus twice the tens into the units, plus the square of the units,* which we find to be 2025.

WRITTEN EXERCISES.

Square the following numbers:

2. 24.	*Ans.* 576.	**8.** 254.	*Ans.* 64516.
3. 35.	*Ans.* 1225.	**9.** 467.	*Ans.* 218089.
4. 36.	*Ans.* 1296.	**10.** 703.	*Ans.* 494209.
5. 44.	*Ans.* 1936.	**11.** 838.	*Ans.* 702244.
6. 57.	*Ans.* 3249.	**12.** 2005.	*Ans.* 4020025.
7. 78.	*Ans.* 6084.	**13.** 4628.	*Ans.* 21418384.

550. The following principles, derived from the above solution, are important and should be committed to memory:

PRINCIPLES.

1. *The square of a number of two figures equals the* TENS^2 + 2 *times* TENS × UNITS + UNITS^2.

2. *The square of a number of three figures equals* HUNDREDS^2 + 2 *times* HUNDREDS × TENS + TENS^2 + 2(HUNDREDS + TENS) × UNITS + UNITS^2.

551. These principles may also be expressed by letters. Let u represent units figure, t tens, h hundreds, and T thousands, and a period between two letters denote multiplication; then we have

$$(t+u)^2 = t^2 + 2t.u + u^2.$$
$$(h+t+u)^2 = h^2 + 2h.t + t^2 + 2(h+t).u + u^2.$$
$$(T+h+t+u)^2 = T^2 + 2T.h + h^2 + 2(T+h).t + t^2.$$
$$2(T+h+t).u + u^2.$$

CUBING NUMBERS.

552. There are **Two Methods** of explaining the principles of cubing numbers, called the *Analytic* or *Algebraic*, and the *Synthetic* or *Geometrical*, method.

1. The object of these methods is to find the law of forming the cube, and thus to prepare for corresponding methods of explaining Evolution.

2. Teachers who prefer the geometrical method will find it in the Supplement.

1. Find the cube of 45 by the analytical method.

ANALYTICAL SOL.—Squaring 45 by the method already given, we have $40^2+2\times(5\times 40)+5^2$. We then multiply this by $40+5$. Five times 5^2 equals 5^3, 5 times $2\times 5\times 40$ equals $2\times 5\times 5\times 40$, or $2\times 5^2\times 40$. Five times 40^2 equals 5×40^2. We next multiply by 40. Forty times $5^2=40\times 5^2$, forty times $2\times 5\times 40$ equals $2\times 5\times 40^2$, forty times 40^2 equals 40^3. Taking the sum of these products, and we have first 5^3; next, *once* $5^2\times 40$ plus *twice* $5^2\times 40$ equals *three* times $5^2\times 40$; next, *twice* 5×40^2 plus *once* 5×40^2 equals *three* times 5×40^2, and next we have 40^3; hence, $45^3=40^3+3\times 5\times 40^2+3\times 5^2\times 40+5^3$. Therefore the cube of 45 equals ***the cube of the tens, plus three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units.***

OPERATION.

$$\begin{array}{rr}
45^2= & 40^2+2\times(5\times 40)+5^2 \\
45\ = & 40+5 \\
\hline
 & 5\times 40^2+2\times 5^2\times 40+5^3 \\
 & 40^3+2\times 5\times 40^2+\quad 5^2\times 40 \\
\hline
45^3= & 40^3+3\times 5\times 40^2+3\times 5^2\times 40+5^3
\end{array}$$

Cube the following numbers:

2. 14.	*Ans.* 2744.	**7.** 32.	*Ans.* 32768.
3. 17.	*Ans.* 4913.	**8.** 53.	*Ans.* 148877.
4. 21.	*Ans.* 9261.	**9.** 87.	*Ans.* 658503.
5. 25.	*Ans.* 15625.	**10.** 235.	*Ans.* 12977875.
6. 29.	*Ans.* 24389.	**11.** 327.	*Ans.* 34965783.

PRINCIPLES.

1. *The cube of a number consisting of two figures equals* TENS^3 + 3 *times* TENS^2 × UNITS + 3 *times* TENS × UNITS^2 + UNITS^3.

2. *The cube of a number consisting of three figures equals* HUNDREDS^3 + 3 *times* HUNDREDS^2 × TENS + 3 *times* HUNDREDS × TENS^2 + TENS^3 + 3 *times* (HUNDREDS + TENS)^2 × UNITS + 3 *times* (HUNDREDS + TENS) × UNITS^2 + UNITS^3.

553. These principles may also be expressed by letters, as follows:

$$(t+u)^3=t^3+3t^2.u+3t.u^2+u^3$$

$$(h+t+u)^3=h^3+3h^2.t+3h.t^2+t^3+3(h+t)^2.u$$
$$+3(h+t).u^2+u^3.$$

EVOLUTION.

554. Evolution is the process of finding a root of a number.

555. A **Root** of a number is *one* of its *equal* factors. Roots are of different degrees; as, *second, third,* etc.

556. The **Square Root,** or *second root,* of a number is *one* of its *two equal* factors. Thus, 8 is the square root of 64, since $8 \times 8 = 64$.

557. The **Cube Root,** or *third root,* of a number is *one* of its *three equal* factors. Thus, 4 is the cube root of 64, since $4 \times 4 \times 4 = 64$.

558. The **Fourth Root** is *one* of the *four equal* factors, the *fifth root* is *one* of the *five equal* factors, etc.

559. The **Symbol of Evolution** is $\sqrt{\ }$; thus, $\sqrt[2]{64}$, or $\sqrt{64}$, denotes the square root of 64; $\sqrt[3]{64}$ denotes the cube root of 64.

560. The **Index** of the root is a small figure placed in the angle of the symbol. The *index* indicates the degree of the root.

Roots are also indicated by the denominator of a fractional exponent; thus, $9^{\frac{1}{2}}$ denotes $\sqrt{9}$; $27^{\frac{1}{3}}$ denotes $\sqrt[3]{27}$, etc.

561. The following principles of involution are given to enable us to determine the number of figures in the root:

PRINCIPLES.

1. *The square of a number contains twice as many figures as the number itself, or twice as many less one.*

$$1^2 = 1 \quad 9^2 = 81 \quad 10^2 = 100 \quad 99^2 = 9801$$

DEM.—The square of 1 is 1, and the square of 9 is 81; hence, the square of a number consisting of *one* figure is a number consisting of *one* or *two* figures. The square of 10, the smallest number of two figures, is 100; the square of 99, the largest number of two figures, is 9801; hence, the square of a number consisting of *two* figures is a number consisting of *three* or *four* figures—that is, *twice two,* or *twice two less one,* etc. The same may be shown for the square of a number consisting of any number of figures.

2. *The cube of a number contains three times as many figures as the number itself, or three times as many, less one or two.*

DEM.—The cube of 1 is 1, and the cube of 9 is 729; hence, the cube of any number consisting of one figure is a number consisting of *one, two,* or *three* figures. The cube of 10 is 1000, a number of four figures; the cube of 99 is 970299, a number of six figures; hence, the cube of a number consisting of two figures contains *four, five,* or *six* figures—that is, *three times* two, or *three times* two *less one* or *two.* The same may be shown for the cube of a number consisting of any number of figures.

$$1^3=1$$
$$9^3=729$$
$$10^3=1000$$
$$99^3=970299$$

EVOLUTION BY FACTORING.

562. When the number is a perfect power and the factors are easily found, the root of a number can be readily obtained by the following

Rule.—*Resolve the number into its prime factors, and for the square root form a product by taking* ONE *of every* TWO *equal factors; for the cube root,* ONE *of every* THREE *equal factors,* etc.

WRITTEN EXERCISES.

1. Find the square root of 324.

SOLUTION.—We first resolve the number into its prime factors. Since the square root of a number is one of its two equal factors, we take *one* of every *two* equal factors and have $2 \times 3 \times 3 = 18$. Hence, the square root of 324 is 18.

NOTE.—We have marked the factors taken with a little star, and it will be well for the student to do the same in his solutions.

OPERATION.

2)324
*2)162
3)81
*3)27
3)9
*3

Solve the following problems:

2.	$\sqrt{225}$.	*Ans.* 15.	**7.**	$\sqrt[3]{19683}$.	*Ans.* 27.
3.	$\sqrt{1024}$.	*Ans.* 32.	**8.**	$\sqrt[3]{46656}$.	*Ans.* 36.
4.	$\sqrt{4096}$.	*Ans.* 64.	**9.**	$\sqrt[4]{5308416}$.	*Ans.* 48.
5.	$\sqrt{9604}$.	*Ans.* 98.	**10.**	$\sqrt[5]{7962624}$.	*Ans.* 24.
6.	$\sqrt{11664}$.	*Ans.* 108.	**11.**	$\sqrt[6]{11390625}$.	*Ans.* 15.

ORAL EXERCISES.

1. The square of a number is 64; what is the number?

2. The cube of a number is 27; what is the number?

3. The cube of a number is 125; what is the number?

4. Twice the square of a number is 98; what is the number?

5. Three times the square of a number is 108; what is the number?

6. One-third of the square of a number equals 48; what is the number?

7. The square of twice a number equals 100; what is the number?

8. The square of $\frac{2}{5}$ of a number equals 64; what is the number?

9. The square of twice a number is 18 more than twice the square of the number; what is the number?

10. Twice the square of a number is 8 more than 6 times the square of half the number; what is the number?

11. $\frac{2}{3}$ of the cube of a number is 10 more than the cube of $\frac{2}{3}$ of the number; what is the number?

12. The square of a number divided by the number equals 7; what is that number?

13. The cube of a number divided by the number equals 64; what is the number?

WRITTEN EXERCISES.

1. The third power of a number is 3375; what is the number? *Ans.* 15.

2. The fourth power of a number is 1296; what is the number? *Ans.* 6.

3. The product of the square and the cube of a number is 7776; what is the number? *Ans.* 6.

4. The product of the square of one number by the cube of another number is 675; what are the numbers?

Ans. 5 and 3.

SQUARE ROOT.

563. There are **Two Methods** of explaining the general process of extracting the square root, called the *Analytic* or *Algebraic Method*, and the *Synthetic* or *Geometrical Method.*

1. Teachers who prefer the geometrical method of explanation will find it in the Supplement.

2. With young pupils who have a difficulty in understanding evolution it may be well to drill them upon the method of doing the work, not requiring them to give the explanation until they are better prepared to understand it.

1. Extract the square root of 2025.

OPERATION.

$$\begin{array}{lrr} t^2+2t.u+u^2=2025 & & (40 \\ t^2=40^2 & 1600 & 5 \\ \hline 2t.u+u^2= & 425 & 45 \\ 2t=40\times 2=80 & & \\ (2t+u).u=(80+5)\times 5=425 & & \end{array}$$

ANALYTICAL SOLUTION.—Since the square of a number *contains twice as many figures as the number itself, or twice as many less one*, the square root of 2025 will consist of two places, and hence will consist of tens and units, and 2025 consists of *tens*^{2} + 2 × *tens* × *units* + *units*^{2}.

The greatest number of tens whose square is contained in 2025 is 4 tens; squaring the tens and subtracting, we have 425, which equals 2 × *tens* × *units* + *units*^{2}. Now, since 2 × *tens* × *units* must be greater than *units*^{2}, 425 must consist principally of twice the tens into the units; hence, if we divide by *twice the tens*, we can ascertain the units. Twice the *tens* equals $40\times 2=80$; dividing, we find the units to be 5; we now find 2 × *tens* × *units* + *units*^{2}, or, what is the same, 2 × *tens* + *units*, both multiplied by *units*, equals $(80+5)\times 5=425$; and subtracting, nothing remains. Hence, the square root of 2025 is 4 tens and 5 units, or 45.

NOTES.—1. When there are three figures in the root, we use the formula for three terms.

2. In practice we determine the number of figures in the root by pointing off the number into periods of two figures each, beginning at the right. We also abbreviate the work by omitting ciphers and condensing the other parts, preserving only the *trial* and *true* divisors. For illustration, see solution in the margin.

OPERATION.

$$\begin{array}{lr} & 10\cdot49\cdot76)324 \\ 3 & 9 \\ \hline 62 & 149 \\ & 124 \\ \hline 644 & 2576 \\ & 2576 \\ \hline \end{array}$$

Rule.—I. *Begin at units, and separate the number into periods of two figures each.*

II. *Find the greatest number whose square is contained in the left-hand period, place it at the right as a quotient, subtract its square from the left-hand period, and annex the next period to the remainder for a dividend.*

III. *Double the root found, and place it at the left for a* TRIAL DIVISOR; *divide the dividend, excluding the right-hand term, by this divisor for the second term of the root.*

IV. *Annex the second term of the root to the trial divisor for the* TRUE DIVISOR, *multiply the result by the second term of the root, subtract the product from the dividend, and bring down the next period for the next dividend.*

V. *Double the root now found for a second* TRIAL DIVISOR, *find the third term of the root as before, and thus proceed until all the periods have been used.*

1. If the product of a true divisor by a term of the root exceeds the dividend, the term of the root must be diminished by a unit.

2. When a cipher occurs in the root, annex a cipher to the trial divisor, bring down the next period, and proceed as before.

3. The square root of a common fraction is evidently the square root of each term. When these terms are not perfect squares, reduce the fraction to a decimal, and extract the root. When a number is not a perfect square, annex periods of ciphers and carry the root on to decimals.

4. By squaring 1, .1, .01, etc. we see that the *square of a decimal contains twice as many decimal places as the decimal;* hence, to extract the square root of a decimal we *point off the decimals into periods of two figures each, counting from the decimal point,* and proceed as in whole numbers.

$1^2 = 1$
$.1^2 = .01$
$.01^2 = .0001$

WRITTEN EXERCISES.

Extract the square root of

2. 289.	*Ans.* 17.	9. 59049.	*Ans.* 243.
3. 676.	*Ans.* 26.	10. 65536.	*Ans.* 256.
4. 784.	*Ans.* 28.	11. 164836.	*Ans.* 406.
5. 1369.	*Ans.* 37.	12. 277729.	*Ans.* 527.
6. 3136.	*Ans.* 56.	13. 390625.	*Ans.* 625.
7. 4096.	*Ans.* 64.	14. 5764801.	*Ans.* 2401.
8. 9216.	*Ans.* 96.	15. 7387524.	*Ans.* 2718.

Find the square root of

16. $\frac{169}{196}$. *Ans.* $\frac{13}{14}$.
17. $\frac{225}{256}$. *Ans.* $\frac{15}{16}$.
18. $\frac{784}{1089}$. *Ans.* $\frac{28}{33}$.
19. $\frac{1444}{2401}$. *Ans.* $\frac{38}{49}$.
20. .2809. *Ans.* .53.
21. .6084. *Ans.* .78.
22. .0729. *Ans.* .27.
23. .059049. *Ans.* .243.
24. .064516. *Ans.* .254.
25. 49.4209. *Ans.* 7.03.
26. 402.0025. *Ans.* 20.05.
27. 2141.8384. *Ans.* 46.28.
28. 3. *Ans.* 1.732+.
29. $\frac{2}{3}$. *Ans.* .81649+.
30. $\frac{16}{111}$. *Ans.* .37966+.

APPLICATIONS OF SQUARE ROOT.

564. The Applications of Square Root to problems involving geometrical figures are extensive.

Principle.—*The side of a square is equal to the square root of its area.*

WRITTEN EXERCISES.

1. A man owns a farm in the form of a square which contains 40 acres; how many rods in length or breadth is it?

SOLUTION.—The 40 acres equal 40×160, or 6400 sq. rd.; extracting the square root, we have 80 rods.

2. Henry has a square piece of land containing 640 acres; what is the length of one side? *Ans.* 320 rods.

3. How many rods of fence are required to enclose a square lot having an area of 9216 sq. ft.? *Ans.* $23\frac{3}{11}$ rods.

4. How many acres of land, in the form of a square, can be enclosed by 320 rods of fence? *Ans.* 40 acres.

5. What would it cost to fence a square lot containing 640 acres at \$4 per rod? *Ans.* \$5120.

6. A man has 300 yards of carpet $\frac{3}{4}$ yd. wide; what is the length of one side of a square room which the carpet will exactly cover? *Ans.* 15 yds.

7. A cabinet-maker has a board 15 ft. long by 1 ft. 8 in.

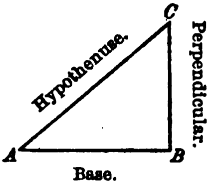
wide; what is the side of the largest square table he can make from it? *Ans.* 5 ft.

8. A general, wishing to form his army into a solid square of 75 men on a side, found he lacked 625 men to complete the square; how many men in his army? *Ans.* 5000.

9. A general, attempting to form an army of 8430 men into a square, found that he had 149 men over; what was the number on a side of the square? *Ans.* 91.

RIGHT TRIANGLES.

565. A **Right Triangle** is a triangle which has one right angle.



566. The **Base** of a triangle is the side on which it stands; as, *AB*.

567. The **Perpendicular** is the side which forms the right angle with the base; as, *BC*.

568. The **Hypothenuse** is the side opposite the right angle; as, *AC*.

569. The **Principles** of right triangles, derived from Geometry, are as follows:

PRINCIPLES.

1. *The square of the hypothenuse equals the sum of the squares of the other two sides.*

2. *Hence, the square of either side equals the square of the hypothenuse, diminished by the square of the other side.*

NOTE.—The smallest integers which can express the relation of the three sides of a right triangle are 3, 4, and 5. We may have an infinite number of right triangles with their sides in this relation. Other relations are 5, 12, and 13; 8, 15, and 17, etc.

WRITTEN EXERCISES.

1. The two sides of a right triangle are 60 and 80 inches, respectively; required the hypothenuse.

SOLUTION.—Hypothenuse $= \sqrt{60^2 + 80^2} = \sqrt{10000} = 100$, *Ans.*

2. Two rafters, each 34 feet long, meet at the ridge of a roof 16 feet above the top of the walls; what is the width of the house? *Ans.* 60.

3. A rope 85 ft. long stretches from the top of a derrick to a point on the ground 40 feet from the foot of the derrick; how high is the derrick? *Ans.* 75 ft.

4. A rectangular lot of land is 1260 rods long and 525 rods broad; what is the distance between two opposite corners? *Ans.* 1365 rods.

5. Two vessels sail from the same port: one sails north 9 miles an hour, the other sails west 12 miles an hour; how far are they apart in 2 days? *Ans.* 720 miles.

6. A ladder 26 ft. long stands close against a building; how far must it be drawn out at the bottom that the top may be lowered 2 feet? *Ans.* 10 ft.

7. A telegraph-pole was broken 12 feet from the bottom, and fell so that the end struck 16 ft. from the foot; required the length of the pole. *Ans.* 32 ft.

8. A ladder 130 ft. long, with its foot in the street, will reach on one side to a window 78 ft. high, and on the other to a window 50 ft. high; what is the width of the street? *Ans.* 224 ft.

SIMILAR FIGURES.

570. Similar Figures are those which have the same form. Thus, circles are similar figures; also squares, etc.

571. The **Principles** of similar figures, derived from Geometry, are as follows:

PRINCIPLES.

1. *The areas of all similar figures are to each other as the squares of their like dimensions.*

2. *Hence, the like dimensions of similar figures are to each other as the square roots of their areas.*

WRITTEN EXERCISES.

1. The area of a rectangle is 320, and one side is 16; what is the area of a similar rectangle, the shorter side being 24?

SOLUTION.—Since the rectangles are similar, their areas are as the squares of their corresponding sides; hence, we have the proportion in the margin. Cancelling and multiplying, we have 720.

OPERATION.

Area of 2d : $320 = 24^2 : 16^2$

Area of 2d $= \dfrac{320 \times 24^2}{16^2} = 720$, *Ans.*

2. I have a lot 25 rd. long and 18 rd. broad; what are the dimensions of a similar lot 4 times as large? *Ans.* 50; 36.

3. There are two circular gardens, one 8 rods in diameter, and the other 32 rods; the second is how many times as large as the first? *Ans.* 16 times.

4. The area of a circle whose diameter is 10 feet is 78.54 square feet; what is the diameter of a circle whose area is 1256.64 square feet? *Ans.* 40 ft.

5. A farmer has a rectangular field 60 rods long and 45 wide; what are the dimensions of another similar field containing 30 acres? *Ans.* 80 rd.; 60 rd.

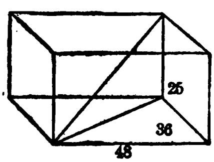
6. If a horse tied to a post in the centre of a field by a rope, 1 ch. $78\frac{1}{4}$ li., can graze upon an acre, what length of rope would allow it to graze upon $5\frac{4}{9}$ acres? *Ans.* 4 ch. $15\frac{11}{12}$ li.

7. If a pipe $\frac{3}{4}$ in. in diameter pours into a cistern 25 gal. in a given time, how much will a pipe 3 in. in diameter pour into the cistern in the same time? *Ans.* 400 gal.

8. If a pipe whose diameter is 1.5 in. fills a cistern in 5 hours, in what time will a pipe whose diameter is 3 in. fill the same cistern? *Ans.* $1\frac{1}{4}$ hours.

SUG.—It pours in four times as much, and fills it in $\frac{1}{4}$ of 5 hours.

9. Required the distance between a lower corner and the opposite upper corner of a room 48 feet long, 36 feet wide, and 25 feet high. *Ans.* 65 feet.



10. Two men, A and B, started from one corner of a square field containing 40 acres: A went directly across to the opposite corner, while B walked round the sides of the field; if A, after reaching the corner, goes along the side to meet B, how far from the corner would they meet? *Ans.* 23.431+ rods.

CUBE ROOT.

572. There are **Two Methods** of explaining the general process of extracting the cube root, called the *Analytic* or *Algebraic Method* and the *Synthetic* or *Geometrical Method.*

Teachers who prefer the geometrical method of explanation will find it in the Supplement.

1. Extract the cube root of 97336.

OPERATION.

$$
\begin{array}{rr|rr}
 & & 97336 & (40 \\
 & 40^3 = & \underline{64000} & \underline{6} \\
\textit{trial divisor}, \ 3\times 40^2 & = 4800 & 33336 & 46 \\
3\times 40\times 6 & = 720 & & \\
6^2 & = \underline{36} & & \\
\textit{true divisor}, & 5556 & \underline{33336} &
\end{array}
$$

SHOWN BY LETTERS.

$$
\begin{array}{l}
t^3 + 3t^2 \times u + 3t \times u^2 + u^3 = 97336)46 \\
\underline{t^3 = 40^3 \qquad\qquad\qquad\qquad = 64000} \\
\qquad 3t^2 \times u + 3t \times u^2 + u^3 = 33336 \\
3t^2 \times 3 \times 40^2 \qquad\quad = 4800 \\
3t \times u = 3 \times 40 \times 5 = \ 720 \\
\qquad \underline{u^2 = 5^2 \qquad\qquad\quad 36} \\
(3t^2 + 3t \times u + u^2) \times u = 5556 \times 6 = 33336
\end{array}
$$

ANALYTIC SOLUTION.—Since the cube of a number consists of *three times as many places as the number* itself, or of *three times as many less one or two,* the cube root of 97336 will consist of two places, or of tens and units, and the number itself will consist of *tens*^{3} + 3 × *tens*^{2} × *units* + 3 × *tens* × *units*^{2} + *units*^{3}.

The greatest number of tens whose cube is contained in 97336 is 4 tens. Cubing the tens and subtracting, we have 33336, which equals 3 × *tens*^{2} × *units* + 3 × *tens* × *units*^{2} + *units*^{3}. Now, since 3 × *tens*^{2} × *units* is much greater than 3 × *tens* × *units*^{2} + *units*^{3}, 33336 consists principally of 3 times *tens*^{2} × *units;* hence, if we divide by 3 times *tens*^{2}, we can ascertain the *units* : 3 times *tens*^{2} equals $3 \times 40^2 = 4800$; dividing by 4800, we find the *units* to be 6. We then find 3 times *tens* × *units* equal to $3 \times 40 \times 6 = 720$, and *units*^{2} = $6^2 = 36$, and, adding these and multiplying by *units*, we have (3 *tens*^{2} + 3 *tens* × *units* + *units*^{2}) × *units*, which equals $5556 \times 6 = 33336$; subtracting, nothing remains; hence, the cube root of 97336 is 46.

2. Find the cube root of 14348907.

We will solve this problem, indicating the solution by means of letters, and abbreviating the operation as in practice. A point like a period indicates the multiplication of the letters.

SHOWN BY LETTERS.

$$\begin{array}{lrr|l}
 & & & \textit{htu} \\
 & & & 14348907(243 \\
h^3 = 200^3 = & & 8000000 & \\
\hline
 & & & 6348907 \\
3h^2 = 3 \times 200^2 & = & 120000 & \\
3h.t = 3 \times 200 \times 40 & = & 24000 & \\
t^2 = 40^2 & = & 1600 & \\
\hline
 & & 145600 & 5824000 \\
\hline
 & & & 524907 \\
3(h+t)^2 = 3 \times 240^2 & = & 172800 & \\
3(h+t)u = 3 \times 240 \times 3 & = & 2160 & \\
u^2 = 3^2 & = & 9 & \\
\hline
 & & 174969 & 524907 \\
\hline
\end{array}$$

OPERATION AS IN PRACTICE.

$$\begin{array}{lr|l}
 & & 14{\cdot}348{\cdot}907(243 \\
2^3 = & 8 & \\
\hline
 & & 6348 \\
2^2 \times 300 = & 1200 & \\
2 \times 4 \times 30 = & 240 & \\
4^2 = & 16 & \\
\hline
 & 1456 & 5824 \\
\hline
 & & 524907 \\
24^2 \times 300 = & 172800 & \\
24 \times 3 \times 30 = & 2160 & \\
3^2 = & 9 & \\
\hline
 & 174969 & 524907 \\
\hline
\end{array}$$

The method employed in actual practice is derived from the other by omitting ciphers, using parts of the number instead of the whole number each time we obtain a figure of the root, etc. It will also be seen that by separating the number into *periods of 3 figures each*, we have the *number of places in the root*, the *part of the number* used in obtaining each figure of the root, etc.

Rule.—I. *Begin at units and separate the number into periods of three figures each.*

II. *Find the greatest number whose cube is contained in the left-hand period, write it for the first term of the root, subtract its cube from the left-hand period, and annex the next period to this remainder for a dividend.*

III. *Square the first term of the root and multiply by* 300 *for a* TRIAL DIVISOR; *divide the dividend by the trial divisor for the second term of the root.*

IV. *To the trial divisor add* 30 *times the product of the second term of the root by the first term, and also the square of the second term; their sum will be the* TRUE DIVISOR.

V. *Multiply the true divisor by the second term of the root, subtract the product from the dividend, and annex the next period for another dividend.*

VI. *Square the root now found, multiply by* 300, *and find the third term of the root as before, and thus continue until all the periods have been used.*

1. If the product of the true divisor by the term of the root exceeds the dividend, the root must be diminished by a unit.

2. When a dividend will not contain a trial divisor, place a cipher in the root and two ciphers at the right of the trial divisor; bring down the next period, and proceed as before.

3. To find the cube root of a common fraction, extract the cube root of both terms. When these are not perfect cubes, reduce to a decimal and then extract the root.

4. By cubing 1, .1, .01, etc. we see that the *cube of a decimal contains three times as many decimal places as the decimal;* hence, to extract the cube root of a decimal we *point off the decimal into periods of three figures each, counting from the decimal point.*

$1^3 = 1$
$.1^3 = .001$
$.01^3 = .000001$

WRITTEN EXERCISES.

Find the cube root of

1. 12167.	*Ans.* 23.	**14.** 1124864.	*Ans.* 104.
2. 32768.	*Ans.* 32.	**15.** 8998912.	*Ans.* 208.
3. 39304.	*Ans.* 34.	**16.** 41063625.	*Ans.* 345.
4. 46656.	*Ans.* 36.	**17.** 43614208.	*Ans.* 352.
5. 68921.	*Ans.* 41.	**18.** 75686967.	*Ans.* 423.
6. 148877.	*Ans.* 53.	**19.** $\frac{2197}{9261}$.	*Ans.* $\frac{13}{21}$.
7. 274625.	*Ans.* 65.	**20.** 185.193.	*Ans.* 5.7.
8. 438976.	*Ans.* 76.	**21.** 629.422793.	*Ans.* 8.57.
9. 551368.	*Ans.* 82.	**22.** 4.	*Ans.* 1.5874+.
10. 614125.	*Ans.* 85.	**23.** 5.	*Ans.* 1.7099+.
11. 2571353.	*Ans.* 137.	**24.** 6.	*Ans.* 1.8171+.
12. 12326391.	*Ans.* 231.	**25.** $\frac{2}{3}$.	*Ans.* .873+.
13. 34965783.	*Ans.* 327.	**26.** 1879080904.	*Ans.* 1234.

NOTE.—For a shorter method of Cube Root, see Supplement.

APPLICATIONS OF CUBE ROOT.

573. The **Applications** of cube root to problems involving geometrical volumes, such as cubes, parallelopipeds, spheres, etc., are extensive.

Principle.—*The edge of a cube is equal to the cube root of the contents of the cube.*

WRITTEN EXERCISES.

1. Required the dimensions of a cubical chest which shall contain 125000 cubic feet. *Ans.* 50 ft.

2. The pedestal of a certain statue is a cubical block whose contents are 175616 cu. ft.; required the number of square feet in one face. *Ans.* 3136.

3. What is the entire surface of a cube whose cubical contents are 74088 cubic feet? *Ans.* 10584 sq. ft.

4. What is the edge of a cube which shall contain as much as a solid 30 ft. 4 in. long, 15 ft. 6 in. wide, and 9 ft. 8 in. high? *Ans.* 16.56 ft.+.

5. What is the depth of a cubical cistern which shall contain 300 gal. (231 cu. in.) of water? *Ans.* 41.07 in.+.

6. A farmer had a cubical bin which contained 60 bushels of grain; what was its depth? *Ans.* 4.21 ft.+.

7. What would it cost to plaster the bottom and sides of a cubical reservoir which contains 100 barrels ($31\frac{1}{2}$ gals.) of water at 6 cents a square foot? *Ans.* $16.85.

SIMILAR VOLUMES.

574. Similar Volumes are such as have the same shape, but differ in size; as, cubes, spheres, etc.

575. A **Dimension** of a volume is a length, breadth, height, diameter, radius, circumference, etc.

576. The **Principles** of similar volumes are derived from geometry.

PRINCIPLES.

1. *Similar volumes are to each other as the cubes of their like dimensions.*

2. *Like dimensions of similar volumes are to each other as the cube roots of those volumes.*

WRITTEN EXERCISES.

1. A man has two balls, one 8 in. in diameter, the other 4 in.; the first is how many times as large as the second?

SOLUTION.—By the principle above we have the proportion 1st : 2d : : $8^3 : 4^3$; and since the 1st term equals the 2d term multiplied by the ratio of the 3d to the 4th, we have 1st term = 2d term multiplied by the ratio of 8^3 to 4^3, which is 2d term $\times (\frac{8}{4})^3$, or 2d $\times 2^3$, or 2d $\times$ 8. Hence, the 1st term is 8 times as large as the 2d.

OPERATION.

1 : 2d : : $8^3 : 4^3$
1st = 2d $\times (\frac{8}{4})^3 =$ 2d $\times 2^3$
1st = 2d $\times$ 8

2. Required the relation of two cubes whose dimensions are 4 in. and 16 in. respectively. *Ans.* 2d is 64 times 1st.

3. If a ball 4 in. in diameter weigh 20 pounds, what will be the weight of a ball 6 in. in diameter? *Ans.* $67\frac{1}{2}$ lb.

4. I have two cubical boxes, one having a length of 9 inches and the other of 12; how many times will the larger contain the smaller? *Ans.* $2\frac{10}{27}$ times.

5. If a haystack 10 ft. in diameter contain 12 tons of hay, what will be the contents of a stack of hay whose diameter is 15 feet? *Ans.* $40\frac{1}{2}$ tons.

6. If a man 5 ft. 4 in. high weighs 160 lb., what is the weight of a man of similar build whose height is 6 ft.? *Ans.* $227\frac{13}{16}$ lb.

7. The sun is 885680 miles in diameter, and the earth 7912 miles; the sun is how many times as large as the earth? *Ans.* About 112^3.

8. There are two balls whose diameters are respectively 4 in. and 5 in.; what is the diameter of a ball whose contents are equal to the contents of both? *Ans.* 5.74 in.—.

SUG.—Cube 4 and 5, take their sum, and then compare this with either of the given balls.

HIGHER ROOTS.

577. Any root whose index contains only the factors 2 or 3 can be extracted by means of the square and cube root, according to the following principle:

PRINCIPLE.

A root of a number equals a root of a root of the number in which the product of the indices of the two latter roots equals the index of the former.

Since the square of the cube of a number equals the sixth power, the sixth root of a number equals the square root of the cube root of the number; and the same is true in any other case.

WRITTEN EXERCISES.

1. Extract the sixth root of 4096.

SOLUTION.—To find the sixth root of 4096, we first extract the square root, which we find to be 64, and then find the cube root of 64, which is 4. Hence, the sixth root of 4096 is 4.

Required the value of

2. $\sqrt[4]{625}$. *Ans.* 5.

3. $\sqrt[6]{729}$. *Ans.* 3.

4. $\sqrt[4]{20736}$. *Ans.* 12.

5. $\sqrt[8]{390625}$. *Ans.* 5.

6. $\sqrt[9]{262144}$. *Ans.* 4.

7. $\sqrt[12]{16777216}$. *Ans.* 4.

NOTE.—For a general rule of Evolution, see *Brooks's Higher Arithmetic.*

MISCELLANEOUS EXERCISES.

1. The square of the square of a number equals 50625; what is the number? *Ans.* 15.

2. The square of the cube of a number equals 46656; what is the number? *Ans.* 6.

3. The square of a number multiplied by the cube of the number equals 4084101; what is the number? *Ans.* 21.

SECTION XI.

MENSURATION.

578. **Mensuration** treats of the measurement of geometrical magnitudes.

579. **Geometrical Magnitudes** consist of the *Line*, *Surface*, *Volume*, and *Angle*.

Teachers will illustrate the rules of Mensuration concretely, so far as possible.

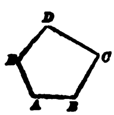
MENSURATION OF SURFACES.

580. A **Surface** is that which has length and breadth without thickness. Surfaces are *plane* or *curved*.

581. A **Plane Surface** is a surface such that if any two of its points be joined by a straight line, every part of that line will lie in the surface.

582. A **Plane Figure** is a plane surface bounded by lines either straight or curved.

583. A **Polygon** is a plane figure bounded by straight lines; as, *ABCDE*.



A Polygon of three sides is called a *Triangle;* of four sides, a *Quadrilateral;* of five sides, a *Pentagon;* of six sides, a *Hexagon*, etc.

584. A **Diagonal** of a polygon is a line joining the vertices of two angles not consecutive.

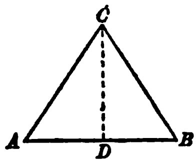
585. The **Perimeter** of a polygon is the sum of its sides.

586. The **Area** of a plane figure is the number of square units in its surface.

NOTE.—The principles of Mensuration are derived from geometry; their application to practical purposes is usually given in arithmetic.

THE TRIANGLE.

587. A **Triangle** is a polygon of three sides and three angles; as, *ABC*.



588. The **Base** is the side upon which it seems to stand; as, *AB*. The *Altitude* is a line perpendicular to the base, drawn from the angle opposite; as, *CD*.

589. An **Equilateral Triangle** is a triangle which has its three sides equal; when two sides are equal it is called *isosceles;* when its sides are unequal it is called *scalene.*

Rule.—*To find the area of a triangle, multiply the base by one-half of the altitude.*

NOTE.—If the three sides are given and not the altitude, *take half the sum of the sides, subtract from it each side separately, multiply the half sum and these remainders together, and take the square root of the product.*

WRITTEN EXERCISES.

1. Required the area of a triangle whose base is 45 rods and altitude 27 rods. *Ans.* 3 A. 127½ P.

2. Required the area of a triangular tract whose base is 645 rods and altitude 484 rods. *Ans.* 975 A. 90 P.

3. Required the area of an equilateral triangle whose sides are each 10 chains. *Ans.* 43.30+ ch.

4. What is the area of a field whose sides are respectively 30, 40, and 50 chains? *Ans.* 60 A.

5. The rafters of a house are 25 ft. long on one side and 20 on the other, and the width of the house is 35 ft.; how many square feet of boards will it require to board up both gables? *Ans.* 489.9 sq. ft.

THE QUADRILATERAL.

590. A **Quadrilateral** is a polygon having four sides and four angles. There are three classes—the *parallelogram*, *trapezoid*, and *trapezium*.

591. A **Parallelogram** is a quadrilateral whose opposite sides are parallel. The *altitude* is the perpendicular distance between its opposite sides.

592. A parallelogram which is right-angled is called a *Rectangle.* When the four sides are equal it is called a *Square.*

593. An oblique-angled parallelogram is called a *Rhomboid.* An equilateral rhomboid is called a *Rhombus.*

Rule.—*To find the area of a parallelogram, multiply the base by the altitude.*

WRITTEN EXERCISES.

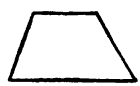
1. What is the area of a parallelogram 30 feet long and 24 feet wide? *Ans.* 80 sq. yd.

2. A has a rectangular lot 180 chains long and 60 chains wide; what is its area? *Ans.* 1080 A.

3. What is the difference in the area of two lots, one being 354 rd. long, 52 rd. wide, and the other 95 chains long and 28 chains wide? *Ans.* 150 A. 152 P.

4. A carpenter had a plank 18 inches wide, from which he wished to saw off 9 square feet; what will be the length of the piece sawed off? *Ans.* 6 ft.

594. A **Trapezoid** is a quadrilateral which has two of its sides parallel. Its *altitude* is the perpendicular distance between its parallel sides.



Rule.— *To find the area of a trapezoid, multiply one-half the sum of the parallel sides by the altitude.*

WRITTEN EXERCISES.

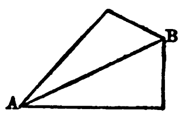
1. Required the area of a trapezoid, one side being 150 in., the other 90 in., and the altitude 48 in. *Ans.* 40 sq. ft.

2. What is the area of a plank 15 feet long, 20 inches wide at one end and 12 inches at the other end? *Ans.* 20 sq. ft.

3. A farm in the form of a trapezoid has its parallel sides 72 ch. and 84 ch. in length, and the perpendicular distance between them is 40 ch.; how large is the farm? *Ans.* 312 A.

4. A farmer has two fields, one in the form of a trapezoid, the two parallel sides being 40 and 44 rods respectively, and the perpendicular distance between them being 28 rods; and the other a rectangle, 42 rods long and 28 rods wide; what is the difference in their areas? *Ans.* They are equal.

595. A **Trapezium** is a quadrilateral which has none of its sides parallel. A diagonal, as AB, divides the trapezium into two triangles.



Rule.—*To find the area of a trapezium, divide the trapezium into two triangles by a diagonal, find the area of each triangle, and take the sum.*

WRITTEN EXERCISES.

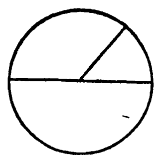
1. What is the area of a trapezium whose diagonal is 145 in., and the altitudes of the triangles, the diagonal being the base, are 32 and 48 in. respectively? *Ans.* 40 sq. ft. 40 sq. in.

2. Required the area of a farm in the form of a trapezium the length of whose sides are respectively 30, 40, 35, and 45 chains, and the length of the diagonal 50 chains.

Ans. 136 A. 77.6+ P..

THE CIRCLE.

596. A **Circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the *centre.*



597. The curved line is called the *circumference,* and a line passing through the centre and ending in the circumference is the *diameter.* Half the diameter is called the *radius.*

Rule.—I. *The circumference of a circle equals the diameter multiplied by* 3.1416.

II. *The diameter of a circle equals the circumference multiplied by* .3183.

WRITTEN EXERCISES.

1. What is the circumference of a wheel whose diameter is 5 feet 8 inches? *Ans.* 17.8024 ft.

2. What is the distance around a circular fish-pond, the diameter of which is 20 rods? *Ans.* 62.832 rd.

3. A man has a garden in the form of a circle, the diameter of which is 28 rods; what is the distance around the garden? *Ans.* 87.9648 rd.

4. What is the diameter of a water-wheel whose circumference is 47.124 feet? *Ans.* 15 feet.

5. What is the diameter of a circular park whose circumference is 2 miles? *Ans.* 203.712 rd.

Rule.—I. *The area of a circle equals the circumference multiplied by one-fourth of the diameter, or the square of the circumference multiplied by* .07958.

II. *The area of a circle equals the square of the radius multiplied by* 3.1416, *or the square of the diameter multiplied by* .785398.

III. *To find the diameter of a circle when the area is given, divide the area by* .785398, *and extract the square root.*

NOTE.—The area will vary slightly in the decimal figures as we use the different rules.

WRITTEN EXERCISES.

1. What is the area of a circle whose diameter is 15 and circumference 47.124? *Ans.* 176.715.

2. What is the area of a circular park whose diameter is 210 rods? *Ans.* 216 A. 76.0518 P.

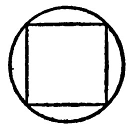
3. What is the area of a circular garden whose circumference is 150 rods? *Ans.* 11 A. 30.55 P.

4. The area of a circle is 176.715; what are the diameter and the circumference? *Ans.* Diam., 15; circ., 47.124.

5. If a cow is fastened to a post in a field, how long must the rope be to allow her to graze over 1 acre? *Ans.* 7.136+ rd.

6. A circular park contains 120 acres; how many rods of fence will be needed to go round it? *Ans.* 491.189 rd.

598. A square is inscribed in a circle when each of its angles is in the circumference.



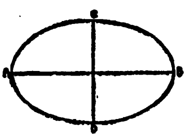
Rule.—*To find the side of an inscribed square, multiply the diameter by* .707106 *or multiply the circumference by* .225079.

1. The diameter of a round stick of timber is 2 ft. 6 in.; what is the side of a square stick that can be cut out of it? *Ans.* 1.767 + ft.

2. How large a square can be cut out of a circular board whose circumference is 250 inches? *Ans.* 56.26975 in.

THE ELLIPSE.

599. An **Ellipse** is a plane figure bounded by a curved line, the sum of the distances from every point of which to two fixed points is equal to the line drawn through these points and terminated by the curve. The two fixed points are called *foci;* the line through the foci is the *transverse* axis; and a line perpendicular to this passing through the centre and terminated by the curve is the *conjugate* axis.



Rule.—*To find the area of an ellipse, multiply half of the two axes together, and that product by* 3.1416.

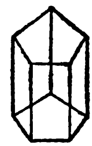
1. What is the area of an ellipse whose transverse axis is 50 inches and conjugate axis is 18 inches? *Ans.* 706.86 sq. in.

2. Required the area of an elliptical mirror whose length is 5.5 feet and breadth 4 feet. *Ans.* 17.2788 sq. ft.

MENSURATION OF VOLUMES.

600. A **Volume** is that which has length, breadth, and thickness.

THE PRISM.



601. A **Prism** is a volume whose ends are equal and parallel polygons and whose sides are parallelograms.

602. The polygons are called *bases*, the parallelograms form the *lateral surface*, and the prism takes its name from the form of its base.

603. The **Parallelopiped** is a prism whose bases are parallelograms. A *cube* is a parallelopiped all of whose sides are squares.

Rule.—I. *The lateral surface of a prism equals the perimeter of the base multiplied by the height.*

NOTE.—To find the entire surface we add the area of the bases.

II. *The contents of a prism equal the area of the base multiplied by the altitude of the prism.*

WRITTEN EXERCISES.

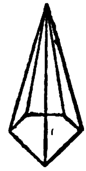
1. What is the lateral surface of a triangular prism, the three sides of whose base are respectively 6, 7, and 8 feet, and height 50 feet? *Ans.* 1050 sq. ft.

2. What is the entire surface of the triangular prism given in the first problem? *Ans.* 1090.66 sq. ft.

3. What are the contents of a square prism whose altitude is 40 feet and the side of the base 4 feet? *Ans.* 640 cu. ft.

4. Required the contents of a triangular prism, the sides of whose base are each 20 inches, and whose altitude is 30 inches. *Ans.* 5196.15 cu. in.

THE PYRAMID.



604. The **Pyramid** is a volume bounded by a polygon and several triangles meeting in a common point. The polygon is called the *base*, and the triangles form the *lateral surface.*

605. The point at the top is called the *vertex;* the distance from the vertex to the base is the *altitude;* the distance from the vertex to the middle of a side is the *slant height.*

Rule.—I. *The lateral surface of a pyramid equals the perimeter of the base multiplied by one-half the slant height.*

II. *The contents of a pyramid equal the area of the base multiplied by one-third of the altitude.*

WRITTEN EXERCISES.

1. What is the lateral surface of a triangular pyramid, the sides of whose base are each 5 ft., and whose slant height is 30 ft.? *Ans.* 225 sq. ft.

2. Required the lateral surface of a pentagonal pyramid, the sides of whose base are each 6 ft., and whose slant height is 50 ft. *Ans.* 750 sq. ft.

3. What will be the cost of painting a hexagonal church-spire at 30¢ a sq. yd., the sides of the base being 6 ft. and slant height 75 ft.? *Ans.* $45.

4. Required the contents of a pyramid whose base is 6 ft. square, and whose altitude is 75 ft. *Ans.* 900 cu. ft.

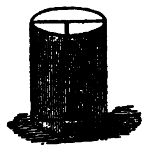
5. Required the contents of a pyramid whose base is a triangle, each side of which is 10 ft., and the altitude of the pyramid is 84 ft. *Ans.* 1212.428 cu. ft.

6. What is the weight of the pyramid in Ex. 5 if it is composed of granite weighing 165 lb. a cubic foot?

Ans. 100 T. 50.62 lb.

THE CYLINDER.

606. The **Cylinder** is a body bounded by a uniformly curved surface of uniform diameter, with circles for its ends. The two circular ends are called *bases*.



607. The **Altitude** of a cylinder is the distance from the centre of one base to the centre of the other.

Rule.—I. *To find the convex surface of a cylinder, multiply the circumference of the base by the altitude.*

II. *The contents of a cylinder equal the area of the base multiplied by the altitude.*

WRITTEN EXERCISES.

1. What is the convex surface of a cylinder, altitude 10 ft. and diameter of base 5 ft.? *Ans.* 157.08 sq. ft.

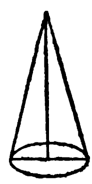
2. What is the convex surface of a cylinder 50 feet long and 16 feet in diameter? *Ans.* 2513.28 sq. ft.

3. Required the contents of a cylinder 40 feet long and 6 feet in diameter. *Ans.* 1130.976 cu. ft.

4. Required the contents of a cylindrical log 15 feet long and $8\frac{2}{3}$ feet in diameter. *Ans.* 884.884 cu. ft.

THE CONE.

608. A **Cone** is a volume whose base is a circle, and whose convex surface tapers uniformly to a point called the *vertex*.



609. The **Altitude** of a cone is the distance from the vertex to the centre of the base, and the *slant height* is the distance from the vertex to the circumference of the base.

Rule.—I. *The convex surface of a cone equals the circumference of the base multiplied by one-half the slant height.*

II. *The contents of a cone equal the area of the base multiplied by one-third of the altitude.*

WRITTEN EXERCISES.

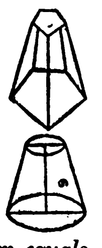
1. What is the convex surface of a cone, the circumference of whose base is 72 inches and slant height 60 inches? *Ans.* 2160 sq. in.

2. I have a conical haystack whose slant height is 9.25 ft., and the diameter of the base is 8.5 ft.; how many square yards of canvas will cover it completely? *Ans.* 13.723 sq. yd.

3. Required the contents of a sugar-loaf, diameter of the base being 6 in. and height 15 in. *Ans.* 141.372 cu. in.

4. How many cubic feet in a conical haystack 8 ft. high, and base 24 ft. in circumference? *Ans.* 122.227 cu. ft.

THE FRUSTUM OF A PYRAMID AND CONE.



610. The **Frustum of a Pyramid** is the part of a pyramid which remains after cutting off the top by a plane parallel to the base.

611. The **Frustum of a Cone** is the part of a cone which remains after cutting off the top by a plane parallel to the base.

Rule.—I. *The lateral or convex surface of a frustum equals the sum of the perimeters or circumferences of the two bases multiplied by one-half the slant height.*

II. *To find the contents of a frustum, take the sum of the two bases and the square root of their product, and multiply this sum by one-third of the altitude of the frustum.*

WRITTEN EXERCISES.

1. Required the lateral surface of the frustum of a square pyramid whose slant height is 20 feet, the side of the lower base 18 feet and upper base 12 feet. *Ans.* 1200 sq. ft.

2. Required the surface of a frustum of a cone whose slant height is 12 feet, diameter of lower base 10 feet and upper base 6 feet. *Ans.* 301.5936 sq. ft.

3. Required the cost, at $12\frac{1}{2}$¢ a sq. yd., of painting the lateral surface of a vat 12 ft. in diameter at the bottom and 10 ft. at the top, the slant height being 14 ft. *Ans.* $6.72.

4. What are the contents of the frustum of a square pyramid the sides of whose bases are 12 and 18 feet, and whose altitude is 18 feet? *Ans.* 4104 cu. ft.

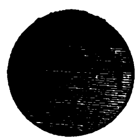
SUG.—$12^2+18^2+\sqrt{12^2\times 18^2}=144+324+216=684$, and this multiplied by 6 equals 4104 cu. ft.

5. What is the value of the wheat, at $0.75 a bushel, contained in a bin the lower base of which is 8 ft. square, the upper base 12 ft. square, and the height 9 ft.? *Ans.* $547.20.

6. What is the amount of timber in a log which measures 60 feet in length, the radius of one base being 8 feet and of the other 5 feet? *Ans.* 8105.328 cu. ft.

7. A vat measures 12 feet in diameter at top, 15 feet at bottom, and is 10 feet deep; how many gallons of water will be required to fill it? *Ans.* 10751.616 + gal.

THE SPHERE.



612. A **Sphere** is a volume bounded by a curved surface, every point of which is equally distant from a point within called the *centre.*

613. The **Diameter** of a sphere is a line passing through its centre and ending in the surface. The *radius* is half the diameter.

Rule.—I. *The surface of a sphere equals the circumference multiplied by the diameter, or the square of the radius multiplied by* 4 *times* 3.1416.

II. *The contents of a sphere equal the cube of the diameter multiplied by* $\frac{1}{6}$ *of* 3.1416.

III. *To find the edge of a cube which may be cut from a given sphere, square the diameter, divide by 3, and extract the square root of the quotient.*

WRITTEN EXERCISES.

1. Required the surface of a sphere whose diameter is 36 inches. *Ans.* 4071.5136 sq. in.

2. Required the surface of a sphere whose diameter is 108 inches. *Ans.* 36643.6224 sq. in.

3. Required the contents of a cannon-ball whose diameter is 9 inches. *Ans.* 381.7044 cu. in.

4. The diameter of the earth is 8000 miles; required its surface and solid contents. *Ans.* Sur., 201062400 sq. mi.

5. What is the weight of a cannon-ball 20 in. in diameter, a cu. ft. of iron weighing about 450 lb. *Ans.* 1090.83+ lb.

6. How many cubic feet of iron in a hollow shell whose outside diameter is 16 inches and inside diameter 12 inches? *Ans.* .717+ cu. ft.

7. What is the edge of a cube which may be cut from a sphere 24 inches in diameter? *Ans.* 13.856+ in.

8. What is the edge of a cube which can be cut from a billiard-ball whose circumference is $9\frac{1}{2}$ inches? *Ans.* 1.746— in.

GAUGING.

614. Gauging is the process of finding the capacity of casks and other vessels.

Barrels and casks differ from cylinders in bulging out in the middle. By ascertaining the approximate mean diameter of the cask or barrel, the capacity can be obtained like that of a cylinder.

Rule.—I. *To find the mean diameter of a barrel or cask, add to the head diameter $\frac{2}{3}$, or, if the staves are not much curved, $\frac{3}{5}$, of the difference between the head and bung diameters.*

II. *To find the capacity in gallons, multiply the square of the mean diameter by the length (both expressed in inches), and this product by* .0034.

WRITTEN EXERCISES.

1. How many gallons in a cask whose head diameter is 28 in., bung diameter 36 in., and length 40 in.? *Ans.* $151\frac{1}{9}$ gal.

2. How many gallons in a barrel of slight curvature, 3 ft. long, the head diameter being 26 in. and the bung diameter 29 in.? *Ans.* 94.595616 gal.

EXERCISES IN MENSURATION.

1. Two towns, 42 miles apart, are on a map located $10\frac{1}{2}$ inches apart; what is the scale on which the map is drawn? *Ans.* $\frac{1}{4}$ in. to the mi.

2. How many feet of boards will cover the gable end of a house 34 ft. wide, the ridge being 18 ft. high? *Ans.* 306 sq. ft.

3. The rafters of a roof are 18 ft. long, and the distance between the eaves is 24 ft.; what is the height of the ridge? *Ans.* 13.41+ ft.

4. I have a triangular building lot whose sides measure 25, 35, and 40 feet respectively; if I sell it at $5 per square foot, what do I receive? *Ans.* $2165.05.

5. How many dozen boxes of perfumery, 2 in. on each side, can be packed in a rectangular box whose dimensions are respectively 1 ft. 6 in., 1 ft., and 8 in.? *Ans.* 18 doz.

6. A cubic inch of gold is hammered out to cover a *square* (100 sq. ft.); what is its thickness? *Ans.* .000069 inch.

7. The cost of a cube of metal, at $12 per cu. in., is $4116; find the cost of gilding it over at 2¢ per sq. in. *Ans.* $5.88.

8. If 50 cubic feet of air are required per person in a well-ventilated room, how many persons can safely remain in a room 60 ft. long, 40 ft. wide, and 12 ft. high? *Ans.* 576 persons.

MISCELLANEOUS EXAMPLES.

FUNDAMENTAL RULES.

1. If 256 is added to the difference of two numbers, the result will be 368; if the smaller number is 729, what is the larger number? *Ans.* 841.

2. If 441 is added to the difference of two numbers, the result will be 657; if the larger number is 540, what is the smaller number? *Ans.* 324.

3. If we subtract a number twice from 1876, the sum of the two results will be 2252; what is the subtracted number? *Ans.* 750.

4. If a number is subtracted from 2085, and then subtracted from the remainder, the result will be 335; what is the subtracted number? *Ans.* 875.

5. A number of marbles divided among 29 boys gives each boy 16 marbles, and leaves a remainder of 26 marbles; how many marbles are there? *Ans.* 490.

6. If three times a number is subtracted from 4236, the remainder will be 2316; what is the number? *Ans.* 640.

7. The product of two numbers is 45260; if one number is 365, what is the other number? *Ans.* 124.

8. If the product of two numbers is increased by 625, the result will be 81625; if one number is 324, what is the other? *Ans.* 250.

9. The product of two numbers, divided by 81, is 700; if one number is 350, what is the other? *Ans.* 162.

10. The product of three numbers is 746496, and the product of the two smaller numbers is 6912; if one of the smaller numbers is 96, what are the others? *Ans.* 72 and 108.

11. The quotient of two numbers, increased by 25, is 97; if the smaller number is 108, what is the larger? *Ans.* 7776.

12. The quotient of two numbers, diminished by 16, is 80; if the smaller number is 72, what is the larger? *Ans.* 6912.

13. The quotient of two numbers, multiplied by the smaller number, is 5120; if the smaller number is 80, what is the larger? *Ans.* 5120.

14. The quotient of two numbers, divided by the smaller number, is 3; if the smaller number is 48, what is the larger? *Ans.* 6912.

15. If two numbers are each divided by 3, the product of these results will be 1215; if one number is 81, what is the other? *Ans.* 135.

16. If two numbers are each divided by 4, the quotient of these results will be 3; if one number is 96, what is the other? *Ans.* 288.

17. If one number is multiplied by 3 and another divided by 4, the product of these results would be 1440; if one number is 24, what is the other? *Ans.* 80.

18. If one number is multiplied by 4 and the other divided by 3, the quotient of the results will be 3; if one number is 25, what is the other? *Ans.* 900.

19. At the rate of 30 problems in an hour for A, and 15 in 45 minutes for B, in what time can they together solve 100 problems? *Ans.* 2 hours.

20. If a boy that steps 28 inches takes 1000 steps in going to school, how many steps will his brother take, whose step is 35 inches? *Ans.* 800 steps.

21. If a number is used three times as a factor, the result will be 1157625; what is the number? *Ans.* 105.

22. If a number is used four times as a factor, the result will be 3111696; what is the number? *Ans.* 42.

23. Find the greatest common divisor and the least common multiple of 36, 48, 60, and 72. *Ans.* 12; 720.

24. Find the greatest common divisor and least common multiple of 847, 1331, and 1573? *Ans.* 121; 121121.

Find the greatest common divisor of

25. 1220 and 2013. *Ans.* 61.

26. 1241 and 1460. *Ans.* 73.

27. 9301 and 13231. *Ans.* 131.

28. 72491 and 103121. *Ans.* 1021.

Find the least common multiple of

29. 357 and 612. *Ans.* 4284.

30. 612 and 663. *Ans.* 7956.

31. 4141 and 6161. *Ans.* 252601.

32. 6161 and 7171. *Ans.* 437431.

33. What is the greatest length of boards I can use in fencing my garden, the sides of which are 168 feet, 182 feet, 280 feet, and 252 feet? *Ans.* 14 ft.

34. Mr. Bainbridge has a field whose sides measure 256, 292, 384, and 400 feet respectively; what is the length of the rails used to fence it if they are all of equal length and the longest that can be used? *Ans.* 4 ft.

35. A farmer makes 675 gallons of cider, and has but 12 barrels, each holding 45 gallons, to store it in; how many more such barrels does he need? *Ans.* 3 barrels.

36. Four men start at the same place to walk around a garden: A can go around in 9 minutes, B in 10 minutes, C in 12 minutes, and D in 15 minutes; in what time will they all meet at the starting-point? *Ans.* 180 minutes.

37. A, B, C, and D start from the same point, A travelling a mile in 18 minutes, B in 24 minutes, C in 30 minutes, and D in 35 minutes; what is the least whole number of miles each may travel that they may return to the starting-point at the same moment? *Ans.* A, 140; B, 105; C, 84; D, 72.

COMMON FRACTIONS.

1. Add $3\frac{1}{2}$ of $\frac{1}{7}$, $\frac{1}{14}$ of $3\frac{1}{2}$, and $\frac{1}{2}$ of $16\frac{2}{3}$ of $\frac{3}{50}$. *Ans.* 1.

2. Add $\frac{1}{10}$ of $\frac{3}{7}$ of $13\frac{2}{9}$, $\frac{1}{15}$ of $6\frac{3}{7}$ of $2\frac{2}{9}$, and $\frac{14}{15}$ of $3\frac{1}{7}$. *Ans.* $4\frac{19}{42}$.

3. Subtract $\frac{2}{3}$ of $\frac{3}{5}$ of $9\frac{1}{6}$ from $\frac{14}{45}$ of $\frac{6}{7}$ of 56. *Ans.* $11\frac{4}{15}$.

4. What is the value of $\frac{3}{4}+\frac{5}{6}+\frac{7}{8}$ minus $\frac{1}{4}+\frac{1}{6}+\frac{9}{10}$? *Ans.* $1\frac{17}{120}$.

5. Multiply $\frac{8}{9}$ of $\frac{1}{2}$ of $\frac{20}{36}$ by $24\frac{9}{15}$ of $\frac{75}{100}$. *Ans.* $4\frac{5}{9}$.

6. Multiply $\frac{21}{22}$ of $\frac{1}{20}$ of $14\frac{2}{3}$ by $\frac{5}{8}$ of $\frac{3}{10}$ of $\frac{1}{13}$ of $9\frac{3}{4}$ of 100. *Ans.* $9\frac{27}{32}$

7. Divide $\frac{4}{5}$ of $\frac{7}{12}$ of $\frac{9}{10}$ by $\frac{5}{6}$ of $\frac{15}{64}$ of $\frac{21}{20}$ of $5\frac{1}{3}$. *Ans.* $\frac{48}{125}$.

8. Divide $\frac{7}{10}+\frac{17}{20}-\frac{19}{30}$ by $\frac{2}{3}+\frac{4}{5}-\frac{5}{6}$. *Ans.* $1\frac{17}{38}$.

9. Divide $\frac{15}{16}\times\frac{17}{21}\times\frac{19}{36}$ by $\frac{38}{63}\times\frac{17}{24}\times\frac{45}{63}$. *Ans.* $1\frac{5}{16}$.

10. What part of $3\frac{3}{4}$ is $2\frac{2}{5}$? Of $7\frac{3}{5}$ is $5\frac{1}{15}$? *Ans.* $\frac{16}{25}$; $\frac{2}{3}$.

11. Divide the sum of $4\frac{7}{8}$ and $5\frac{2}{3}$ by their difference. *Ans.* $13\frac{6}{19}$.

What is the value

12. Of $\frac{24}{25}\times\frac{75}{81}\times\frac{9}{64}\times\frac{256}{289}$? *Ans.* $\frac{32}{289}$.

13. Of $\frac{49}{64}\times\frac{27}{128}\times\frac{144}{729}\times\frac{48}{343}$? *Ans.* $\frac{1}{224}$.

14. Of $\frac{729}{961} \times \frac{992}{1089} \times \frac{231}{1024} \times \frac{124}{693}$? *Ans.* $\frac{27}{968}$.

15. Of $\frac{289}{441} \times \frac{231}{340} \times \frac{700}{841} \times \frac{870}{999} \times \frac{1998}{935}$? *Ans.* $\frac{20}{29}$.

16. Of $(\frac{1}{3} + \frac{8}{15}) \times \frac{3}{11} + (\frac{4}{17} + \frac{5}{6}) \times 3$? *Ans.* $3\frac{827}{1870}$.

17. Of $(\frac{10}{11} + \frac{15}{19}) \times (\frac{21}{22} - \frac{35}{48}) \times \frac{627}{710} \times \frac{44}{49}$? *Ans.* $\frac{17}{56}$.

18. Of $\frac{25}{57} \times \frac{42}{65} \times \frac{19}{20} \div \frac{7}{13}$? *Ans.* $\frac{1}{2}$.

19. Of $7\frac{7}{10} \times 4\frac{6}{11} + (4\frac{1}{2} - 3\frac{4}{5}) - (5\frac{6}{7} + 6\frac{3}{11})$? *Ans.* $23\frac{439}{770}$.

20. Of $732\frac{1}{11} - (33\frac{1}{3} + 37\frac{1}{2}) - (6\frac{1}{4} + 6\frac{7}{12}) \times 49\frac{15}{16}$? *Ans.* $20\frac{415}{1056}$.

21. Of $(\frac{24}{5} - \frac{15}{11}) \times \frac{77}{81} \div (6\frac{2}{3} - 5\frac{3}{4})$? *Ans.* $3\frac{31}{55}$.

22. Of $(\frac{54}{63} + 3\frac{4}{7}) \times 9\frac{7}{9} \div (6\frac{2}{3} - 4\frac{1}{12})$? *Ans.* $16\frac{16}{21}$.

23. Of $(77\frac{20}{21} - 44\frac{3}{7}) \times (6\frac{5}{8} - 5\frac{3}{4}) \div (8\frac{2}{3} + 7\frac{3}{4} - 3\frac{7}{12})$? *Ans.* $2\frac{2}{7}$.

24. Of $\dfrac{\frac{4}{7} \text{ of } \frac{3}{8}}{5\frac{1}{14} - 4\frac{51}{56}} - \dfrac{\frac{5}{11} \text{ of } \frac{13}{15}}{7\frac{15}{22} - 6\frac{1}{2}}$? *Ans.* 1.

25. Of $\dfrac{6\frac{1}{4} + 5\frac{1}{2}}{6\frac{1}{4} - 5\frac{1}{2}} + \dfrac{6\frac{1}{4} \times 5\frac{1}{2}}{6\frac{1}{4} \div 5\frac{1}{2}}$? *Ans.* $45\frac{11}{12}$.

26. Of $\dfrac{1 + \frac{1}{5}}{1 - \frac{1}{5}} \div \dfrac{1 \times \frac{1}{5}}{1 \div \frac{1}{5}}$? *Ans.* $37\frac{1}{2}$.

27. Of $\dfrac{1\frac{1}{2} \times 1\frac{1}{2} \times 1\frac{1}{2} \times 1\frac{1}{2} - 1}{1\frac{1}{2} \times 1\frac{1}{2} \times 1\frac{1}{2} - 1}$? *Ans.* $1\frac{27}{38}$.

28. Of $\left(\dfrac{\frac{1}{3} + \frac{3}{10}}{6\frac{1}{3}} + \dfrac{5\frac{5}{7}}{4\frac{4}{9}}\right) \div 11 \times \dfrac{8\frac{1}{12}}{15\frac{2}{5}}$? *Ans.* $\frac{6}{25}$.

29. Of $\dfrac{1 - \frac{1}{5}}{2} \times \dfrac{2 - \frac{1}{5}}{3} \times \dfrac{3 - \frac{1}{5}}{4} \div \dfrac{4 - \frac{1}{5}}{5}$? *Ans.* $\frac{21}{95}$.

30. Of $\left(\dfrac{\frac{1}{3} + \frac{2}{5}}{3\frac{1}{5}} - \dfrac{8\frac{5}{8}}{115}\right) \div \left(3 \times \dfrac{3}{81\frac{9}{37}}\right)$? *Ans.* $1\frac{47}{120}$.

31. What must I pay for $14\frac{12}{17}$ yards of cloth, if $47\frac{3}{11}$ yards cost \$$378\frac{6}{7}$? *Ans.* \$$117\frac{6}{7}$.

32. What must a farmer pay for $17\frac{3}{4}$ tons of hay, if $5\frac{5}{6}$ tons cost \$$70\frac{7}{9}$? *Ans.* \$$215\frac{11}{30}$.

33. What must I pay for $10\frac{4}{5}$ barrels of rice, if $7\frac{3}{5}$ barrels cost \$$72\frac{5}{8}$? *Ans.* \$$103\frac{1}{2}$.

34. If \$$24\frac{3}{4}$ will buy $25\frac{2}{3}$ bushels of corn, how many dollars will buy $50\frac{5}{9}$ bushels? *Ans.* \$$48\frac{3}{4}$.

35. If $26\frac{8}{11}$ bushels of wheat are given for $6\frac{8}{15}$ tons of hay, how many bushels must be given for $46\frac{1}{5}$ tons of hay? *Ans.* 189 bu.

36. If $7\frac{3}{8}$ yards of cloth are given for $12\frac{2}{15}$ bushels of corn, how much corn must be given for $19\frac{1}{21}$ yards of cloth? *Ans.* 32 bu.

37. If at a certain time of day a pole 63 feet long casts a shadow $36\frac{3}{4}$ feet long, what is the length of a pole which casts a shadow $33\frac{3}{8}$ feet long? *Ans.* $57\frac{3}{8}$ feet.

38. A cistern has a capacity of $289\frac{1}{11}$ gallons, and has a pipe discharging into it $25\frac{1}{3}$ gallons per hour, and there is a leak through which it loses $5\frac{1}{7}$ gallons per hour; how long will it take to fill the cistern? *Ans.* $14\frac{7}{22}$ hours.

39. Mr. Williams, laying in a stock of goods, invests $\frac{1}{3}$ of his money in flour, $\frac{1}{4}$ in sugar and molasses, $\frac{1}{12}$ in tea and coffee, and the remainder, $540, in sundry other groceries; what was the whole amount invested? *Ans.* $1620.

DECIMAL FRACTIONS.

1. What common fraction equals .00096? *Ans.* $\frac{3}{3125}$.

2. Express .06125 as a common fraction. *Ans.* $\frac{49}{800}$.

3. Add $\frac{.2}{2.7}$, $11\frac{\frac{1}{2}}{5.4}$, $2\frac{7}{45}$, and .1. *Ans.* $13\frac{19}{45}$.

4. Add $\frac{.4}{5}$, $\frac{2\frac{5}{6}}{5\frac{2}{3}}$, $\frac{21}{48}$, and $\frac{2\frac{4}{9}}{1\frac{7}{18}}$. *Ans.* $2\frac{311}{400}$.

Find the value of

	Ans.
5. $.0075 \times 4.008$.	.03006.
6. $13\frac{22}{35} \div (28 + \frac{1}{5} + .7)$.	$\frac{53}{110}$.
7. $118\frac{1}{4} \div .04\frac{1}{2}$.	$2627\frac{7}{9}$.
8. $14\frac{\frac{2}{3}}{27} \div 11\frac{1.75}{13.5}$.	$1\frac{469}{1808}$.
9. $\frac{2\frac{1}{4}}{3} + \frac{3}{16} - \frac{1\frac{1}{2} \times 2\frac{1}{3}}{\frac{7}{4} \text{ of } \frac{16}{5}}$.	$\frac{5}{16}$.
10. $\frac{7.5}{37\frac{1}{2}}$ of $\frac{2.5}{2\frac{2}{3}} + 2\frac{19}{36}$.	$2\frac{53}{72}$.
11. $3\frac{6\frac{2}{3}}{11\frac{3}{7}} + \frac{5\frac{5}{9}}{28\frac{4}{7}} - .83\frac{1}{3}$.	$2\frac{17}{18}$.
12. $\frac{3.1515 + 3.08\frac{2}{3}}{12\frac{12}{25} - .005} + \frac{3.5}{.35}$.	$10\frac{1}{2}$.
13. $\frac{5}{7}$ of $.3 + .25$ of $\frac{8}{21}$.	$\frac{13}{42}$.
14. $2.400\frac{1}{3} \div 1.56\frac{1.25}{6\frac{1}{4}}$.	$1.53\frac{47}{71}$.
15. $\frac{2\frac{1}{3} + 4\frac{5}{12}}{1\frac{3}{4} \times 3\frac{1}{3}} + \frac{1}{6}$ of $.6\frac{5}{7}$.	10.

16. Multiply 1.25 of $.8\frac{3}{4}$ of $9\frac{6}{35}$ by .8 of $\frac{\frac{7}{8}}{.2}$. *Ans.* $35\frac{7}{64}$.

17. Divide £240.75 by $\frac{3}{4}$ of $\frac{24.5}{28}$ of $\frac{5}{7}$ of 1.8. *Ans.* £$285\frac{1}{3}$.

18. What number multiplied by 7.6 will give $6\frac{2}{3}$ for a product? *Ans.* $\frac{50}{57}$.

19. Divide seven millionths by twelve and a half ten-millionths. *Ans.* 5.6.

20. Add $\frac{1}{4}$, $\frac{8\frac{1}{2}}{36}$, .375, and .5, and multiply the result by $6.2\frac{1}{2}$. *Ans.* $8\frac{73}{144}$.

21. Subtract $\frac{3}{7}$ of 12 from $\frac{5}{8}$ of $\frac{8.75}{.4}$, and divide the remainder by $\frac{5}{7}$. *Ans.* $11\frac{301}{320}$.

22. Multiply the sum of .5 and $\frac{3.5}{14}$ by the difference between $\frac{8\frac{1}{2}}{17}$ and $\frac{10}{26\frac{2}{3}}$. *Ans.* $\frac{3}{32}$.

23. Multiply $\frac{62\frac{1}{2}}{1000}$ by 25 millionths, and divide the product by 125 hundred-thousandths. *Ans.* .00125.

Find the value

24. Of $.9\times\frac{18}{35}\times\frac{7}{24}\times\frac{16}{27}$. *Ans.* .08. **25.** Of $\frac{229}{1728}\times1.44$. *Ans.* $.1908\frac{1}{3}$.

26. Of $(6.05+3\frac{3}{4}-.004\frac{4}{5})\div.4$. *Ans.* 24.488.

27. Of $28\frac{.25}{.7}\div1134\frac{\frac{2}{15}}{\frac{3.5}{7.5}}$. *Ans.* $.02\frac{1}{2}$.

28. Of $\left(\frac{8-.4}{2}+\frac{16-.8}{4}\right)\times7\frac{7}{10}$. *Ans.* 58.52.

29. Of $.006006\div.024\frac{19.8}{24.2}$. *Ans.* .242.

30. Of $(.2\times.02)-(.01\times.001)\times10\frac{2}{5}$. *Ans.* .003896.

31. Of $13\frac{13}{189}\div\left(50-\frac{100}{3}+\frac{7\frac{1}{3}}{4.5}\right)$. *Ans.* $\frac{5}{7}$.

32. Of $(6.125+8.75-9.1235)\div.0125$. *Ans.* 460.12.

33. Of $\frac{7-3.004}{.2+7.3}\times\frac{5-.08}{4.8}\div1\frac{1}{5}$. *Ans.* .4551.

34. Of $(.101+.5\frac{1}{4}-.3\frac{1}{3})\times\frac{.04\frac{1}{6}}{.1\frac{2}{3}}\div.8\frac{4.2}{.6}$. *Ans.* .05225.

BILLS AND ACCOUNTS.

Let pupils make out bills and accounts from the following statements:

1. David Rowe & Co. bought of Nathaniel Rudolph 10 bbl. Yellow Sugar, 3031 lb., @ 4¢; 2 tierces Lard, 713 lb., @ 9½¢; 2 tierces Rice, 1216 lb., @ 7¢; what was the amount of the bill? *Ans.* $274.09½.

2. Required the footing of the following bill: 2 doz. Smith's Bitters, @ $7.25; 2 doz. Laudanum, 2 oz., @ $2.25; 2 doz. Brown's Syrup, @ $1.75; 2 doz. Fancy Soap, @ 67¢; 2 doz. Golden Liniment, @ $1.85. *Ans.* $27.54.

3. Mrs. Brown bought of Tyndale & Co., Jan. 1, 1895, the following articles: 1 Soup Tureen, $2.50; 2 Sauce Tureens, @ 75¢; 1 doz. Butter Plates, 50¢; 2 Glass Pitchers, @ 62½¢; 3 Oval Glass Dishes, @ 50¢; 4 Cov'd Dishes, @ 1.25; required the bill receipted. *Ans.* $12.25.

4. Mr. James Thornton of Peoria, Ill., bought of Hood, Bonbright & Co. the following articles, Oct. 14, 1894: 3 doz. Ladies' Berlin Gloves, @ $2.25; 1 doz. do. Berlin ½ Gauntlets, $3.75; 1 doz. Colored Buck do., $15; 1 doz. Ladies' Black Jouvin Kid Gloves, $16; 2 doz. Gents' Buck Driving Gauntlets, @ $16.50; 2 doz. Gents' White Kid Gloves, @ $11; make out bill for amount, deducting 20%. *Ans.* $77.20.

5. Mrs. Jane Wilson of Trenton, N. J., presented the following bill to Thomas Horne, March 1, 1895: Board for 4 weeks, @ $9; fuel and light, 4 weeks, @ $1.50; washing, 5½ doz., @ $1. Mr. Horne presented the following bill to Mrs. Wilson at the same date: Feb. 15, 15 lb. Tea, @ 45¢; Feb. 19, 10 lb. Coffee, @ 25¢; 50 lb. Granulated Sugar, @ 5¢; Feb. 26, 28 lb. Butter (½ firkin), @ 35¢, and 4½ doz. Eggs, @ 42¢. Make out both bills, receipting the smaller and crediting its amount upon the other. *Ans.* Bal. $24.06.

DENOMINATE NUMBERS.

1. How many days from Jan. 1, 1880, to Jan. 1, 1900? *Ans.* 7305.

2. If a boy averages 32 inches each step, how many steps will he take in walking 1 mile? *Ans.* 1980 steps.

3. How many acres in the form of a square can be enclosed by 160 rods of fence? *Ans.* 10 acres.

4. A lot of silver weighs 432 lb. Avoirdupois; how much does it weigh by Troy weight? *Ans.* 525 lb.

5. April 10th falls on Wednesday; on what day of the week does the following 10th of September fall? *Ans.* Tuesday.

6. How many powders, each containing 5 grs. of quinine, can be made from 1 lb. Apothecaries'? From 1 lb. Av.? *Ans.* 1152; 1400.

7. On May 20th the sun rises at Philadelphia at 4 h. 40 min. A. M., and sets at 7 h. 13 min. P. M.; how long is it between sunrise and sunset? *Ans.* 14 h. 33 min.

8. A traveller finds his watch one hour slow; in which direction has he travelled, and how far directly east or west? *Ans.* 15° east.

9. At 56 lb. per bushel what will be the weight of 84 bu. 3 pk. 6 qt. of corn? *Ans.* $4756\frac{1}{2}$ lb.

10. If the distance between two places on the same parallel is 60°, and it is 12 M. at the eastern point, what is the time at the western point? *Ans.* 8 A. M.

11. If a bushel of wheat weighs 60 lb., how many sacks holding 2 bu. each will be required to hold 9 T. 7 cwt. 20 lb.? *Ans.* 156.

12. How far apart are two places whose difference of time is 2 h. 30 min., if 1° equals 60 miles? *Ans.* 2250 miles.

13. A farmer sowed 8 bu. 2 pk. 2 qt. of wheat, and harvested from it 443 bu. 3 pk. 4 qt. of grain; how much was raised from 1 bushel? *Ans.* $51\frac{115}{137}$ bu.

14. A teacher started for Europe on July 4, 1892, and returned on the following 10th of May, 1893; how many days was she away? *Ans.* 310.

15. The area of a rectangular field is 121 A. 100 P., and one side is 40.50 chains; what is the length of the other side of the field? *Ans.* $30\frac{5}{162}$ ch.

16. What is the difference between the standard and local time of Boston, long. 71° 3′ 30″? *Ans.* 15 min. 46 sec.

17. A car contains 21,643 lb. of wheat; find the value of the load at 92¢ per bushel, 1 bushel = 60 lb.? *Ans.* $331.86 –.

18. When it is 9 o'clock P. M. Friday, standard time, in San Francisco, 122° 4′ 15″ W., what is the time in the city of London? *Ans.* 5 A. M. Saturday.

19. Franklin Park, Boston, contains 525 acres of land; what is it worth at $0.37½ a square foot? *Ans.* $8575875.

20. When it is noon at Paris it is 6 h. 49 min. 45 sec. A. M. at Philadelphia; what time will it be at Paris when it is noon at Philadelphia? *Ans.* 5 h. 10 min. 15 sec. P. M.

21. At the rate of 10½ miles an hour, how far will a ship sail from 4 h. 20 min. A. M. to 7 h. 45 min. P. M.? *Ans.* 161⅞ mi.

22. How many silver coins, each weighing 412½ gr., can be coined from a bar of silver weighing 16 lb. 8 oz. Avoir.? *Ans.* 280.

23. What is the value of a pile of wood 19.5 feet long, 8.4 feet wide, and 6.3 feet high, at $4.25 per cord? *Ans.* $34.26.

24. Two men were born on the first day of January, one in the year 1756, the other in 1822; both died 37 years after they were born, upon the first of January; required the difference of their ages. *Ans.* 1 day.

25. A train whose rate was 35 miles an hour went a certain distance in 12 h. 40 min.; how long would it take a train whose rate was 40 miles to go the same distance? *Ans.* 11 h. 5 min.

26. A grocer has a false balance by which he gives but 15 oz. to the pound; what is the real value of a barrel of sugar (250 lb.) for which he receives $28? *Ans.* $26.25.

27. A man was born the 29th of February, 1824, and died the 3d of March, 1860; how many birthdays did he see, and what was his exact age? *Ans.* 10; age, 36 yr. 4 da.

28. A coal-dealer bought 350 tons of coal, weighing 2240 lb. each, at $3.50 a ton, and sold it at $4.25 a ton, each ton weighing 2000 lb.; what was his profit? *Ans.* $441.

29. If a telegram is sent from Philadelphia, 75° 10′ W., to San Francisco, 122° 24′ 15″ W., at 9 h. 30 min. P. M., at what hour, local time, will it be received at San Francisco; at what hour standard time? *Ans.* 6 h. 21 min. 3 sec. P. M.; 6 h. 30 min. 40 sec.

30. A goldsmith bought an ingot of gold at $192 per lb., and sold it at $16 an ounce, using Avoirdupois weight both times; if the true weight of the ingot was 8 lb. Troy, how much did he gain by the fraud, gold being worth $16 an ounce? *Ans.* 421\frac{53}{175}$.

31. Two men start from different places on the equator, and travel toward each other till they meet; on comparing their watches with the time of the place of meeting, it is found that the first is 45 minutes slow and the second 1 hr. 15 min. fast; how far apart were the points at which they started? *Ans.* 2074.8 mi.

32. A balloon started from Paris with despatches for Tours, and travelled 143 miles 256 rods, which it made at the rate of 51 miles 80 rods an hour; starting at 4 A. M., when did it alight? *Ans.* 6 h. 48 min. $21\frac{3}{41}$ sec. A. M.

33. The distance from a certain toll-gate east to a tavern is $3\frac{4}{11}$ miles; from the toll-gate west to a school-house is $45\frac{10}{33}$ rods; half way between the tavern and the school-house is a creek 100 yards wide; how far from the toll-gate to the middle of the creek; also to the farther bank of the creek?

Ans. 1 mi. 195 rd. $2\frac{11}{12}$ yd.; 1 mi. 204 rd. $3\frac{5}{12}$ yd.

PRACTICAL MEASUREMENTS.

1. How many acres are there in a square lot each of whose sides is 20 chains? *Ans.* 40 A.

2. A man has a lot of land 32 rods long which contains 1 acre; required its width. *Ans.* 5 rods.

3. A rectangular field containing 8 acres is 32 rods wide; what is the length of the field? *Ans.* 40 rods.

4. A city lot containing 800 square yards has a front of 60 feet; what is its depth? *Ans.* 120 feet.

5. How many yards of fence will be needed to enclose a rectangular field 60 rods long, containing 20 acres? *Ans.* $1246\frac{2}{3}$ yd.

6. Find the area in acres of a right triangle, the length of the sides being 18 rods, 24 rods, and 30 rods. *Ans.* 1 A. 56 P.

7. A turnpike 40 feet wide was run through a township; how many acres per mile did it occupy? *Ans.* $4\frac{28}{33}$ A.

8. A farmer has a 10-acre field in the form of a right triangle; if one side measures 200 yards, what is the length of the other? *Ans.* 484 yards.

9. How many cakes of ice, 4 ft. long by 2 ft. wide, can be cut from a rectangular pond 60 ft. long by 40 ft. wide? *Ans.* 300 cakes.

10. How many square inches in the entire surface of a cube whose dimensions are 15 inches? *Ans.* 1350 sq. in.

11. Find the entire surface of a block of marble 8 ft. long, 18 in. wide, and 4 in. thick. *Ans.* $30\frac{1}{3}$ sq. ft.

12. How many flagstones, $5\frac{1}{2}$ ft. long by 3 ft. wide, will be needed to lay a sidewalk $\frac{1}{2}$ a mile long and 6 ft. wide? *Ans.* 960.

13. A courtyard, 42 ft. by 32 ft., is to be paved with flagstones measuring 6 ft. by 4 ft.; what will be the cost at $3.75 a flagstone? *Ans.* $210.

14. A farm 1 mile square is divided into square fields, each containing 40 acres; how many miles of fence will it require to enclose all the fields? *Ans.* 10 miles.

15. A man has a lot 320 ft. long and 210 ft. wide; how many square yards in a sidewalk 6 ft. wide around this lot just outside of it? *Ans.* $722\frac{2}{3}$ sq. yd.

16. Find the area of a gravel-walk 6 ft. wide just inside a fence enclosing a field 320 ft. long and 210 ft. wide. *Ans* $690\frac{2}{3}$ sq. yd.

17. A farmer wishes to enclose a lot 350 ft. long by 280 ft. wide with a tight board fence 6 feet high; how many square feet of boards will it require? *Ans.* 7560 sq. ft.

18. How many sheets of tin 6 in. by 4 in. will be required to cover a roof 60 ft. by 36 ft., no allowance being made for overlapping? *Ans.* 12960 sheets.

19. Mary's hoop was 3 ft. in diameter; how many yards would it travel in making 100 revolutions? *Ans.* 314.16 yd.

20. There is a circular park 100 rods in diameter, and within it is a circular lake 10 rods in diameter; what is the area of the park exclusive of the lake? *Ans.* 48 A. 95.46 P.

21. A cubic foot of water weighs 1000 oz.; what will a gallon (231 cu. in. = 1 gal.) of water weigh? *Ans.* $133\frac{49}{72}$ oz.

22. Find the weight of a quart of mercury, considering mercury 13 times as heavy as water. *Ans.* $434\frac{133}{288}$ oz.

23. A tank 8 ft. long and 5 ft. wide requires $21\frac{7}{9}$ sq. yd. of lead to line its sides and bottom; how deep is it? *Ans.* 6 ft.

24. Find the weight of a plank 22 ft. 6 in. long, 10 in. wide, and $2\frac{1}{2}$ in. thick, at $40\frac{1}{2}$ lb. to the cubic foot. *Ans.* $158\frac{13}{64}$ lb.

25. How many bushels (2150.42 cu. in. = 1 bu.) will a bin hold, the dimensions of which are 10 ft. long by $7\frac{1}{2}$ ft. wide by 5 ft. deep? *Ans.* 301.33 bu.

26. Mr. Wilson has in his house a cistern 8 ft. 4 in. long, 6 ft. 9 in. wide, and 4 ft. 6 in. deep; what is its capacity in gallons? *Ans.* $1893\frac{39}{77}$ gal.

27. How many Belgian blocks, averaging 6 in. × 12 in. on the surface, will be required to lay a pavement on the roadway of a street 500 yards long and 15 yards wide? *Ans.* 135,000 blocks.

28. How many bricks, 8 in. × 4 in., will be required to lay a pavement on a sidewalk 7 feet wide, extending along 4 lots, each having 18 ft. 6 in. front? *Ans.* 2331 bricks.

29. A has a circular garden and B a square one; the distance around each is 64 rods; which contains the most land, and how much? *Ans.* A's, 69.939 P.

30. Mr. Glass has a cylindrical cistern 8 ft. 6 in. in diameter and 5 ft. 9 in. deep; how many hogsheads (63 gal.) of water does it contain? *Ans.* 38.74+ hhd.

31. The house to which the above cistern belongs is 40 ft. long and 34 ft. wide, the eaves projecting 6 in. on each side, and all the water falling on the roof is conducted to the cistern; to what depth would it be filled by a shower in which $\frac{7}{8}$ of an inch of rain falls? *Ans.* 1 ft. $10\frac{1}{8}$+ in.

32. I have a yard 100 ft. square which I wish to pave, making a flagged walk 6 ft. wide around the outside, at 75¢ a square yard, and paving the rest with bricks, at \$9 ℔ M., allowing 4 bricks to the square foot; what will be the whole cost? *Ans.* \$466.78.

33. I have a bin 8 ft. 3 in. long, 3 ft. 6 in. wide, and 4 ft. 9 in. deep, filled with corn in the ear; how many bushels are there (Art. 340), and how many bushels when shelled, if 2 bushels on the ear make 1 bushel when shelled? *Ans.* $87\frac{39}{80}$ bu.; $43\frac{89}{100}$ bu.

34. A room contained 3 windows, which were curtained with brocatelle $\frac{1}{2}$ of a yard wide; 10 yards were required for each window, @ \$1.50, and the curtains were lined with silk $\frac{5}{8}$ of a yard wide, @ \$.87$\frac{1}{2}$; how many yards of silk were required, and what was the whole cost of the curtains? *Ans.* 24 yd.; \$66.

PERCENTAGE AND ITS APPLICATIONS.

1. A base-ball club won 18 games and lost 12 games; what per cent. of its games did it win? *Ans.* 60%.

2. A lady lost $750, which was 25% of what she then had; how much had she at first? *Ans.* $3750.

3. I sold a horse so that $\frac{4}{5}$ of the gain equalled $\frac{2}{15}$ of the cost; what was the gain per cent.? *Ans.* $16\frac{2}{3}$%.

4. A wheelman sold his old bicycle for $80, and lost $16\frac{2}{3}$%; what did it cost him? *Ans.* $96.

5. I bought goods amounting to $500 for $5\frac{1}{2}$% off for cash; what did I pay for them? *Ans.* $472.50.

6. On July 11, 1864, a U. S. greenback was worth but 38 cents in gold; what was the premium on gold? *Ans.* $163\frac{3}{19}$%.

7. A sold 20% of his land to B, and 25% of the remainder to C, and then had 120 acres; how much had he at first? *Ans.* 200 A.

8. I paid $225 for goods on which a discount of 10% was allowed; what was the gross price? *Ans.* $250.

9. What is the list price of an article for which I paid $60, after a discount of 20% was deducted? *Ans.* $75.

10. A piano marked $850 was sold for 25 and 10% off for cash; what was the selling price? *Ans.* $573.75.

11. What per cent. of the list price is paid by a buyer who receives a discount of 20 and 10 per cent.? *Ans.* 72%.

12. What is the list price of an article for which I pay $15.30, after a discount of 15 and 10% is deducted? *Ans.* $20.

13. What per cent. of the list price is paid by a buyer who receives a discount of 20, 10, and 5%? *Ans.* 68.4%.

14. A merchant marks an article $1.50, and sells it at a discount of 20%; what is his gain per cent. if the article cost him 95 cents? *Ans.* $26\frac{6}{19}$%.

15. An organ costing $250 was sold at an advance of 20%; if the marked price was $400, what deduction per cent. was made? *Ans.* 25%.

16. What price must be marked on cloth costing $2.50 a yard, so that a profit of 20% may be made when the cloth is sold at 20% less than the marked price? *Ans.* $3.75.

17. A man sold a piano for \$450, which was 10% less than he asked for it, and his asking price was 20% more than the piano cost him; what was its cost? *Ans.* \$416$\frac{2}{3}$.

18. A bookseller wishes to mark up his dictionaries, which he is now selling at \$8, so that he can deduct 20% and yet receive the present price; what must be the marked price? *Ans.* \$10.

19. The sum of A's and B's money is \$2800, and 25% of A's equals 33$\frac{1}{3}$% of B's; how much has each? *Ans.* A, \$1600; B, \$1200.

20. A bought stock at 25% below par, and sold it at 20% above par; how much did he invest if he gained \$1920? *Ans.* \$3200.

21. Mr. Thompson asked for flour 20% more than cost, but sold it for 90% of the price asked; what did he gain per cent.? *Ans.* 8%.

22. What must I ask for goods which cost 40 cents a yard that, after falling 33$\frac{1}{3}$%, I may gain 10% on the value? *Ans.* 66¢.

23. If I retail flour at a gain of 25%, and sell at wholesale for 4% less than at retail, what is my gain per cent. at wholesale? *Ans.* 20%.

24. B's loss at wholesale was 5%, and his retail price was 20% more than his wholesale price; what was his gain at retail? *Ans.* 14%.

25. C's retail gain is 12$\frac{1}{2}$%, and his retail price is 5% more than his wholesale price; what is his gain at wholesale? *Ans.* 7$\frac{1}{7}$%.

26. A farmer bought 20 turkeys for \$12, and lost 20% of them; how must the remainder be sold to gain 12$\frac{1}{2}$% by the transaction? *Ans.* 84$\frac{3}{8}$¢ apiece.

27. D lost 20% of a cargo of flour, and sold the remainder for a gain of 40%; did he gain or lose on the investment, and how much per cent.? *Ans.* Gained 12%.

28. A barrel of vinegar leaked away 25%; what per cent. must I gain on the remainder that I may gain 10 per cent. on the cost of the vinegar? *Ans.* 46$\frac{2}{3}$%.

29. A sold two dwellings for \$3600 each: on one he gained 20%, and on the other he lost 20%; what was his gain or loss by the transaction? *Ans.* Loss, \$300.

30. A's money is 25% more than B's money; then B's money is how many per cent. less than A's? *Ans.* 20%.

31. A lady sold her piano for $360, and cleared $\frac{1}{6}$ of this money; what per cent. would she have gained by selling it for 5% more than she received? *Ans.* 26%.

32. What is my annual rate of profit on a ten-acre wood-lot which cost $100, the yearly growth of wood averaging 1 cord to the acre, and the market price $4.25 a cord, the cutting and hauling costing 75¢ a cord? *Ans.* 35%.

33. Miss Willard sold her house and lot for $5000, receiving $\frac{12}{13}$ as much for the house as for the lot; on the lot she gained 4%, and on the house she lost 4%; what was the gain or loss? *Ans.* No gain or loss.

34. My agent bought some horses for me: he paid $250 for keeping, his commission was $375, and entire bill $15625; what was the rate? *Ans.* 2½%.

35. I sold goods on commission at 6% through a broker, who charged me 2%; my commission, after paying the brokerage, was $468; required the sum paid to my employers. *Ans.* $10998.

36. An agent buys goods on commission at 2½%, charging 1½% for the money; the agent charges his employer $224; what is the amount of goods bought? *Ans.* $5600.

37. An agent receives 4% commission and 2½% for guaranteeing payment; he remits to his principal $7480; what does the agent receive? *Ans.* $520.

38. I send my agent at Havana $2562.50 to purchase cigars; how much will he invest, deducting his commission of 2½% on his purchase? *Ans.* $2500.

39. A merchant instructs his agent in Chicago to buy pork to the amount of $3500; how much must he remit to settle the bill, the charges being $15 and the agent's commission 1½%? *Ans.* $3567.50.

40. A real-estate agent, having sold a house, paid to the owner $4387.50 as the proceeds of the sale, after deducting 2½% as his commission; for what was the property sold? *Ans.* $4500.

41. If an agent's commission is $143.20 when he sells $5728 worth of goods, how much would it be when he sells $12,575 worth? *Ans.* $314.37½.

42. A creditor receives on a debt of $5720 a dividend of 60%, of which he pays his attorney 5%; he afterward receives 20%

more, of which he pays his attorney 6%; what is the net amount he receives? *Ans.* $4335.76.

43. A gentleman left an estate to be divided equally among 5 heirs, subject to an inheritance tax of 5%, which caused a deduction of $2560 from the whole amount; what did each receive? *Ans.* $9728.

44. An agent's commission for a certain time was $364; if his sales had amounted to $1500 more, his commission would have been $401.50; what was the amount of his sales? *Ans.* $14,560.

45. A landlord allows his agent 5% on his gross rentals, and receives a net rental of $9560.80; what is the value of his property if the gross rental is 8% of the value? *Ans.* $125,800.

46. A landlord received $992.60 as the net rental of a house after the agent had paid $147.40 for repairs and charged 5% commission on the gross rental; what was the gross rental? *Ans.* $1200.

47. A man owns 80 shares of stock ($50): the company declares a 5% dividend, payable in stock; how many shares will he then own? *Ans.* 84 shares.

48. Mrs. Clark received $300, payable in stock, as her share of a 4 per cent. dividend; how many shares, at $50 each, did she then own? *Ans.* 156 shares.

49. Mr. Wallace received 10 shares and $45 in money as his share of a 5% dividend; how many shares, at $50 each, did he then own? *Ans.* 228 shares.

50. I exchanged 56 shares of bank stock ($50), at 2½% discount, for 40 shares of gas stock ($100), at 1½% premium, paying the balance in cash; how much cash did I pay? *Ans.* $1330.

51. Harold bought 30 shares of stock, $50 each, at 97%; he sold ⅓ of it at 99½%, and the remainder at 101½%; what did he gain by the transaction? *Ans.* $57.50.

52. A broker exchanged 700 shares of stock ($100), at 5% discount, for United States bonds ($100), at 5% premium, paying $70 in money; how many did he get? *Ans.* 634 bonds.

53. Mr. Fish bought a number of shares of bank stock ($50), the discount at 5% being $200; ¼ of it he sold at par and the rest at 7% advance; what was the average gain on each share? *Ans.* $5.12½.

54. Mr. Westlake bought Pennsylvania R. R. stock ($50) at $49\frac{7}{8}$ and sold it at $53\frac{1}{4}$; after paying brokerage he found he had a profit of $237.50; how many shares did he buy? *Ans.* 76.

55. Mrs. Warner has $10,000 Reading Railroad 6's, quoted at $103\frac{3}{4}$; would she increase or diminish her annual income if she should sell them and buy with the proceeds 7% State bonds at $110\frac{1}{8}$? *Ans.* Increase, $58.

56. I bought a lot 50 ft. front and 85 ft. deep, at a ground-rent of $5.40 per ft. front; what would be the cost of the property, the ground-rent being 6% of it? *Ans.* $4500.

57. Wishing to meet a note for $5000, I directed my broker to sell sufficient West Phila. Pass. Railway stock ($50) to cover the note and brokerage; if the stock was selling at $78\frac{5}{8}$, how many shares must he sell, and what is the surplus? *Ans.* 64 sh.; $24.

58. I sold 25 shares of Philadelphia National Bank ($100) at $156\frac{1}{2}$, and directed my broker to invest the proceeds in Norristown R. R. stock ($50) at 99; what is the amount of investment, after deducting brokerage? *Ans.* 39 shares; $40.37½ surplus.

59. How many shares of North Penn. R. R. ($50), at 49, must be sold, that the proceeds, invested in Pennsylvania State 6's, at $115\frac{1}{2}$, may give an income of $600, $\frac{1}{4}$% brokerage being charged on sale and purchase? *Ans.* 237 shares; $8.37½ surplus.

60. Mr. Jackson sold $15,000 Union Pacific 7's at $101\frac{1}{8}$, and invested part of the proceeds in Illinois 6's at $117\frac{1}{2}$, sufficient to produce an income of $750, and deposited the remainder (brokerage $\frac{1}{4}$%) in bank; what was his bank deposit? *Ans.* $412.50.

61. I had some California 7's, which brought me in an income of $546, but, preferring an investment nearer home, I decided to exchange them for Philadelphia 6's; if the California bonds were worth 117 and the Philadelphia 105, how much must I add to my investment to secure the same income, brokerage not considered? *Ans.* $429.

62. I made $5000 by a speculation, and, wishing to invest it permanently, I bought $2000 6% bonds at $117\frac{3}{8}$, and invested all the remainder possible in $4\frac{1}{2}$% bonds at $110\frac{1}{4}$ (denominations as low as $50); what surplus remained, brokerage $\frac{1}{4}$%, and what was the annual income? *Ans.* Sur., $50.75; Inc., $225.75.

INTEREST, DISCOUNT, ETC.

1. The amount of a certain principal for a certain time at 4% is \$819, and at 8% for the same time is \$988; required the time and the principal. *Ans.* $6\frac{1}{2}$ yr.; \$650.

2. The amount of B's money for 8 years at 6% is \$5100 more than the interest of his money for 10 years at 8%; required B's money. *Ans.* \$7500.

3. The amount of \$250 for 6 years at 10% is to be divided between C and D, so that C shall have three times as much as D; what does each receive? *Ans.* C, \$300; D, \$100.

4. A, B, and C together have \$1200, of which A has twice, and B three times, as much as C; what is the interest of each for 5 years at 6%? *Ans.* A, \$120; B, \$180; C, \$60.

5. If the interest of \$2500 for 4 years, at 10 per cent., be divided into two parts, which are as 2 to 3, it will respectively give $\frac{1}{3}$ of B's and $\frac{1}{2}$ of A's money; how much has each? *Ans.* Each \$1200.

6. The interest on $\frac{1}{2}$ of A's and $\frac{1}{3}$ of B's fortune for 5 years, at 6 per cent., is \$240; what is the fortune of each, provided $\frac{1}{2}$ of A's equals $\frac{1}{3}$ of B's? *Ans.* A, \$800; B, \$1200.

7. A's money is four times B's, and the sum of the interest received by both for 3 years, at 8 per cent., is \$600; how much money has each? *Ans.* A, \$2000; B, \$500.

8. The interest for 4 years, at 5 per cent., on the money Martin owes is \$40, and the interest for the same time and rate per cent. on the money due him is \$70; how much more has he due than he owes? *Ans.* \$150.

9. The interest on the money A paid for a farm, house, and store for 8 years, at 5 per cent., equals \$18,000; what was the cost of each, provided the farm cost three times as much as the house, and the house twice as much as the store? *Ans.* Store, \$5000.

10. A man wishes to place such a sum of money on interest at 6 per cent. that it will give an annual interest of \$360 for a poor sister; required the amount invested. *Ans.* \$6000.

11. Two-thirds of A's fortune, plus $\frac{3}{4}$ of B's, being on interest for 6 years, at 5 per cent., amounts to \$7800; what is the fortune of each, if $\frac{2}{3}$ of A's equals $\frac{3}{4}$ of B's? *Ans.* \$4500; \$4000.

12. A gives his note for $850, payable in 2 yr. 8 mo. without interest; at the end of 8 mo. he wishes to pay the note; what should the holder of the note receive, money worth 6%? *Ans.* $758.93 –.

13. A man owes $600, of which one-third is to be paid in one year and the remainder in two years; what is the present value of the note, money worth 6 per cent.? *Ans.* $545.82.

14. What is the present worth of $2400, one-fourth due in 8 mo., one-third in 1 year, and the remainder in 18 mo., money being worth 6 per cent.? *Ans.* $2249.07.

15. A man owes $1800, $\frac{1}{3}$ of which is due in 1 year, $\frac{1}{2}$ of the remainder in 2 years, and the remainder in 3 years; required the present value, money worth 6%. *Ans.* $1610.225.

16. Required the cost of a 3 mo. draft on Philadelphia, exchange at $\frac{3}{4}$% discount, which will pay a debt of $950, money worth 6%. *Ans.* $928.15.

17. A Baltimore merchant wishes to pay a debt of $1500 in Detroit by a sight draft on the First National Bank, Baltimore; if exchange on Baltimore is $\frac{1}{8}$% premium at Detroit, what must be the face of the draft? *Ans.* $1498.13 –.

18. If the Baltimore merchant in the previous problem buy, instead of a sight draft, a 90-day draft, what will the time draft cost? *Ans.* $1474.91.

19. A Boston merchant sends to a creditor in Savannah a sight draft on Boston for $1498.13; what was the debt, exchange on Boston being at a premium of $\frac{1}{8}$%? *Ans.* $1500.

20. My agent sold $6000 worth of goods on commission at 3%, and remits to me the proceeds in a draft bought at $\frac{1}{2}$% premium, which I sell at $\frac{3}{4}$% premium; what did the goods bring me in? *Ans.* $5834.47.

RATIO AND PROPORTION.

1. A piece of cloth measured with a yard-stick 1 in. too short appears to be 25 yd. long; what is its true length? *Ans.* $24\frac{11}{36}$ yd.

2. A train which runs 40 miles an hour leaves New York at 8.45 A. M.; how far will it have run at 3.15 P. M.? *Ans.* 260 miles.

3. A can do a piece of work in 6 days and B in 8 days; if A's wages are \$2.50 a day, how much should B receive a day? *Ans.* \$1.87½.

4. If 27 men build 54 rods of wall in 6 days, how many rods will 32 men build in 9 days? *Ans.* 96 rods.

5. If 5 men can mow 45 acres of grass in 6 days, in how many days will 12 men mow 90 acres? *Ans.* 5 days.

6. How many hours must 9 men work so that they may do as much work in 16 days as 12 men can do in 15 days of 8 hours each? *Ans.* 10 hours.

7. In what time will 8 masons build a wall 84 feet long, working 10 hours a day, if 12 masons build a wall 96 feet long in 8 days, working 8 hours a day? *Ans.* $8\frac{2}{5}$ days.

8. 25 men can build the walls of a house in 24 days; after they have worked 8 days, how many men must join them to finish it in 10 days? *Ans.* 15 men.

9. If a garrison of 450 men has provisions for 10 months, how many men must leave at the end of 4 months, so that the provisions remaining may last 9 months longer? *Ans.* 150 men.

10. If 300 cats kill 300 rats in 300 minutes, how many cats will kill 100 rats in 100 minutes?

11. If it costs \$1845 to pave a street 750 ft. long and 60 ft. wide, what will be the cost of paving a street 1250 ft. long and 80 ft. wide? *Ans.* \$4100.

12. If 8 men can reap a field of 40 acres in 12 days, working 8 hours a day, how long will it take 10 men to reap a field of 50 acres, working 9 hours a day? *Ans.* $10\frac{2}{3}$ days.

13. If 11 men, working 9 hours a day, can reap a field of 30 acres in 6 days, in how many days can 9 men, working 10 hours a day, reap a field 550 yards long and 420 yards wide? *Ans.* $10\frac{1}{2}$ days.

14. If 12 men, working 10 hours a day, can hoe 50 acres in 25 days, how long will it take 30 boys, working 8 hours a day, to hoe 75 acres, 8 men being equal to 12 boys? *Ans.* $28\frac{1}{8}$ days.

15. In a cotton-factory it was found that 6 men do as much as 8 boys, and 6 boys do as much as 9 girls; how many girls will be required to do as much as 54 men? *Ans.* 108 girls.

16. A can do 2 times as much as B in a day, B can do 3 times

as much as C, C can do 2 times as much as D, and D can do $\frac{1}{3}$ as much as E; in what time can E do as much as A does in 24 days? *Ans.* 96 days.

17. How much will 150 horses cost, if 10 horses are worth 24 cows, 3 cows are worth 18 sheep, 16 sheep are worth 15 pigs, and 20 pigs are worth $100? *Ans.* $10125.

18. If John can read a book of 320 pages, each page containing 32 lines and each line averaging 14 words, in 8 hours, how long will it take him to read a book of 480 pages, each page containing 35 lines and each line 16 words? *Ans.* 15 hours.

19. A, B, and C enter into partnership; A puts in $680, B $720, C $600; they gain $600 in 6 months; what does each receive? *Ans.* A, $204; B, $216; C, $180.

20. A and B agree to do a piece of work for $310; A sends 20 men for 12 days, and B sends 25 boys for 15 days; what shall each receive, if 3 men do as much as 5 boys? *Ans.* A, $160; B, $150.

21. A, B, and C entered into partnership and gained $740: A had $1200 in trade 9 mo., B $1400 in trade 8 mo., and C $1500 in trade 10 mo.; what was the gain of each? A, $216; B, $224; C, $300.

22. A borrowed $1200 for 6 mo. and $1500 for 8 mo.: at the end of 4 months he paid $2000; when, in equity, should the remainder be paid? *Ans.* 1 yr. 4 mo.

23. A, B, and C gained in partnership $3600; A's stock was $5000, B's $6000, and C's gain $1400; required C's stock and A's and B's gain. *Ans.* C, $7000; A, $1000; B, $1200.

24. Two men, A and B, gain $12\frac{1}{2}\%$ on their stock, and then 20% of A's gain equals $520, and $\frac{2}{3}$ of B's stock equals $\frac{3}{4}$ of A's; what is the stock of each? *Ans.* A, $20,800; B, $23,400.

25. A, B, and C enter into partnership with a capital of $240,000: A's stock was $65,000, B's gain $3400, and C's gain $3600; required A's gain and B's and C's stock. *Ans.* A, $2600; B, $85,000; C, $90,000.

26. A's capital was in trade 10 mo., B's 15 mo., and C's 18 mo.: A's gain was $1250, B's gain was $1500, C's gain $1350, and the whole capital invested was $6000; what was the capital of each partner? *Ans.* A, $2500; B, $2000; C, $1500.

27. A, B, and C form a partnership for carrying on a nursery: A contributes $800, B $600, and C 10 acres of land on which to

establish the nursery; their first year's profits are $1500, of which C receives $660; what are A's and B's gain, and the value of C's land per acre? *Ans.* A's, $480; B's, $360; $110 per acre.

28. An eccentric old schoolmaster made a will which read as follows: "I bequeath $4059 to my two sons in the proportion of $\frac{5}{6}$ to $\frac{7}{8}$, which amounts are respectively equal to $\frac{9}{10}$ and $\frac{11}{12}$ of the amounts I bequeath to my two daughters;" required the share of each. *Ans.* $1980; $2079; $2200; $2268.

INVOLUTION AND EVOLUTION.

1. The 4th power of a number divided by the square of the number equals 49; what is the number? *Ans.* 7.

2. The square of a number divided by $\frac{2}{3}$ of the number equals 27; what is the number? *Ans.* 18.

3. A number divided by 6 gives double the square root of the number; what is the number? *Ans.* 144.

4. The square of a number multiplied by one-half of the number equals 32; what is the number? *Ans.* 4.

5. How many rods long is a square field containing 90 acres? How many yards of fence will enclose it? *Ans.* 120 rd.; 2640 yd.

6. The surface of all the faces of a cube equals 54 sq. in.; what is the length of the side of the cube? *Ans.* 3 in.

7. How far apart are the opposite corners of a square farm that contains 250 acres? *Ans.* 282.84+ rods.

8. How much shorter distance does a man walk in going diagonally across a rectangular field, 90 feet by 120 feet, than by going from one corner around to the opposite corner? *Ans.* 60 ft.

9. What is the distance from the lower corner to the opposite upper corner of a room 20 feet long by 16 ft. wide by 10 ft. high? *Ans.* 27.49+ ft.

10. A house is 36 feet from the ground to the eaves; how long must a ladder be to reach to the eaves if its foot is placed 15 feet from the house? *Ans.* 39 ft.

11. A ladder 85 feet long stands close against a building; how far must it be drawn out at the base that the top may be lowered 10 ft.? *Ans.* 40 ft.

12. A tree was broken 45 feet from the top, and fell so that the end struck the ground 27 feet from the foot; required the original height of the tree. *Ans.* 81 feet.

13. A ladder 65 ft. long, standing with its foot in the street, will reach on one side to a window 52 ft. high, and on the other to a window 60 ft. high; what is the width of the street? *Ans.* 64 ft.

14. If a horse tied to a post in the centre of a field by a rope 39.215 yards long can graze over an acre, how long a rope will enable him to graze over $2\frac{7}{9}$ acres? *Ans.* $65.358\frac{1}{3}$ yd.

15. If a pipe 2 in. in diameter pour 40 gal. into a cistern in a given time, how much will a pipe $2\frac{1}{2}$ in. in diameter pour in in the same time? *Ans.* $62\frac{1}{2}$ gal.

16. A has a circular garden whose diameter is 15 rods, and B has a circular garden whose area is $2\frac{7}{9}$ times as great; what is the diameter of B's garden? *Ans.* 25 rods.

17. Mary has a circular garden, and Sarah a square one, and each of them contains 4 acres; how much greater is the distance around one than around the other? *Ans.* 11.514 rods.

18. A and B start from one corner of a park a mile square: A goes around the sides, and B goes straight across to the opposite corner, where they meet; how much farther did A travel than B? *Ans.* 187.452 rods.

19. A general, wishing to draw up his division into a square, found on the first trial he lacked 144 men to complete the square; he then diminished the side of the square by 2 men, and had 204 men over; how many men were there in the division? *Ans.* 7600.

20. A gentleman has a box whose edges are in the proportion of 2, 3, and 4, and its contents are 3000 cubic inches; what are the dimensions of the box? *Ans.* 10, 15, 20.

21. If a ball 7 inches in diameter weighs $13\frac{1}{3}$ lb., what is the diameter of a ball which weighs $106\frac{2}{3}$ lb., provided the materials of both are the same? *Ans.* 14 in.

22. There are three balls whose diameters are respectively 3 in., 4 in., and 5 in.; required the diameter of a ball of the same material weighing as much as the three. *Ans.* 6 in.

MENSURATION.

1. How many square yards in the entire surface of a cube whose solid contents are 216 cubic yards? *Ans.* 216 sq. yd.

2. Required the length of a hand-rail for a flight of stairs of 18 steps, each step being 7 in. high and $9\frac{1}{3}$ in. wide. *Ans.* $17\frac{1}{2}$ ft.

3. The entire surface of a square prism is 1650 sq. in.: one side of the base is 15 in.; what is the lateral surface? *Ans.* 1200 sq. in.

4. The lateral surface of a square prism is 600 sq. ft., the altitude is 15 ft.; what are the contents? *Ans.* 1500 cu. ft.

5. What is the difference between the area of a circle 12 inches in diameter and that of the inscribed square? *Ans.* 41.097+.

6. How many square inches in the surface of a frame 3 in. wide around a looking-glass 8 in. in diameter? *Ans.* 103.67+ sq. in.

7. A gentleman has a block in the form of a parallelopiped which is 48 in. long, 36 in. wide, and 24 in. high; what is the entire surface of the block? *Ans.* 52 sq. ft.

8. Two steamers sail from the same port: one sails north at the rate of 15 miles an hour, and the other east at the rate of 12 miles an hour; how far apart are they in a day? *Ans.* 461.016 mi.

9. What are the contents in liquid measure of a tin pan whose height is 5 inches, upper diameter 17 inches, and lower diameter 9 inches? *Ans.* $2\frac{74}{77}$+ gal.

10. What is the length of a tape that will wind spirally around a cylinder 52 feet long and 3 feet in circumference, provided it passes around the cylinder once every 4 feet? *Ans.* 65 feet.

11. A rectangular field containing $7\frac{1}{2}$ acres is 30 rods wide; how much will it cost to build a fence from one corner to the diagonally opposite corner at $2 a rod? *Ans.* $100.

12. Having a square yard which contains $\frac{2}{5}$ of an acre, I make a gravel-walk around it which occupies $\frac{15}{64}$ of the whole yard; what is the width of the walk? *Ans.* 8 ft. 3 in.

13. A circular flower-bed, 15 feet in diameter, is surrounded by a walk 5 feet wide; how many square feet of surface does the walk contain? *Ans.* 314.16 sq. ft.

14. A coal-dealer has a wagon which holds exactly one ton of Schuylkill gray-ash coal (36 cu. ft.); if the wagon-bed is $6\frac{1}{2}$ ft. long and 4 ft. wide, what is its depth? *Ans.* $16\frac{8}{13}$ in.

15. How many square feet of boards will it take to cover the gable ends of a house 34 ft. wide, 45 ft. high to the ridge-pole, and 30 ft. high to the eaves? *Ans.* 510 sq. ft.

16. If a crib 20 ft. long, 6 ft. wide, and 8 ft. high is filled with corn in the ear, how many bushels will there be when it is shelled if 2 bu. in the ear produce 1 bu. shelled? *Ans.* $307\frac{1}{5}$ bu.

NOTE.—In Ex. 16 notice that corn in the ear is measured by heaped bushels, Art. 340.

17. The outer edge of a walk surrounding a circular plot of ground measures 120 feet, the inner edge measures 81 feet; how many square feet in the walk? *Ans.* 623.83 – sq. ft.

18. What is the expense of sodding a plot of ground 45 yd. long and 95 ft. wide with sods 15 in. × 24 in., the sods when laid costing $1.50 per hundred? *Ans.* $76.95.

19. How much will it cost to fence a rectangular garden 20 rods long and 15 rods wide, with pickets 4 inches wide and 3 inches apart, at $9 ℔ M.? *Ans.* $17.82.

20. What will be the cost of flooring, at $33.25 per M., of a three-story house, the inside measure being 58 ft. × 34 ft., deducting 15 ft. 6 in. by 8 ft. 3 in. for the stairs? *Ans.* $183.95.

21. How much will it cost to roof a warehouse, 48 ft. × 60 ft., with slate, the height of the ridge being 10 ft. and the eaves projecting 6 inches, at $14.75 per square (100 sq. ft.)? *Ans.* $469.05.

22. A lady, having a lot of ground 40 rods square, planted 2 acres with corn, 150 square rods with vegetables, 12 rods square with flowers, and the remainder she kept to pasture a cow; how much of the lot was pasture? *Ans.* 6 A. 26 P.

23. The earth taken from 4 cellars was used in grading a lot of ground; if the cellars were 30 × 21 ft., 28 × 18 ft., 24 × 16 ft., and 32 × 24 feet respectively, and 5 ft. deep, how many loads (cu. yd.) did it take? *Ans.* $423\frac{1}{3}$ loads.

24. A yard 36 feet square has in the centre a fountain, the basin of which is 12 feet in diameter: there is a flower-bed, 4 feet wide, around three sides of the yard; what will be the expense of paving the remainder at $2.25 per square yard? *Ans.* $195.73.

25. What will be the cost of a thousand tiles in the shape of a rhombus 15 in. on a side, a line drawn from an obtuse angle per-

pendicular to the opposite side meeting it 9 in. from the acute angle, at 75¢ a square foot? *Ans.* \$937.50.

26. In a circular grass-plot whose diameter is 50 yards there is a gravel-walk, 1 yard wide, running round it 1 yard within the edge; what will be the cost of sodding the lot at 12¢ per square yard? *Ans.* \$217.90.

27. A circular room, of which the diameter is 25 ft. and height of wall 14 ft., is covered with a hemispherical dome; what is the cost of plastering the wall and dome at 25¢ per sq. ft.? *Ans.* \$520.32$\frac{3}{4}$.

28. A cast-iron garden-roller is 40 in. long, 22$\frac{1}{2}$ in. in diameter, and the iron is $\frac{3}{4}$ in. thick; required its weight, if a cubic inch of iron weighs 4$\frac{1}{2}$ ounces. *Ans.* 576.532$\frac{11}{16}$ lb.

29. If a cannon-ball, 6 in. in diameter, is melted and cast in a conical mould 6 in. in diameter at the base, what is the length of the cone? *Ans.* 12 in.

30. A railroad embankment has an average depth, for 5 miles, of 12 feet, width at top 8 yards, and at bottom 12 yards; what was the cost of carting at 15¢ per load (cu. yd.)? *Ans.* \$52800.

31. An ice-house is 40 ft. long, 30 ft. wide, and 20 ft. deep; what area of ice 6 in. thick will be required from a pond to fill it? what would it weigh at 56 lb. the cu. ft.? *Ans.* 48000 sq. ft.; 1344000 lb.

32. A company wish to excavate a canal 25 miles long, with an average width of 9 yards and an average depth of 5 ft. 6 in.; how long will it take 500 men to do the work, if each man averages 12 cubic yards a day? *Ans.* 121 days.

33. In a tin funnel, one part conical, the slant height of the conical part is 4 in., the circumference at one end 10 in., and at the other end 1 in.; the other part is cylindrical, the length being 5 in.; required the number of sq. in. of tin in it. *Ans.* 27 sq. in.

34. A has a circular field 72 rods in diameter enclosed with a wire fence, and B has a square field requiring the same length of fence to enclose it; what is the difference in the areas of the two fields? *Ans.* 873.75 P.

35. What costs the excavation for a cellar 5$\frac{1}{2}$ ft. deep under the main part of a dwelling-house, 32×24 ft., and an excavation, 1$\frac{1}{2}$ ft. wide and 2 ft. deep, for the walls of a wing 16 ft. square, at 50¢ per cu. yd.? *Ans.* \$80.72$\frac{2}{9}$.

SUPPLEMENT.

[The *Supplement* contains additional matter for advanced classes, or for schools requiring more than the standard course in arithmetic.]

615. The following methods of greatest common divisor and least common multiple are often used:

1. Find the greatest common divisor of 42, 84, and 126.

OPERATION.

$$\begin{array}{r|r}2 & 42-84-126 \\ \hline 3 & 21-42-63 \\ \hline 7 & 7-14-21 \\ \hline & 1-2-3\end{array}$$

$$2 \times 3 \times 7 = 42$$

SOLUTION.—We write the numbers one beside another, as in the margin. Dividing by 2, we see that 2 is a factor of each number; it is therefore a factor of the G. C. D. (Prin. 1). Dividing the quotients by 3, we see that 3 is a factor of each number, and therefore a factor of the G. C. D.; and in the same way we see that 7 is a factor of the G. C. D. Now, since the quotients 1, 2, and 3 are prime to each other, 2, 3, and 7 are all the common factors; hence their product, which is 42, is the G. C. D.

2. Find the least common multiple of 12, 30, and 70.

OPERATION.

$$\begin{array}{r|r}2 & 12-30-70 \\ \hline 3 & 6-15-35 \\ \hline 5 & 2-5-35 \\ \hline & 2-1-7\end{array}$$

$$2 \times 3 \times 5 \times 2 \times 7 = 420$$

SOLUTION.—Placing the numbers one beside another and dividing by 2, we see that 2 is a factor of each of them; it is therefore a factor of the L. C. M. (Prin. 3); dividing the quotients that will contain it by 3, we see that 3 is a factor of some of the numbers; it is therefore a factor of the L. C. M. Dividing the next quotients by 2, we see that 5 is a factor of some of them; hence 5 is a factor of the L. C. M., and, the quotients having no other common factor, we see that the factors of the given numbers are 2, 3, 5, 2, and 7; hence their product, which is 420, is the L. C. M.

SIMPLE INTEREST BY CANCELLATION.

616. The following Six Per Cent. Method by *Cancellation* will be found convenient and practical:

Rule for Months.—*Point off two places in the principal, divide by 2, and multiply by the number of months.*

For the Int. for 2 mo. is $\frac{1}{100}$ of the principal, and for 1 mo. it is $\frac{1}{2}$ of $\frac{1}{100}$ of the principal; hence the above rule.

Rule for Days.—*Point off three places in the principal, divide by 6, and multiply by the number of days.*

For the Int. for 2 mo., or 60 days, is $\frac{1}{100}$ of the principal, and for 6 da. it is $\frac{1}{10}$ of $\frac{1}{100}$, or $\frac{1}{1000}$ of the principal, and for 1 da. it is $\frac{1}{6}$ of $\frac{1}{1000}$ of the principal; hence the above rule.

1. What is the Int. of $2400 for 38 mo. at 7%?

SOLUTION.—We point off two places in $2400 to divide by 100, then divide by 2, and multiply by 38, then divide by 6 to find the Int. at 1%, and multiply by 7 to find the Int. at 7%; reducing by cancellation, we have $532.

OPERATION.

$$\frac{\overset{2}{\$\cancel{24}.00} \times 38 \times 7}{\cancel{2} \times \cancel{6}} = \$532$$

2. What is the Int. of $4800 for 63 da. at 6%? at 4½%?

SOLUTION.—The Int. for 1 da. is $\frac{1}{6}$ of $\frac{1}{1000}$ of the Prin.; hence we point off three places in $4800, divide by 6 to find the Int. for 1 da., and multiply by 63 to find the Int. for 63 days; cancelling and multiplying, we have $50.40.

OPERATION.

$$\frac{\overset{.800}{\$\cancel{4}.\cancel{800}} \times 63}{\cancel{6}} = \$50.40$$

NOTE.—To find the Int. at 4½%, divide by 6 and multiply by 4½, expressing the work and cancelling.

ANNUAL INTEREST.

617. Annual Interest is the simple interest of the principal and of each year's interest from the time of its accruing until settlement.

618. Annual Interest is sanctioned by some States when the note is written "with interest payable annually."

1. *Annual Interest* allows interest on the *unpaid interest* of a debt as well as upon the debt itself.

2. The neglect to collect the annual interest on a note drawn "with interest payable annually" is, in some States, regarded as a waiving of the contract requiring it.

1. What is the amount due on a note of $500, at 6%, for 3 yr. 8 mo., interest payable annually?

SOLUTION.—The interest on $500 for 1 year is $30, and for 3 yr. 3 mo. is $97.50. The first year's interest is on interest 2 yr. 3 mo., the 2d year's interest, 1 yr. 3 mo., and the 3d year's interest only 3 mo., which is equivalent to 1 year's interest on interest for 2¼ yr. + 1¼ yr. + ¼ yr., or 3¾ yr. The interest of $30 for 3¾ yr. equals $6.75; adding the interest on the principal, the interest on the interest, and the principal, we have $604.25, the amount due.

OPERATION.

2¼ yr. + 1¼ yr. + ¼ yr. = 3¾ yr.
$500 × .06 = $30, int. for 1 yr.

$30 × 3¼ = $97.50, int. on prin. for 3¼ yr.
$30 × .22½ = 6.75, int. on int. for 3¾ yr.
500 principal.
$604.25, amount.

Rule.—I. *Find the interest on the principal for the given time and rate; also find the interest on each year's interest for the time it has remained unpaid.*

II. *The sum of these interests will be the annual interest, and this, added to the principal, will be the amount due.*

WRITTEN EXERCISES.

2. Find the annual int. of \$2700 for 3 yr. at 6%. *Ans.* \$515.16.

3. Find the annual int. of \$5000 for 7 yr. at 4½%. *Ans.* \$1787.625.

4. Find the annual int. of \$780 for 3 yr. 5 mo. at 5%. *Ans.* \$141.53⅞.

5. What is the annual interest and the amount of \$874.50 for 5 yr. 6 mo. 18 da. at 4%? *Ans.* \$211.98−; \$1086.48−.

6. What is the difference between the annual interest and the simple interest of \$7500 for 5 yr. 8 mo. at 6%? *Ans.* \$360.

7. What is the amount due Jan. 31, 1895, on a note for \$2500, dated July 10, 1890, interest payable annually at 4%, if the yearly interest has been regularly paid? *Ans.* \$2556.94⅔.

COMPOUND INTEREST.

619. Compound Interest is interest on both principal and interest when the interest is not paid when due.

1. Compound interest assumes that if the borrower does not pay the interest when due, it is proper that he should pay interest for it until paid. While it seems just, it has not the sanction of law.

2. Many savings-banks allow compound interest upon balances remaining on deposit for a full interest period.

1. What is the compound interest of \$500 for 2 yr. at 4%?

Solution.—Multiplying by the rate per cent., we find the interest for 1 year to be \$20; adding this to the principal, we find the amount to be \$520, which is the principal for the 2d year. Multiplying the new principal by the rate, we find the interest for the 2d year to be \$20.80, and adding this to the 2d principal, we find the amount for 2 years to be \$540.80, from which subtract the first principal, and the remainder, \$40.80, is the compound interest. Hence the following

OPERATION.

\$500	
.04	
20.00	= Int. 1st year.
500	
520.00	= Am't 1st year.
.04	
20.80	= Int. 2d year.
520	
540.80	= Am't 2d year.
500	
40.80	= C. Int. for 2d yr.

Rule.—I. *Find the amount of the principal for the first period of time for which interest is reckoned, and make this the principal for the second period.*

II. *Find the amount of this principal for the next period, and thus continue till the end of the given time.*

III. *Subtract the given principal from the last amount, and the result will be the compound interest.*

WRITTEN EXERCISES.

2. Find the comp. int. of $375 for 3 yr. at 6%. *Ans.* $71.63.

3. Find the comp. int. of $800 for 5 yr. at 7%. *Ans.* $322.04.

4. Find the comp. int. of $643.80 for 3 yr. 6 mo. at 4%. *Ans.* $94.87.

5. What is the compound interest of $2050.50 for 2 yr. 6 mo. at 6%, payable semi-annually? *Ans.* $326.59.

6. What is the amount of $350 for 1 yr. 9 mo. at 8 per cent., payable quarterly? *Ans.* $402.04.

7. What is the amount of $600 for 10 yr. at 6%, compound interest?

SOLUTION.—In the table (page 415), under 6%, and opposite 10 yr., we find the amount of $1 to be $1.7908477; $1.7908477 × 600 = $1074.51.

8. What is the amount of $640 for 12 yr. at 7 per cent., compound interest? *Ans.* $1441.40.

9. What is the amount of $400 for 18 yr. at 4 per cent., compound interest? *Ans.* $810.33.

10. What is the difference between the simple and compound interest of $300 for 3 yr. 3 mo. 3 da. at 3%? *Ans.* $1.03.

AVERAGING ACCOUNTS.

620. Averaging an Account is the process of finding the mean or equitable time for the payment of the balance of the account.

Averaging accounts is a case of Equation of Payments when there are both debits and credits in the account.

1. In the following account, required the balance and the time when due:

DR. JOSEPH ADAMS. CR.

1894.				1894.			
July 10	To merchandise,	300	00	July 20	By Cash,	150	00
Aug. 20	" "	400	00	Sept. 16	" "	300	00
Oct. 28	" "	600	00	Oct. 20	" "	500	00

OPERATION.

Due.	Time.	Items.	Products.	Due.	Time.	Items.	Products.
1894.				1894.			
July 10	00	$300	00000	July 20	10	$150	1500
Aug. 20	41	400	16400	Sept. 16	68	300	20400
Oct. 28	110	600	66000	Oct. 20	102	500	51000
		1300	82400			950	72900
		950	72900				
		350	9500				

$9500 \div 350 = 27\frac{1}{7}$ da.

SOLUTION.—Select the date of the item first due as the focal date, and find the time the others are due after it; then multiplying each item by the corresponding time, and taking the sums of the products, we find that if paid on the 10th of July the *Dr.* items must suffer a discount of $1 for 82400 days, and the *Cr.* items must suffer a discount of $1 for 72900 days. Subtracting the two sums, we find that the *Dr.* side must suffer a discount of $1 for 9500 days more than the *Cr.* side, hence $350, the balance of the items, must suffer a discount of $\frac{1}{350}$ of 9500 days, which is $27\frac{1}{7}$ days. Hence, the balance is due 27 days after July 10th, or on August 6th. Hence we have the following

Rule.—I. *Find when each item is due, take the earliest date as the focal date, find the difference between the focal date and the remaining dates, and multiply each item by the corresponding difference.*

II. *Balance the columns of products and also the columns of items, and divide the former by the latter; the quotient added to the focal date will give the equated time.*

III. *If the two balances be on opposite sides of the account, the quotient obtained must be subtracted from the focal date.*

1. Other dates than the earliest might be selected as the focal date. If we reckon from the last date, we have the interest instead of the discount.

2. Instead of *products* we may obtain the *interest* at any per cent. on each item, and divide the balance of interest by the interest on the balance of the account for one day; the quotient will be the number of days to be added to or subtracted from the focal date.

WRITTEN EXERCISES.

2. What is the balance of the following account, and when is it due? *Ans.* Balance, $750; due June 16th.

DR. WILLIAM MONROE. CR.

1893.			1893.		
June 8	To Mdse.,	$400	July 17	By Cash,	$200
July 10	" Sundries,	600	Aug. 12	" "	400
Aug. 15	" Mdse.,	450	Sept. 16	" Sundries,	100

8. What is the balance of the following account, and when is it due? *Ans.* Balance, $655; due April 20th.

DR. CHARLES JACKSON. CR.

1892.			1892.		
Jan. 1	To Sund. on 3 mo.	$575	Mar. 20	By Cash,	$200
Jan. 18	" " 3 mo.	480	Apr. 10	" "	450
Feb. 20	" " 3 mo.	800	Apr. 30	" Merchandise,	300
Mar. 16	" Cash,	250	May 12	" Cash,	500

SETTLEMENT OF ACCOUNTS.

621. An **Account Current** is a written statement of the debit and credit items of business transactions between two parties.

622. The **Adjustment** of an account is the determining of the balance due at a specified date.

623. An account is **Settled** upon *payment* of the adjusted balance or by *carrying* it to another account.

In finding the cash balance, interest should be allowed on each item for the time between the day it is due and the day of settlement.

Rule.—I. *Find the interest on each item from the time it becomes due to the date of settlement.*

II. *Add the interest to the item if due before the date of settlement, and subtract it when the item is due after the date of settlement. The difference of the sums of the results on both sides of the account will be the cash balance.*

1. An account may be *adjusted* by averaging it, and finding the amount of the balance from the time it becomes due till the time of settlement.

2. In *averaging* an account, we find *at what date the balance* is due; in *adjusting* an account, we find *what balance* is due at a *specified date.*

WRITTEN EXERCISES.

1. Required the cash balance of the following account, Dec. 12, 1894, interest at 6 per cent. *Ans.* 302.75\frac{5}{8}$.

GEORGE PATTERSON, IN ACCOUNT WITH TAYLOR & SON.

1894.				1894.			
June 7	To Mdse. on 3 mo.	750	00	Aug. 15	By Cash,	400	00
July 12	" " on 3 mo.	350	00	Sept. 18	" "	300	00
Sept. 5	" " on 3 mo.	500	00	Nov. 30	" "	600	00

2. Required the cash balance of the following account, Aug. 8, 1891, interest at 6 per cent. *Ans.* $279.29.

WILLIAM WHITE, IN ACCOUNT WITH HENRY BLACK.

1891.				1891.			
Mar. 16	To Mdse., 2 mo.	850	00	May 28	By Cash,	400	00
Apr. 20	" " 2 mo.	750	00	July 12	" "	800	00
May 24	" " 3 mo.	375	00	July 20	" "	300	00
June 12	" " 3 mo.	500	00	Aug. 4	" "	700	00

GEOMETRICAL METHOD OF EVOLUTION.

624. The **Geometrical Method** of explaining evolution is here given for teachers who may prefer it:

SQUARE ROOT.

1. Find the square root of 1225.

OPERATION.

$$\begin{array}{rr} & 1225(30 \\ 30^2 = & 900 \quad 5 \\ 30 \times 2 = 60 & \overline{325}\ \overline{35} \\ (60+5) \times 5 = & \underline{325} \end{array}$$

FIG. 1.

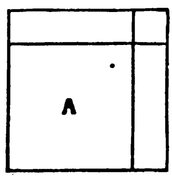


FIG. 2.

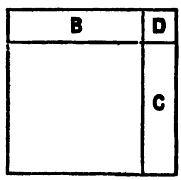
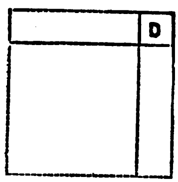


FIG. 3.



GEOMETRICAL SOL.—Let Fig. 1 represent a square which contains 1225 square units; then our object is to find the number of linear units in the edge. Since the square of a number consists of *twice as many places as the number itself, or twice as many less one*, the square root of 1225 will consist of two places, and hence will consist of tens and units.

The greatest number of tens whose square is contained in 1225 is 3 tens. Let A, Fig. 1, represent a square whose sides are 30 units; its area will be 30^2, or 900 square units. Subtracting 900 from 1225, we find remaining a surface containing 325 square units. By inspection we find this surface to consist principally of the two rectangles B and C, Fig. 2, each of which is 30 units long, and since they nearly complete the square, their area is nearly 325 units; hence if we divide 325 by their length, we will find their width. The length of both is $30 \times 2 = 60$; dividing 325 by 60, we find their width to be 5 units. Adding the length of the little corner square, Fig. 3, whose sides are 5 units, we find the entire length of the surface remaining after the removal of the square A is $60 + 5 = 65$ units, and, multiplying this by the width, we find the whole area of the remainder to be $65 \times 5 = 325$ square units. Subtracting 325 square units from the

square units left after subtracting 900 square units, nothing remains; therefore the side of the square whose area is 1225 square units is 35 units; hence the square root of 1225 is 35.

NOTES.—1. When there are three figures in the root, after removing the first two rectangles and small square we have two rectangles and a small square remaining.

2. Square root can also be explained by building up the square instead of separating it into its parts.

CUBE ROOT.

2. Find the cube root of 91125.

OPERATION.

$$\begin{array}{r|l} & 91125(40 \\ 40^3 = & 64000 \quad \underline{5} \\ & 27125 \quad 45 \\ 3 \times 40^2 = 4800 & \\ 3 \times 40 \times 5 = 600 & \\ 5^2 = \underline{25} & \\ 5425 & 27125 \end{array}$$

GEOMETRICAL SOL.—Let Fig. 1 represent the cube which contains 91125 cubic units; then our object is to find the number of linear units in its edge. The number of terms in the root, found as before, is two. The greatest number of tens whose cube is contained in the given number is 4 tens. Let A, Fig. 1, represent a cube whose sides are 40; its contents will be $40^3 = 64000$. Subtracting 64000 from 91125, we

FIG. 1.

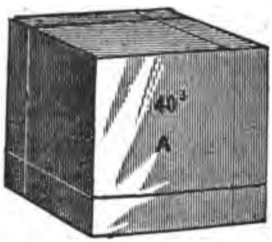


FIG. 2.

FIG. 3.

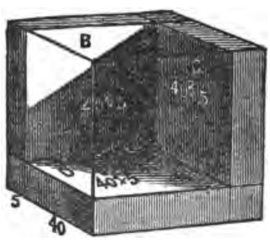
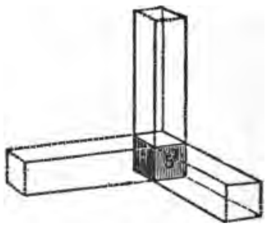


FIG. 4.



find a remainder of 27125 cubic units, which, by removing the cube A from Fig. 1, leaves a solid represented by Fig. 3.

Inspecting this solid, we perceive that the greater part of it consists

of the three rectangular slabs, B, C, and D, each of which is 40 units in length and breadth; hence, if we divide 27125 by the sum of the areas of one face of each, regarded as a base, we can ascertain their thickness. The area of a face of one slab is $40^2 = 1600$, and of the three, $3 \times 1600 = 4800$, and dividing 27125 by 4800, we have a quotient of 5; hence the thickness of the slab is 5 units.

Removing the rectangular slabs, there remain three other rectangular solids, E, F, G, as shown in Fig. 2, each of which is 40 units long and 5 units thick; hence the surface of a face of each is $40 \times 5 = 200$ square units, and of the three is $3 \times 40 \times 5 = 600$ square units.

Finally removing E, F, and G, there remains only the little corner cube H, Fig. 4, whose sides are 5 units and the surface of one of its faces $5^2 = 25$ square units. We now take the sum of the surfaces of the solids remaining after the removal of the cube A, and multiply this by the common thickness, which is 5, and we have their solid contents equal to $(4800 + 600 + 25) \times 5 = 27125$ cubic units, which, subtracted from the number of cubic units remaining after the removal of A, leaves no remainder. Hence the cube which contains 91125 cubic units is $40 + 5$, or 45 units on a side.

NOTE.—This can also be explained by building up the cube instead of separating it into its parts.

SHORT METHOD OF CUBE ROOT.

625. The abbreviation consists in obtaining the successive trial divisors by a law which enables us to use our previous work.

1. Extract the cube root of 14706125.

OPERATION.

```
                        14·706·125(245
                         8
12      t. d.           6706
 24
  16
1456          T. D.
  16                    5824
1728      t. d.         882125
 360
  25
176425 T. D.            882125
```

SOLUTION.—We find, as before, the number of figures in the root, and the first term of the root, cube, subtract, and bring down the first period.

We then find, as before, the trial divisor, 12, by taking three times the square of the first term. Dividing, we find the second term of the root to be 4. We then, as before, take three times the product of the first and second terms, and the square of the second term, and add these to the trial divisor as a *correction* to obtain the *true divisor*, 1456. We then multiply 1456 by 4, and subtract and bring down the next period.

We then, to find the next *trial divisor*, take the *square* of the *last term*, which is 16, and add it to the previous *true divisor* and the *two corrections*, which were added to the previous trial divisor, and we have 1728 as the next trial divisor.

Then, to find the *true divisor*, we add three times the product of the

last term of the root into the previous part of the root, and also the square of the last term, and have 176425 for the *true divisor*. Multiplying by 5, we have 882125.

The method is indicated in the following formula:

1. TRUE DIVISOR = TRIAL DIVISOR + PRODUCT + SQUARE.
2. TRIAL DIVISOR = SQUARE + TRUE DIVISOR + CORRECTIONS.

The method is readily explained either by the blocks or the algebraic formula.

WRITTEN EXERCISES.

Extract the cube root of

2. 12326391.	*Ans.* 231.	**5.** 633839.779.	*Ans.* 85.9.
3. 41063625.	*Ans.* 345.	**6.** 16348384872.	*Ans.* 2538.
4. 1879080904.	*Ans.* 1234.	**7.** 16503.467336.	*Ans.* 25.46.

ARITHMETICAL PROGRESSION.

626. An **Arithmetical Progression** is a series of numbers which vary by a common difference; as, 3, 5, 7, 9, etc.

627. The **Common Difference** is the difference between any two consecutive terms; thus, in the above series the common difference is 2.

628. The **Quantities** considered are five, any three of which being given, the others may be found.

QUANTITIES CONSIDERED.

1. The first term. **3.** The common difference.
2. The last term. **4.** The number of terms.
5. The sum of all the terms.

629. Given the first term, the common difference, and the number of terms, to find the last term.

1. The first term is 4, the common difference 2, and number of terms 12; required the last term.

SOLUTION.—The first term is 4, the second term equals 4 plus *once* the common difference, the third term equals 4 plus *twice* the common difference, etc.; hence the twelfth term equals the first term plus *eleven* times the common difference, which equals $4 + 2 \times 11 = 26$.

OPERATION.

$$\begin{aligned} 2d &= 4 + 2 &= 6 \\ 3d &= 4 + 2 \times 2 &= 8 \\ 4th &= 4 + 2 \times 3 &= 10 \\ \text{Hence } 12th &= 4 + 2 \times 11 &= 26 \end{aligned}$$

Rule. *The last term equals the first term, increased by the common difference multiplied by the number of terms less one.*

NOTE.—In a descending series we must subtract instead of adding.

WRITTEN EXERCISES.

2. Find the 10th term of the series 3, 6, 9, etc. *Ans.* 30.

3. Find the 20th term of the series 2, 5, 8, etc. *Ans.* 59.

4. Find the 24th term of the series 4, 8, 12, etc. *Ans.* 96.

5. Find the 22d term of the series 3, $5\frac{1}{3}$, $7\frac{2}{3}$, etc. *Ans.* 52.

6. Required the 57th term of a descending series, the 1st term being 70 and common difference $\frac{3}{4}$. *Ans.* 28.

7. A man bought 40 yards of muslin at $\frac{1}{2}$ cent for the first yard, 1 cent for the second, $1\frac{1}{2}$ for the third, and so on; what did the last yard cost? *Ans.* 20 cents.

630. Given the last term, the common difference, and the number of terms, to find the first term.

1. Required the first term, the last term being 24, the number of terms 12, and the common difference 2.

SOLUTION.—From the Rule in Case I. we have $24 = 1\text{st term} + 11$ times 2; hence we find first term $= 24 - 11 \times 2 = 2$.

OPERATION.

$$24 = 1\text{st} + 2 \times 11$$
$$1\text{st} = 24 - 2 \times 11 = 2$$

Rule.—*The first term equals the last term, diminished by the common difference multiplied by the number of terms less one.*

WRITTEN EXERCISES.

2. Required the first term, the last term being 85, common difference 5, and number of terms 16. *Ans.* 10.

3. A woman bought 40 yards of muslin at the rate of 20 cents for the last yard, $19\frac{1}{2}$ for the next to the last, and so on; what did the first yard cost? *Ans.* $\frac{1}{2}$ cent.

4. A man traveled for 12 days, traveling $1\frac{1}{2}$ miles farther each day, and on the last day he went 22 miles; how far did he travel the first day? *Ans.* $5\frac{1}{2}$ miles.

631. Given the first term, the last term, and the number of terms, to find the common difference.

1. What is the common difference if the first term is 4, the last 76, and the number of terms 25?

SOLUTION.—By Case I. we have $76 = 4 + (24$ times the *common difference*); hence, the *common difference* equals $(76 - 4) \div 24$, which equals 3.

OPERATION.

$$76 = 4 + 24 \times \text{diff.}$$
$$\text{diff.} = \frac{76 - 4}{24} = 3$$

Rule.—*The common difference equals the difference of the extremes divided by the number of terms less one.*

WRITTEN EXERCISES.

2. What is the common difference if the first term is 4, the last 400, and the number of terms 100? *Ans.* 4.

3. The amount of $100 at 6% for 20 years is $220; what is the annual interest? *Ans.* $6.

4. The youngest of 11 children is 62 and the oldest 82 years old, their ages being in arithmetical progression; what is the common difference of their ages? *Ans.* 2 years.

632. Given the first term, the last term, and the common difference, to find the number of terms.

1. What is the number of terms, if the first term is 76, the last term 4, and the common difference 3?

SOLUTION.—By Case II. we have $76 = 4 + (\text{No. of terms} - 1) \times 3$; from which we have $(\text{No. of terms} - 1) \times 3 = 76 - 4$, and $\text{No. of terms} - 1 = (76 - 4) \div 3$, or $\text{No. of terms} = (76 - 4) \div 3 + 1 = 25$.

OPERATION.

$$76 = 4 + (n - 1) \times 3$$
$$(n - 1) \times 3 = 76 - 4$$
$$n = \frac{76 - 4}{3} + 1 = 25$$

Rule.—*The number of terms equals the difference between the extremes divided by the common difference, plus one.*

WRITTEN EXERCISES.

2. What is the number of terms if the first term is 4, the last 400, and the common difference 4? *Ans.* 100.

3. In what time will $100 at 6 per cent. simple interest amount to $220? *Ans.* 20 years.

4. A laborer received 50 cents the first day, 60 cents the second, and so on till he received $2.80 a day; how many days did he work? *Ans.* 24 days.

633. Given the first term, the last term, and the number of terms, to find the sum of the series.

1. Given the first term 3, the last term 19, and the number of terms 5, to find the sum of the terms.

SOLUTION.—To derive the rule, we find by Case III. the common difference to be 4. Writing the series in its natural, and then in an inverted, order, we take the sum of the two series, and we have *twice the sum*, equal to 22 taken 5 times —that is, $(3 + 19) \times 5$; hence, the

OPERATION.

$$\text{Sum} = 3 + 7 + 11 + 15 + 19$$
$$\text{Sum} = 19 + 15 + 11 + 7 + 3$$
$$2 \times \text{Sum} = 22 + 22 + 22 + 22 + 22$$
$$2 \times \text{Sum} = 22 \times 5 = (3 + 19) \times 5$$
$$\text{Sum} = \frac{3 + 19}{2} \times 5 = 55$$

sum equals $\frac{1}{2}$ of $(3+19)\times 5$, which equals 55. Now, $(3+19)$ is the *sum of the extremes*, and 5 is the *number of terms;* hence we have the following

Rule.—*The sum of an arithmetical series equals half the sum of the extremes multiplied by the number of terms.*

WRITTEN EXERCISES.

Find the sum

2. Of 12 terms of the series 2, 4, 6, etc. *Ans.* 156.

3. Of 32 terms of the series 3, 6, 9, etc. *Ans.* 1584.

4. Of 40 terms of the series 4, 7, 10, etc. *Ans.* 2500.

5. Of 45 terms of the series 2, $5\frac{1}{4}$, $8\frac{1}{2}$, etc. *Ans.* $3307\frac{1}{2}$.

6. How far can I walk in 10 days, going 15 miles the first day, and increasing the rate 5 miles a day? *Ans.* 375 mi.

7. 100 oranges are placed in a row 2 yards apart, the first being 2 yards from a basket; how far will a boy travel, starting from the basket, to gather them singly into the basket?

Ans. 11 mi. 152 rd. 4 yd.

8. A body will fall $16\frac{1}{12}$ feet in 1 second, 3 times as far the next, 5 times as far the third, etc.; how far will it fall in a minute? *Ans.* 10 mi. $309\frac{1}{11}$ rd.

GEOMETRICAL PROGRESSION.

634. A **Geometrical Progression** is a series of numbers which vary by a common multiplier; as, 2, 6, 18, 54, etc.

635. The **Rate**, or *Ratio*, is the common multiplier; thus, in the above series, the rate is 3.

636. In an **Ascending** series the rate is greater than a unit; in a **Descending** series the rate is less than a unit.

637. The **Quantities** considered are five, any three of which being given, the others may be found.

QUANTITIES CONSIDERED.

1. The first term. **3.** The number of terms.

2. The last term. **4.** The rate.

5. The sum of the terms.

638. Given the first term, the rate, and the number of terms, to find the last term.

1. The first term equals 2, the rate 4, and the number of terms 6; required the last term.

SOLUTION.—The 2d term equals 2×4; the 3d term equals 2×4 multiplied by 4, or 2×4^2, which is the 1st term into the second power of the rate; the 4th term equals 2×4^2 multiplied by 4, or 2×4^3, which is the 1st term into the 3d power of the rate; hence, the 6th term equals the first term into the 5th power of the rate, or 2×4^5, which equals 1024. Hence

OPERATION.

$$2d = 2 \times 4$$
$$3d = 2 \times 4^2$$
$$4th = 2 \times 4^3$$
$$\text{hence } 6th = 2 \times 4^5 = 1024$$

Rule.—*The last term equals the first term multiplied by the rate raised to a power one less than the number of terms.*

NOTE.—The table of compound interest, page 415, can be derived from this case, 1 plus the rate per cent. being the *rate*.

WRITTEN EXERCISES.

2. Find the 8th term of the series 3, 6, 12. *Ans.* 384.

3. Find the 9th term of the series 4, 8, 16, etc. *Ans.* 1024.

4. Find the 10th term of the series 1, $\frac{1}{2}$, $\frac{1}{4}$, etc. *Ans.* $\frac{1}{512}$.

5. Find the 10th term of the series 1, $\frac{1}{3}$, $\frac{1}{9}$, etc. *Ans.* $\frac{1}{19683}$.

6. Find the 12th term of the series 24, 12, 6, etc. *Ans.* $\frac{3}{256}$.

7. Find the 13th term of the series, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{9}$, etc. *Ans.* $\frac{2048}{531441}$.

8. The first term of a descending series is 24 and the rate is $\frac{1}{2}$; what is the 10th term? *Ans.* $\frac{3}{64}$.

9. A merchant doubles his capital every 4 yr.; if he begins with \$1000, how much has he at the end of 20 yr.? *Ans.* \$32,000.

10. A man bought 20 cows, agreeing to pay for them all as much as the last cow would cost, at the rate of 1 cent for the first, 2 cents for the second, 4 cents for the third, etc.; what did they cost? *Ans.* \$5242.88.

639. Given the first term, the rate, and the last term or number of terms, to find the sum of the terms.

1. The first term is 2, the rate is 4, and number of terms 6; required the sum of the terms.

SOLUTION.—Writing the series expressing the sum, and then multiplying by the rate, and taking the difference of the two series, we have 3 *times* the *sum* equals $8192 - 2$, and the sum equals $\frac{1}{3}$ of $8192 - 2$, which is 2730. In this solution we observe that 8192 is the last term multiplied by the rate, and that this is diminished by the first

OPERATION.

$$\begin{array}{rl} \text{Sum} = & 2 + 8 + 32 + 128 + 512 + 2048 \\ \text{Sum} \times 4 = & \quad\; 8 + 32 + 128 + 512 + 2048 + 8192 \\ \hline 3 \times \text{the sum} = & 8192 - 2 \\ \text{Sum} = & \dfrac{8192 - 2}{3} = 2730, \ \textit{Ans.} \end{array}$$

term, and the difference divided by the rate minus one; hence we have the following

Rule.—*To find the sum, multiply the last term by the rate, subtract the first term, and divide the remainder by the rate diminished by unity.*

NOTE.—In a decreasing series we subtract the last term multiplied by the rate from the first term, and divide by 1 minus the rate.

WRITTEN EXERCISES.

Find the sum

2. Of 2, 4, 8, etc. to 8 terms. *Ans.* 510.

3. Of 1, 3, 9, etc. to 9 terms. *Ans.* 9841.

4. Of 3, 6, 12, etc. to 10 terms. *Ans.* 3069.

5. Of 16, 8, 4, etc. to 7 terms. *Ans.* $31\frac{3}{4}$.

6. Of $3\frac{3}{4}$, $\frac{3}{2}$, $\frac{3}{5}$, etc. to 6 terms. *Ans.* $6\frac{561}{2500}$.

7. A mother gave her daughter 1 cent at birth, doubling it on each succeeding birthday; how much was the daughter worth when she became 21 years of age? *Ans.* $20971.51.

8. A lady, thinking $2 a yard too much for a silk dress containing 15 yards, agreed to pay 1 cent for the first yard, 2 cents for the second, etc.; which price was the greater, and how much? *Ans.* 2d, $297.67 more.

9. A man wishing to buy a horse refused to give $250, but agreed to pay 1¢ for the first nail in his shoes, 2¢ for the 2d, 4¢ for the third, etc.; what did the horse cost, there being 32 nails in his shoes? *Ans.* $42949672.95.

INFINITE SERIES.

640. An **Infinite Series** is a series in which the number of terms is unlimited.

641. In a descending infinite series we may consider the last term to be zero; hence the above rule becomes

Rule.—*The sum of an infinite series equals the first term, divided by a unit minus the rate.*

1. What is the sum of the infinite series 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc.?

SOLUTION.—In this series the first term is 1, and the rate $\frac{1}{2}$, and the last term is regarded as zero; hence, we have the sum of the series equal to 1 divided by $1-\frac{1}{2}$, or $1 + \frac{1}{2}$, which is 2.

OPERATION.

$$\text{Sum} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2, \textit{ Ans.}$$

WRITTEN EXERCISES.

2. Sum of the infinite series $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, etc.? *Ans.* 1.

3. Sum of the infinite series $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, etc.? *Ans.* $\frac{1}{2}$.

4. Sum of the infinite series $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, etc.? *Ans.* $\frac{1}{2}$.

5. A ball falls 8 ft. to the floor and bounds back 4 ft., then falling bounds back 2 ft., and so on; how far will it move before coming to rest? *Ans.* 24 feet.

6. A hound and fox, 10 rods apart, run so that when the hound runs 10 rods the fox runs 1 rod, etc.; how far will the hound run to catch the fox? *Ans.* $11\frac{1}{9}$ rods.

THE METRIC SYSTEM.

642. THE **Metric System** of weights and measures is used by most civilized nations except the United States and Great Britain.

The old system of weights and measures in our country is irregular, difficult to learn, and inconvenient to apply. The same is true with the old systems of all nations. Originating by chance, rather than by science, they lacked the simplicity of law, and were, therefore, irregular and chaotic.

In 1795, France adopted a system of weights and measures, called the Metric System, based upon the decimal method of notation, all the divisions and multiples being by 10. It was regarded as so great an improvement upon the old methods that it has since been introduced into Spain, Belgium, Portugal, Switzerland, Holland, Italy, Germany, Austria, Sweden, Denmark, Greece, Mexico, Brazil, and by most of the South American States, and in the most of these countries its use is compulsory. In 1864 the British Parliament passed an act permitting its use throughout the empire whenever parties should agree to use it.

The introduction of the Metric System into this country had been long recommended by scientific men and by such statesmen as Madison, Jefferson, John Quincy Adams, etc. In 1866, through the influence of Charles Sumner, Congress authorized its use in the United States, and provided for its introduction into the post-offices for the weighing of letters and papers. To facilitate its adoption, a convenient standard of comparison was furnished, by making the new five-cent piece five grams in weight and one-fiftieth of a meter, or two centimeters, in diameter.

The *advantages* of the Metric System are numerous and important:

1. It is easily learned; a school-boy can learn it in a single afternoon.

2. It is easily applied, all the operations being the same as in simple numbers.

3. It does away with addition, subtraction, multiplication, division, and reduction of compound numbers.

4. It will facilitate commerce, giving the nations a universal system of weights and measures.

643. The **Metric System** of weights and measures is based upon the decimal system of notation.

644. In this system we first establish the unit of each measure, and then derive the other denominations by taking decimal multiples and divisions of the unit.

645. Names.—We first name the unit of any measure, and then derive the other denominations by adding prefixes to the unit name.

646. The *higher denominations* are expressed by prefixing to the name of the unit

Deka,	**Hekto,**	**Kilo,**	**Myria.**
10	100	1000	10,000

The *lower denominations* are expressed by prefixing to the name of unit

Deci,	**Centi,**	**Milli.**
$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

647. Units.—The following are the different units, with their English pronunciation:

Measure.	Unit.	Pronunciation.	Measure.	Unit.	Pronunciation.
LENGTH,	**Meter**	(meter).	CAPACITY,	**Liter**	(leeter).
SURFACE,	**Are**	(air).	WEIGHT,	**Gram**	(gram).
VOLUME,	**Stere**	(stair).	VALUE,	**Dollar.**	

1. Many of the denominations given in the system are not generally used. Those in common use are printed in bold-faced type.

2. The fractional parts of the standard units are indicated by abbreviations beginning with a *small letter;* the multiples of the unit by those beginning with a *capital.*

MEASURES OF LENGTH.

648. The **Meter** is the *unit of length.* It is the ten-millionth part of the distance from the equator to the poles, and equals 39.37 inches, or 3.28 feet.

TABLE.

10 **Millimeters** (mm)	= 1 **Centimeter**, cm	= .3937079 in.
10 Centimeters	= 1 Decimeter, dm	= 3.937079 in.
10 Decimeters	= 1 **Meter**, m	= 39.37079 in.
10 Meters	= 1 Dekameter, Dm	= 32.80899 ft.
10 Dekameters	= 1 Hektometer, Hm	= 19.88423 rd.
10 Hektometers	= 1 **Kilometer**, Km	= .621382 mi.
10 Kilometers	= 1 Myriameter, Mm	= 6.21382 mi.

1. The *meter* is very nearly 3 feet 3 inches and 3 eighths of an inch in length, which may be easily remembered as the *rule of three threes*.

2. Cloth, etc. are measured by the *meter;* very small distances, by the *millimeter;* great distances, by the *kilometer*.

3. The 5-cent piece of 1866 is very nearly $\frac{1}{50}$ of a *meter* in diameter; hence, its diameter is about $\frac{1}{5}$ of a *decimeter*, or 2 *centimeters*. It was ordered to be $\frac{1}{50}$ of a meter in diameter, but owing to the composition of the alloy it was necessary to make its diameter a little greater; 48.6 nickel 5-cent pieces laid side by side measure one meter.

4. A *decimeter* is about 4 inches; a *kilometer*, about 200 rods, or $\frac{5}{8}$ of a mile; a *millimeter*, about $\frac{1}{25}$ of an inch. The *inch* is about $2\frac{1}{2}$ centimeters; the *foot*, 3 decimeters; the *rod*, 5 meters; the *mile*, 1600 meters or 16 hektometers.

MEASURES OF SURFACE.

649. The **Are** is the *unit of surface* used to measure land. The *are* is a *square dekameter*. It equals 119.6 sq. yd., or 0.0247 acre.

TABLE.

10 Milliares (ma)	= 1 **Centiare**, ca	= 1.196 sq. yd.
10 Centiares	= 1 Deciare, da	= 11.960 sq yd.
10 Deciares	= 1 **Are**, A	= 119.6034 sq. yd.
10 Ares	= 1 Dekare, Da	= .24714 A.
10 Dekares	= 1 **Hektare**, Ha	= 2.4714 A.
10 Hektares	= 1 Kilare, Ka	= 24.714 A.
10 Kilares	= 1 Myriare, Ma.	= 247.14 A.

1. The *are, centiare*, and *hektare* are the denominations principally used, as these are exact squares. The *centiare* is a square whose side is 1 meter; the *hektare* is a square whose side is 100 meters.

The *are* = 100 square meters. The *centiare* = 1 square meter.

The *hektare* = 10,000 square meters.

2. The *deciare* is not a *square*—it is merely the tenth of an are; the *dekare* is not a square—it is merely ten ares.

3. A *hektare* equals nearly $2\frac{1}{2}$ acres; a *centiare* equals nearly $1\frac{1}{5}$ sq. yd. A *acre* is very nearly 40 ares.

MEASURES OF OTHER SURFACES.

650. All surfaces besides land are measured by the *square meter, square decimeter,* etc. The measures are shown by the following table:

TABLE.

100 Sq. millimeters ($^{sq\ mm}$)	= 1 **Sq. centimeter,** $^{sq\ cm}$	= .155 + sq. in.
100 Sq. centimeters	= 1 Sq. decimeter, $^{sq\ dm}$	= 15.5 + sq. in.
100 Sq. decimeters	= 1 **Sq. meter,** $^{sq\ m}$	= 1.196 + sq. yd.

The sq. kilometer = 247.114 A. = .3861 sq. mi. is also used.

MEASURES OF VOLUMES.

651. The units of volume are the *cubic meter* and its divisions. The measures are shown by the following table:

TABLE.

1000 Cubic millimeters ($^{cu\ mm}$)	= 1 **Cubic centimeter,** $^{cu\ cm}$.
1000 Cubic centimeters	= 1 Cubic decimeter, $^{cu\ dm}$.
1000 Cubic decimeters	= 1 **Cubic meter,** $^{cu\ m}$.

In measuring wood the cubic meter is called a *stere.* 3.6 *steres,* or 36 *decisteres,* very nearly equal the common *cord.*

MEASURES OF CAPACITY.

652. The **Liter** is the *unit of capacity.* It equals a *cubic decimeter;* that is, a cubic vessel whose edge is one-tenth of a meter.

653. This measure is used for measuring liquids and dry substances. The *liter* is a cylinder, and holds 2.1135 pints wine measure, or 1.816 pints dry measure.

TABLE.

10 Milliliters (^{ml})	= 1 **Centiliter,** ^{cl}	= .61027 cu. in.
10 Centiliters	= 1 Deciliter, ^{dl}	= 6.1027 cu. in., or .845 gi.
10 Deciliters	= 1 **Liter,** ^{l}	= .908 qt., or 1.0567 qt.
10 Liters	= 1 Dekaliter, ^{Dl}	= 9.08 qt., or 10.567 qt.
10 Dekaliters	= 1 **Hektoliter,** ^{Hl}	= 2.8375 bu., or 26.418 gal.
10 Hektoliters	= 1 Kiloliter, ^{Kl}	= 28.375 bu., or 264.18 gal.

1. The *liter* is principally used in measuring *liquids,* and the *hektoliter* in measuring grains, etc.

2. The *liter* equals nearly $1\frac{1}{16}$ liquid quarts, or $\frac{9}{10}$ of a dry quart, or nearly $\frac{1}{35}$ of a bushel measure.

3. The *hektoliter* is about $2\frac{5}{6}$ bushels, or $\frac{5}{6}$ of a barrel. 4 *liters* are a little more than a *gallon;* 35 *liters*, very nearly a *bushel.*

MEASURES OF WEIGHT.

654. The **Gram** is the *unit of weight.* It is the weight of a cubic centimeter of distilled water at the temperature of melting ice. The *gram* equals 15.432 Troy grains.

TABLE.

10 **Milligrams** (mg)	= 1 **Centigram**, cg	= .15432+ gr.
10 Centigrams	= 1 Decigram, dg	= 1.54324+ gr.
10 Decigrams	= 1 **Gram**, g	= 15.43248+ gr.
10 Grams	= 1 Dekagram, Dg	= .35273 + oz.
10 Dekagrams	= 1 Hektogram, Hg	= 3.52739+ oz.
10 Hektograms	= 1 **Kilogram**, Kg	= 2.20462+ lb.
10 Kilograms	= 1 Myriagram, Mg	= 22.04621+ lb.

1. The *gram* is used in weighing letters and mixing and compounding medicines, and in weighing all very light articles. The new 5-cent coin (dated 1866) weighs 5 grams.

2. The *kilogram* is the ordinary unit of weight, and is generally abbreviated into *kilo.* It equals about $2\frac{1}{5}$ pounds avoirdupois. Meat, sugar, etc. are bought and sold by the *kilogram.*

3. In weighing heavy articles two other weights—the *quintal* (100 kilograms) and the *tonneau* (1000 kilograms)—are used. The *tonneau* is between our *short ton* and *long ton.*

4. The *avoirdupois ounce* is about 28 *grams;* the *pound* is a little less than $\frac{1}{2}$ a *kilo.*

WRITTEN EXERCISES.

1. If a letter weighs 2.5 grams, how many such letters will it take to weigh a kilogram? *Ans.* 400 letters.

2. A lady bought 11.5 meters of silk for a dress, at the rate of $4.75 a meter; what did it cost her? *Ans.* $54.625.

3. My butcher's bill one month was 87.5 kilograms of beef, at $18\frac{3}{4}$ cents a kilo; what was the bill? *Ans.* 16.40\frac{5}{8}$.

4. How much must I pay for 56.25 liters of coal oil, at the rate of $18\frac{3}{4}$ cents a liter? *Ans.* $10.546+.

5. A kilogram weighs 2.2046 lb.; what is the weight of 56½ tonneaux? *Ans.* 124559.9 lb.

6. A bought 2500 ares of land, at \$4.50 an are, and sold it for \$525 a hektare; what was the gain? *Ans.* \$1875.

7. If 15 steres of wood cost \$22.50, what must I pay for 24.5 steres at the same rate? *Ans.* \$36.75.

8. If a kilogram of sugar is worth 21¾ cents, how many kilos can I buy for \$100? *Ans.* 459.77 + kilos.

9. The height of a pole is 68.325^{m}; how long would it take a worm to climb to its top, at the rate of 15^{m} a day? *Ans.* 4.555 da.

10. A kilometer is about ⅝ of a mile; how many kilometers from Lancaster to Philadelphia, 70 miles? *Ans.* 112^{Km}.

11. How much must I pay for $23\frac{3}{4}^{m}$ of silk, at 8 francs 25 centimes a meter? *Ans.* 195.94 – fr.

12. What cost 3 kilares, 7 hektares, 6 deciares of land, at \$275.25 a hektare? *Ans.* \$10185.90.

13. It is about 100 miles from Philadelphia to New York; how many kilometers is it? *Ans.* 160^{Km}.

14. How much will it cost to excavate $12\frac{4}{5}^{cu\ m}$ of earth, at \$37.25 a cubic meter? *Ans.* \$476.80.

15. What is the width of the Atlantic in kilometers, the width being about 3000 miles? *Ans.* 4800^{Km}.

16. What must I pay for 25^{s}, 2^{ds}, and 5^{cs} of wood, at the rate of \$2.65 a stere? *Ans.* \$66.91¼.

17. How long will it take a man to walk from Philadelphia to New York, at the rate of 8^{Km} an hour? *Ans.* 20 hours.

18. Two vessels are 432^{Km} apart, and sail toward each other, each at the rate of 18^{Km} an hour; in how many hours will they be together? *Ans.* 12 hours.

19. A block 3.5^{m} long, $.75^{m}$ wide, and $.8^{m}$ thick cost \$12; what would a cubic meter of marble cost at the same rate? *Ans.* \$5.71 +.

20. A man bought 7000^{g} of jewels, at 40 francs a gram, and sold them at \$15 a pennyweight; how much was gained or lost? *Ans.* \$13475.

PROBLEMS ON IMPORTS.

1. An importer bought 428.5^m of silk in France, at 18 francs a meter, sent it to the United States, paying 25 cents a meter shipping and duty, and sold it for $5.25 a meter; what was his gain? *Ans.* $653.89.

2. A man bought a valuable gem in France which weighed 325.75^g, @ 10.25 francs; the duty on it was $6.25; how must he sell it a gram to clear $150? *Ans.* $2.46.

3. I bought 125.75^l of wine in France, at 45.25 francs a liter, paid $1.25 a liter duty and freight, and sold it at $12.50 a liter; how much did I gain? *Ans.* $316.48.

4. An importer bought 625.5^l of French brandy, at 7.55 francs a liter, paid 15 cents a liter duty and freight, and sold it in New York at $1.65 a liter; how much did he gain? *Ans.* $26.80.

5. A man bought 200^m of cloth in France, at 16.25 francs a meter; he paid 12½ cents a yard duty and freight, and sold it in Boston at $4.62½ a yard; what was the gain? *Ans.* $357.

6. An importer bought 480^g of jewels, at 12.25 francs a gram, paid $5.25 an ounce shipment and duty, and sold them in Philadelphia at $102.75 an ounce; what was the gain? *Ans.* $369.78.

7. A wine-merchant bought 180^l of brandy in Havre, at 32½ decimes a liter; he paid 2½ decimes a liter shipment and $2.25 a gallon duty, and sold it in New York at $6.75 a gallon; what was his gain? *Ans.* $92.40.

NOTES ON TABLES OF WEIGHTS AND MEASURES.

655. The following notes on the Tables of Weights and Measures will be found of interest:

TROY WEIGHT.

I. Name.—The term *Troy* is said to be derived from *Troyes*, the name of a town in France where this weight was first used in Europe. It was brought from Cairo in Egypt during the Crusades of the 12th century.

II. Terms.—The term *pound* is from the Latin *pendo*, to *bend* or *weigh*. The term *ounce* is from the Latin *uncia*, a *twelfth part*, the ounce being one-twelfth part of a pound. The *pennyweight* was the weight of the old English silver penny. The term *grain* originated in the custom of using a number of grains of wheat for the weight of a penny. These grains

were taken from the middle of the ear and well dried, thirty-two at first, and afterward twenty-four, being used to make a pennyweight.

III. SYMBOLS.—The symbol *oz.* is thought to be from the Spanish word *onza*, signifying *ounce*, though Webster derives it from the use of the termination ʒ to express abbreviation, which was afterward changed to *z*; *lb.* is from *libra*, the Latin for pound. *Pwt.* is a combination of *p.* for *penny* and *wt.* for *weight.*

APOTHECARIES' WEIGHT.

I. NAME.—The name arises from the weight being used by *apothecaries.*

II. TERMS.—The term *scruple* is from the Latin *scrupulus*, a *little stone.* The term *dram* is from the Greek *drachma, a piece of money.*

III. SYMBOLS.—The symbols have been supposed to be modifications of the figure 3, suggested by there being 3 scruples in a dram. Another supposition is that they are from inscriptions upon the ancient monuments of Egypt.

IV. Physicians use the Roman notation in writing prescriptions, using the small letters, preceded by the symbols, and writing *j* for *i* when it terminates a number. Thus, 12 gr. is written gr. xij.; 2 scruples, ℈ij. ℞ is an abbreviation for *recipe, take; ā* or *āā* (from the Greek ἀνά) means, *of each; ss.* for *semis*, or *half*, as, ℈ivss. means $4\frac{1}{2}$ scruples; *P.* for *particula*, or *little part; P. æq.* for *equal parts; q. p., quantum placet, as much as you please.*

AVOIRDUPOIS WEIGHT.

I. NAME.—The term *Avoirdupois* is probably from the French *avoir du poids, to have weight.*

II. TERMS.—The term *ton* is from the Saxon *tunne*, a *cask.* The origin of the other terms has already been given. The symbol *cwt.* is from *centum, hundred,* and *weight.* The term *dram* has been used for $\frac{1}{16}$ of an ounce, but is obsolete, fractions of an ounce being used in business transactions.

IV. In Great Britain 28 lb. equal 1 qr., 112 lb. equal 1 cwt., and 2240 lb. equal 1 ton. These are called the *long hundred* and *long ton;* they were formerly used in this country, but are now used only at the custom-houses in invoices of English goods, in ocean freights, in the wholesale iron and plate trade, and in wholesaling and freighting coal from the coal-mines of Pennsylvania.

V. The following denominations are frequently used:

6}	lb. of	petroleum	make	1 gallon.	100	lb. of	raisins	make	1 cask.
25	"	powder	"	1 barrel.	196	"	flour	"	1 barrel.
56	"	butter	"	1 firkin.	200	"	pork, beef, or fish,		1 barrel.
84	"	"	"	1 tub.	240	"	lime	"	1 cask.
100	"	grain or flour	"	1 cental.	280	"	salt at N. Y. S. works	make	1 barrel.
100	"	dry fish	"	1 quintal.					
100	"	nails	"	1 keg.	600	"	rice	make	1 barrel.

LONG MEASURE.

I. TERMS.—The term *inch* is from *uncia*, a *twelfth; foot* is from the human foot; *yard* was a *rod* or *shoot; rod* is from a *measuring-stick* or *rod; furlong*, which has now become obsolete, is from *fur, furrow*, and *lang, long*, the *length of a furrow; mile* is from *mille passuum*, 1000 paces; *span* is the space measured from the end of the thumb to the end of the little finger extended; *cubit*, from the elbow to the end of the middle finger; *fathom*, the length of the two arms extended. The ancient yard of England is said to have been determined by the length of the arm of King Henry I.

II. UNIT.—The *standard unit* of length is the *yard*, from which all other measures of length, and also those of capacity, weight, etc., are derived. It is identical with the Imperial yard of Great Britain, which, under William IV., was declared to be fixed by dividing a pendulum, which vibrates seconds in a vacuum at the level of the sea, at 62° Fahr., in the latitude of London, into 391393 equal parts, and taking 360000 of these parts for the yard. Subsequent scientific experiments have proved that such a standard is impracticable.—*Brooks's Philosophy of Arithmetic.*

III. THE MILE.—The *geographic* or *nautical mile* is equal to 1 minute of one of the great circles of the earth; hence it equals $\frac{1}{60}$ of $\frac{1}{360}$ of the circumference of the earth, which equals about 1.15 statute miles. The English mile is the same as that of the United States. 3 statute miles make a *land league;* 3 nautical miles make a *nautical league.*

IV. OTHER MEASURES.—The following denominations are frequently used: In clockmaking, 6 *points* = 1 *line* and 12 *lines* = 1 *inch;* in measuring the foot, 3 *barleycorns* or *sizes* = 1 inch; in measuring the height of horses, 4 inches = 1 *hand*, the measure being taken directly over the fore shoulder; 1 *span* = 9 inches; 1 *common cubit* = 18 inches, and 1 *sacred cubit* = 21.888 inches; 1 *pace* = 3.3 feet; a *knot* is equal to a nautical mile. Formerly 40 rods made 1 *furlong*, and 8 furlongs 1 mile, but these are seldom used.

SURVEYORS' LINEAR MEASURE.

I. NAME.—*Gunter's chain* is named after the reputed inventor, Edmund Gunter, an English mathematician, born 1581.

II. The denomination *rods* is seldom used by surveyors, distances being represented in chains and links. Since each link is $\frac{1}{100}$ of a chain, the number of links is generally expressed as a decimal; thus, 5 chains and 47 links are written 5.47 chains. Engineers generally use a chain 100 feet long.

SURFACE OR SQUARE MEASURE.

I. TERMS.—*Perch* is from the French *perche*, a pole; *acre* was primarily an open ploughed or sowed field.

II. In the parts of the Mississippi Valley settled by the French the old French *arpent* ($\frac{5}{6}$ of an acre) is still used as the unit of measure.

LIQUID OR WINE MEASURE.

I. Name.—It is called *Wine Measure* because wine was measured by it, while beer was measured by another measure.

II. Terms.—*Gill* is from Low Latin *gilla*, a *drinking-glass*; *pint* is from the Anglo-Saxon *pyndan*, *to shut in*, *to pen*, or from the Greek *pinto*, *to drink*; *quart* is from the Latin *quartus*, a *fourth*. The derivation of *gallon* is not clear; in the French a *galon* is a *grocer's box*.

III. A pint of water weighs nearly one pound; hence the old adage, "A pint's a pound, the world around."

IV. Ale, beer, and milk were formerly sold by a *gallon* of 282 cu. in., the subdivisions being *quarts* and *pints*. The measure was greater than wine measure, as beer was less costly than wine. This measure is now seldom used.

V. The Imperial gallon of Great Britain contains 277.274 cu. in., and is equal to 1.2 United States gallon.

APOTHECARIES' FLUID MEASURE.

I. Terms.—*Minim* is from the Latin *minimus*, *the least*, the minim being the smallest fluid measure used. Several of the other terms are formed by prefixing *fluid* to the terms of Apothecaries' Weight.

II. Symbols.—*Cong.* is the abbreviation of *congius*, the Latin for gallon. *O.* is the initial of *octarius*, the Latin for *one-eighth*, the pint being one-eighth of a gallon. Drops are indicated in a physician's prescription by *gtt.*, for the Latin *guttæ*.

DRY MEASURE.

I. *Peck* is supposed to be a corruption of *pack*, or to be derived from the French *picotin*, a *peck*.

II. The *Chaldron*, consisting in some places of 36 bu. and in others of 32 bu., is used in some parts of the United States for measuring coal, but is being discontinued here, as it has been in England. One-half of a peck, or 4 quarts, is called a *dry gallon*.

In buying and selling grain and seeds, etc. the Boards of Trade of the principal cities of the United States use the number of pounds given in the following table as the equivalent of a bushel:

TABLE OF POUNDS IN A BUSHEL.

Barley	48	Corn, shelled . .	56	Peas	60
Beans	60	Corn in the ear .	70	Rye	56
Buckwheat . . .	48	Malt	34	Timothy-seed . .	45
Clover-seed . . .	60	Oats	32	Wheat	60

TIME MEASURE.

Terms.—*Second* and *minute* are parts of an hour, corresponding to the parts of a degree in Circular Measure. *Hour* is derived from the Latin *hora*, originally a definite space of time fixed by natural laws; a *day*,

derived from the Saxon *daeg*, is the time of the revolution of the earth upon its axis; a *week* is a period of uncertain origin, but which has been used from time immemorial in Eastern countries; a *month*, from Saxon *monadh*, from *mona*, the moon, is the time of one revolution of the moon around the earth; a *year*, from the Saxon *gear*, is the time of the earth's revolution around the sun; a *century* comes from the Latin *centuria*, a collection of a hundred things.

THE CALENDAR.

656. The time from any day of one month to any day of another month in the same year is readily found by the following table:

TABLE

SHOWING THE NUMBER OF DAYS FROM ANY DAY OF ONE MONTH TO THE SAME DAY OF ANY OTHER MONTH IN THE SAME YEAR.

From any day of	To the same day of											
	Jan.	Feb.	Mar.	Apr.	May.	June	July.	Aug.	Sept.	Oct.	Nov.	Dec.
January . . .	365	31	59	90	120	151	181	212	243	273	304	334
February . .	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August . . .	153	184	212	243	273	304	334	365	31	61	92	122
September . .	122	153	181	212	242	273	303	334	365	30	61	91
October . . .	92	123	151	182	212	243	273	304	335	365	31	61
November . .	61	92	120	151	181	212	242	273	304	334	365	30
December . .	31	62	90	121	151	182	212	243	274	304	335	365

METHOD OF USING THE TABLE.—Suppose we wish to find the number of days from March 10th to November 16th. We find March in the first column, and November at the top, and at the intersection we find 245, to which adding 6 days we have 251, the number of days required. The table being constructed for February 28 days, the proper allowance must be made for leap year.

TERMS.—January is derived from *Janus*, the god of the year, to whom this month was sacred. February is from *februa*, the Roman festival of expiation, celebrated on the 15th of this month. January and February were added to the Roman calendar by Numa, Romulus having previously divided the year into 10 months. March is from *Mars*, the god of war and reputed father of Romulus. It was the first month of the Roman calendar. April is probably from the Latin *aperire*, to open, from the opening of the buds or the bosom of the earth in producing vegetation. May is from *Maia*, the mother of Mercury, to whom the Romans offered

sacrifices on the first day of this month. June is from *Juno*, the sister and wife of Jupiter, to whom it was sacred. July was named by Mark Antony after *Julius* Cæsar, who was born in this month. It was previously called Quintilis. August was named after *Augustus* Cæsar, who entered upon his first consulate in this month. It was formerly called Sextilis, or sixth month. September, October, November, and December are respectively named from the Latin numerals, *Septem*, *Octo*, *Novem*, and *Decem*, as, when the year began in March, they were the seventh, eighth, ninth, and tenth months, as their names indicate.

ADJUSTMENT OF THE CALENDAR.

The Rule for Leap Year given on page 177 is explained as follows:

EXPLANATION.—I. Since each year consists of 365 days 5 h. 48 min. 49.7 sec., if we reckon 365 days as one year, the time lost in the calendar in one year is 5 h. 48 min. 49.7 sec., and the time lost in four years is 23 h. 15 min. 18.8 sec.—that is, *one day*, lacking only 44 min. 41.2 sec.; hence, the first error can be corrected by adding *one* day every *four* years, making the year to consist of 366 days.

II. If every fourth year be reckoned as leap year, since we add 44 min., etc. too much, the time *gained* in the calendar in four years is 44 min. 41.2 sec., and in 100 years it will be 18 h. 37 min. 10 sec.—that is, *one* day, lacking 5 h. 22 min. 50 sec.; hence, the second error may be corrected by deducting one day from each centennial leap year, thus calling each centennial year a common year of 365 days.

III. Again, if every centennial year be reckoned as a common year, since we do not add enough, the time lost in 100 years will be 5 h. 22 min. 50 sec., and in 400 years it will be 21 h. 31 min. 20 sec.; hence, the time lost in 400 years will be *one* day lacking 2 h. 28 min. 40 sec., and this error may be rectified by making every fourth centennial year a leap year. In the same way we may make the calendar correct for any number of years.

NOTE.—The reckoning of time by the ancients was very inaccurate. The calendar was reformed by Julius Cæsar, 46 B. C., who made the year to consist of 365¼ days, adding one day every fourth year. This correction assumed the year to consist of 365 days 6 hours, which is 11 min. 10.3 sec. too much; hence his correction introduced a slight error, which in 1582 had amounted to about 14 days. In 1582, Pope Gregory corrected the error which arose from the above correction by striking out 10 days from the calendar, so as to bring the vernal equinox on March 21, as it was at the time of the Council of Nice, 325 A. D., calling the 5th of October the 15th, and ordaining that henceforth only those centennial years should be leap years which are divisible by 400. This change was soon adopted by most Catholic countries. Great Britain did not make the change till 1752, when by Act of Parliament the 3d of September was made the 14th, the error then amounting to 11 days. Russia and the other countries of the Greek Church still adhere to the Julian Calendar, their dates being about 12 days behind ours. The dates are distinguished as Old Style and New Style, marked O. S. and N. S. respectively. In the Old Style the *civil* or *legal* year commenced on the 25th of

March, while the *historical* year commenced on the 1st of January, and dates between those days were marked with the number of both years; as, Jan. 30, $164\frac{8}{9}$. The New Style made the civil year commence also on the 1st of January.

UNITED STATES MONEY.

I. NAME.—United States money is so called because it is the money of the United States. It is called *Federal Money* because it was the money of the Federal Union. It was adopted by Act of Congress, Aug. 8, 1786.

II. TERMS.—The term *dollar* is supposed to be from *Dale* or *Daleburg*, a town where it was first coined; *dime* is from the French *disme*, meaning *a tenth; cent* is from the Latin *centum*, *a hundred; mill* is from the Latin *mille*, *a thousand; eagle* is from the name of the national bird. The *cent* was proposed by Robert Morris and named by Thomas Jefferson.

IV. COINS.—The coins are of *gold*, *silver*, *nickel*, and *bronze*. The *gold coins* are the *double-eagle*, *eagle*, *half-eagle*, *quarter-eagle*, and *one dollar*. The *silver coins* are the *dollar*, *half-dollar*, *quarter-dollar*, and *dime*. The *nickel coin* is the *five-cent piece*. The *bronze coin* is the *cent*. The gold three-dollar piece, the silver twenty-cent piece, half-dime, and three-cent piece, the nickel three-cent piece and cent, the bronze two-cent piece, and the old copper cent and half-cent, although still occasionally seen in circulation, are no longer coined. The mill has never been a coin; it is merely a convenient name for the tenth part of a cent.

V. COMPOSITION.—The gold and silver coins consist of 9 parts of pure metal and 1 part alloy. The alloy of the silver coin consists of pure copper; the alloy of the gold coin consists of silver and copper, the silver not to exceed $\frac{1}{10}$ of the alloy. The nickel coins contain $\frac{1}{4}$ nickel and $\frac{3}{4}$ copper. The bronze coins consist of 95 parts copper and 5 parts tin and zinc.

VI. Gold coins are a legal tender for any amount; silver coins of the present coinage, for any amount not exceeding $5 in any one payment; bronze and nickel coins, for any amount not exceeding 25 cents in any one payment.

ENGLISH MONEY.

I. NAME.—The term *Sterling* is supposed to be derived from *Easterling*, the name given to early German traders, who came from the East to England. Their money was called Easterling Money, which was contracted into *Sterling Money*.

II. TERMS.—The term *farthing* is a modification of "four things," the old English penny being marked with a cross so deeply impressed that it could be broken into two or four pieces, called respectively *half-penny* and *four things*. The *pound* as a measure of value was derived from the pound as a measure of weight, 240 pence formerly weighing a pound. The *guinea* is so called because the gold of which it was first made came from Guinea, in Africa.

III. SYMBOLS.—The symbols £, s., d., qr. are the initials of the Latin words *libra*, *solidus*, *denarius*, and *quadrans*, signifying respectively pound, shilling, penny, and quarter.

V. COINS.—The coins are of three classes—*gold*, *silver*, and *copper*. The *gold coins* are the *sovereign* (= £1) and *half-sovereign* (= 10 s.), *guinea* (= 21 s.) and *half-guinea* (=10 s. 6 d.). The *silver coins* are the *crown* (= 5 s.), the *half-crown* (= 2 s. 6 d.), the *florin* (= 2 s.), the *shilling*, and the *six-penny*, *four-penny*, and *three-penny* pieces. The *copper coins* are the *penny*, *half-penny*, and *farthing*. The guinea and half-guinea, crown and half-crown, though still seen in circulation, are no longer coined.

LUMBERMEN'S NOTATION.

657. Lumbermen in marking lumber employ a modification of the Roman Method of Notation. The first four characters are like the Roman. The others are as follows:

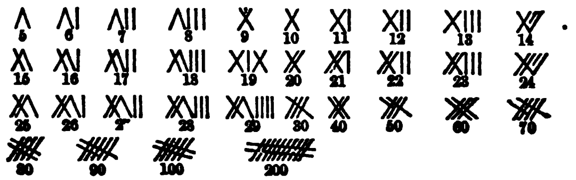


TABLE OF COMPOUND INTEREST.

Amount of $1 at Compound Interest in any number of years not exceeding 20.

Yr.	2½ per cent.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.0250 0000	1.0300 0000	1.0400 0000	1.0500 000	1.0600 000	1.0700 000
2	1.0506 2500	1.0609 0000	1.0816 0000	1.1025 000	1.1236 000	1.1449 000
3	1.0768 9062	1.0927 2700	1.1248 6400	1.1576 250	1.1910 160	1.2250 430
4	1.1038 1289	1.1255 0881	1.1698 5856	1.2155 063	1.2624 770	1.3107 960
5	1.1314 0821	1.1592 7407	1.2166 5290	1.2762 816	1.3382 256	1.4025 517
6	1.1596 9342	1.1940 5230	1.2653 1902	1.3400 956	1.4185 191	1.5007 304
7	1.1886 8575	1.2298 7387	1.3159 3178	1.4071 004	1.5036 303	1.6057 815
8	1.2184 0290	1.2667 7008	1.3685 6905	1.4774 554	1.5938 481	1.7181 862
9	1.2488 6297	1.3047 7318	1.4233 1181	1.5513 282	1.6894 790	1.8384 592
10	1.2800 8454	1.3439 1638	1.4802 4428	1.6288 946	1.7908 477	1.9671 514
11	1.3120 8666	1.3842 3387	1.5394 5406	1.7103 394	1.8982 986	2.1048 520
12	1.3448 8882	1.4257 6089	1.6010 3222	1.7958 563	2.0121 965	2.2521 916
13	1.3785 1104	1.4685 3371	1.6650 7351	1.8856 491	2.1329 283	2.4098 450
14	1.4129 7382	1.5125 8972	1.7316 7645	1.9799 316	2.2609 040	2.5785 342
15	1.4482 9817	1.5579 6742	1.8009 4351	2.0789 282	2.3965 582	2.7590 315
16	1.4845 0562	1.6047 0644	1.8729 8125	2.1828 746	2.5403 517	2.9521 638
17	1.5216 1826	1.6528 4763	1.9479 0050	2.2920 183	2.6927 728	3.1588 152
18	1.5596 5872	1.7024 3306	2.0258 1652	2.4066 192	2.8543 392	3.3799 323
19	1.5986 5019	1.7535 0605	2.1068 4918	2.5269 502	3.0255 995	3.6165 275
20	1.6386 1644	1.8061 1123	2.1911 2314	2.6532 977	3.2071 355	3.8696 845

1,124,864(

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300 x 4 = 1200 3 124 864